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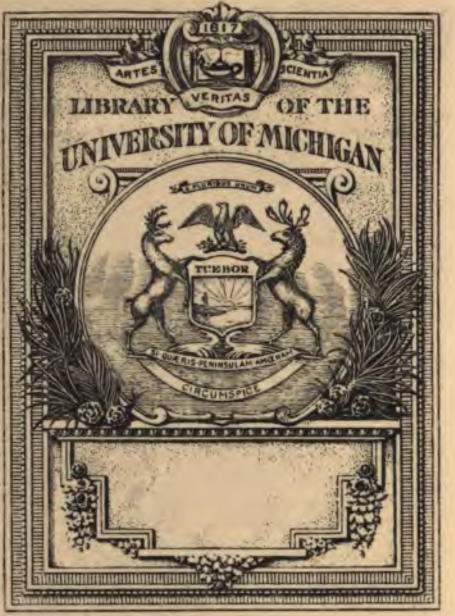

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define the Magnitude, Denſity, and mutua
ſtances of the Primary Planets, and other
things: That we may go on in Order, we
in this Fourth Book, give the Theory of th
condary Planets, which go round the Pri
ones. Altho' the Motion of every Primar
net be ſimple enough, as being (like a Proj
Motion) compounded of an uniform progr
Motion in a Right-line Tangent to its (
and another motion of deſcent by whic
attracted) it tends towards the Sun, thei
menſe diſtance from one another rendrin
effect of their mutual Action or Gravity on
other inſenſible; that of the Secondary Pl
will be quite otherwiſe. For tho' every o
'em tends moſtly to its reſpective Primary P
yet it is urg'd towards the Sun by the ſam
celerating Gravity as its Primary, when a
ſame diſtance, by a leſſer Gravity when
greater diſtance, and by a greater Gravity
at a leſs diſtance. From this different tend
to two Bodies, namely, to the Sun and to it
mary, the Motion of a Secondary Plane
comes very complex, and has ſuch Inequa
as the Aſtronomers hop'd to account for by
Hypotheſes, ſooner than the Philoſophers
explain them from Phyſical Cauſes, till Sir
Newton was ſo happy as to do it.

SECT

SECTION I.

Of the Errors which the Sun produces in the Motion of a Satellite, when its Orbit is concentric to its Primary Planet.

PROPOSITION I.

IF a Primary Planet revolving about the Sun carries with it a Satellite, this Satellite will be ſo nov'd about the Primary, as to be continually accele- ated from its Quadrature with the Sun to its next Conjunction or Oppoſition; but from the Syzygy to the Quadrature it will be retarded: Therefore the Satel- ite will move ſwifter near the Syzygies, and ſlower ear the Quadratures.

Becauſe it has been ſhewn in *Book* 1. that in heſe three Bodies, *viz.* The Sun, a Primary 'lanet, and its Satellite, the attractions of any wo on the third were to one another recipro- ally as the Squares of the Diſtances from it; it s manifeſt that the Caſe of this Propoſition is he ſame with that of *Prop.* 60. *Book* 1. There- ore if, as here [in *Fig.* 1.] *S* denotes the Sun, *T* Primary Planet revolving in the Orb *ETE*, *L* Satellite deſcribing the Orbit *BCGD* round the 'rimary, in which the Points *B* and *G* denote e Syzygies with the Sun, *C* and *D* the Quadra- ures: If *SK*, the middle Diſtance of a Satellite om the Sun, be put for the accelerating At- action of that Satellite from the Sun in that iddle diſtance, and *L* be ſuppos'd any Place of e Satellite in its Orbit, upon *LS* produc'd ke *SA*, which muſt be to *SK* in a duplicate atio of *SK* to *SL*; and this *SA* will denote

the attraction of the Satellite *L* when it
wards *S.* Join *T L*, and draw parallel to i
meeting *S T* in *M.* It is certain that the
ction *S A* may be reſolv'd into the attra
A M and *M S*, with the directions of theſe
of which that which is expreſs'd by *M S*, is re
to the atraction *M N*, taking away the attr
N S, common to the Satellite and its Primar
net; as it has been amply ſhewn in the ſaid 6
of *B.* 1. The Satellite therefore is thus acted
by a triple Attraction: The firſt and princ
that by which *T* draws *L*; the ſecond is a
in the direction *A M*, that is, with the dir
L T parallel to *A M*. Whence it happens, th
Force compounded of theſe two is alſo di
towards *T*; and if the Body *L* was acted up
ly by a Force compounded of thoſe two, it
then deſcribe Areas about *T* proportionable
Times, by *Prop.* 11. *B.* 1. But the Satellit
is alſo urg'd by a third Force, which is as
whoſe direction is from *M* towards *N*;
from *L* towards *F*, (ſuppoſing *L F* paral
M N;) which as it does not tend to *T*,
ther does the whole Force compounded of
three, namely, that by which *L* is truly
tend to *T*. Therefore the Satellite *L* wi
deſcribe Areas about *T* proportionable
Times; but this Force which is expreſs
M N will diſturb the deſcription of Areas
portionable to the Times; for in *C G* the
drant of the Semicircle *C G D* it will acce
the motion of the Body *L* about *T*, wh
moves from *C* to *G*; but after the conju
at *G* in the Quadrant *G D*, when the dir
of the Right-line *L F* from *L* to *F* is for
the Force acting along, it retards the M
When the Satellite is come to the Quad

in *D*, *M N* vaniſhes, (becauſe then *S K*, *S L*, and alſo *S A* are equal) and therefore the effect of the Force expreſs'd by it is nothing; therefore now the Satellite *L* acted on by the other Forces deſcribes, with the Radius that carries it, Areas proportionable to the Times. But when *L* goes thro' the Quadrant *D B*, *S M* goes off from *S N*, and the Force diſturbing the deſcription of the Area by the Radius *T L* that carries the Satellite (and hindring it from being proportionable to the Time which is as *M N*) has its direction from *M* to *N*, or from *L* to *F*, (that is backward) and therefore retards the Motion of the Satellite in the Quadrant: but after the oppoſition in *B*, *L F* tends forward and retards the Motion, till in the Quadrature at *C*, *M N* vaniſhes again, therefore its Effect ceaſes. Again, ſince the Force *M N* hindring the Radius *T L* from deſcribing an Area proportionable to the Time (whilſt the Satellite is going from *C* to *G*, or from *D* to *B*) is continually increas'd, and is greateſt in *G* and *B*, and from thence is continually diminiſh'd till the Satellite come to *D* or *C*, where this Force vaniſhes; it is plain that the Motion of the Satellite ſo diſturb'd is the ſwifteſt (*cæteris paribus*) in the Syzygies *B* and *G*, and ſloweſt in the Quadratures *C* and *D*. *Q.E.D.*

PROPOSITION II.

Things being as before, the Orbit which the Satellite L *deſcribes round its Primary* T, *is more curve in* C *and* D, *the Quadratures with the Sun, than in the Syzygies* G *and* B.

For the Satellite *L*, becauſe it moves ſwifter in the Syzygies than in the Quadratures, will leſs deflect from a Right-line (which it wou'd deſcribe by the Force impreſs'd only) in the firſt than in the laſt; that is, the Orbit of the Satellite is leſs curve in the Conjunction and Oppoſition than in the Quadratures. But this greater Curvity is alſo owing to another cauſe: the Force *M N* (of which we ſpoke in the foregoing Propoſition) in the Conjunction and Oppoſition at *G* and *B* is contrary to the Force by which the Body *T* draws *L*, and therefore diminiſhes it; whence the Body *L* is leſs puſh'd towards *T* at *G* and *B*, than at *C* and *D*; and therefore will leſs deflect from a Right-line in the firſt than in the laſt. Therefore the Satellite's Way is more curve in the Quadratures *C* and *D*, than in the Syzygies *B* and *G*.

COROLLARY.

From what has been ſaid it follows, That the Satellite *L* (*cæteris paribus*) recedes farther from the Primary *T* in the Quadratures than in the Syzygies. [*Fig.* 2.] Therefore the Orbit of the Satellite about its Primary, is in a manner Elliptical, the Primary being in its Center, the greater Axis being between the Quadratures with the Sun, the leſſer between the Syzygies; and ſo the Orbit is more curve in the firſt than in the laſt caſe. And the encreaſe of the celerity of the Satellite near the Syzygies will, in this Orbit, be different from what it wou'd have been in the circular Orbit of the foregoing Propoſition, becauſe the direction of the Force *M N* accelerating or retarding the Motion of the Satellite, is different in this from what it is in that.

What has been ſaid here of the Figure of the Orbit of a Satellite about its Primary, is only true, if, excluding the extrinſecal Force of the Sun, its Orbit is concentrical with the Primary Planet. For if it be excentrical, it may happen by reaſon of that excentricity, that the Satellite *L*, ſhall be farther from the Primary *T*, and ſeen from it ſhall move ſlower in the Syzygies than in the Quadratures; namely, when the Satellite has its greateſt Apſis in its Conjunction or Oppoſition with the Sun. But here alſo what was before demonſtrated will have place (*cæteris paribus*); for even then by reaſon of the ſaid Force *M N* the Satellite will move faſter, and come nearer to its Primary than it wou'd have done without it.

This aforeſaid Elliptic Figure of the Orbit of a Satellite was obſerv'd in the Moon by that moſt ingenious Aſtronomer Dr. *Halley*, who in the Year 1679, firſt of any body, publiſh'd it in his Treatiſe concerning the Emendation of the Moon's Theory. But the Hiſtory of the Academy of Sciences in *France*, publiſh'd in 1698, ſays, that Mr. *Picard* had made the ſame Obſervation long before, namely in 1668.

SECTION II.

Of the Errors which the Sun produces in the Motion of a Satellite, when its Orbit is excentric to its Primary Planet.

IF the Orb of the leſſer Body *L* [*Fig*.3.] be not a Circle concentric to the great Body *T*, but an Ellipſis, in one of whoſe *Foci* the Body *T* is plac'd, the Forces of the greateſt Body *S* will ſo diſturb that Body *L*, that inſtead of a conſtant and unmov'd Ellipſis, which (by *Prop*. 39. *B*. 1.) it ought to deſcribe, it will deſcribe an Ellipſis whoſe excentricity will continually be chang'd; and that Orbit of the Body *L* ſo diſturb'd will be the ſame, as if it ſhou'd deſcribe its Ellipſis at the ſame time that the Axis of the Ellipſis (and the Ellipſis itſelf) is by an angular Motion carried forward or backward upon the Center of the Primary, the Plane of the Orbit continuing the ſame; that is, in the Aſtronomical Phraſe, The Motion of a Satellite that deſcribes an excentric Orbit is ſo diſturb'd by the Sun, that whilſt the Satellite goes thro' its Orbit, the Apſes of its Orbit are ſometimes carried *in conſequentia*, and ſometimes *in antecedentia*. To prove this, the thing muſt be farther fetch'd and ſome *Lemma's* laid down.

PROPOSITION III.

HOw to find the Law of the Centripetal Force, by whoſe Action a Body may be mov'd in any Trajectory revolving about the center of the Forces, whilſt another Body is mov'd in a ſimilar and equal Trajectory which is at reſt.

Let *ALP* (*fig.* 3.) be the Trajectory at reſt, which the Body *T* tending to the Center of the Forces ſcribes, going from *A* thro' *L* towards *P*. It is requir'd to find the Law of the Centripetal Force tending to *T*, by means of which another Body ſhall be carried in the Perimeter of the figure *a l p* ſimilar and equal to the firſt, whilſt at the ſame time the ſaid figure *a l p* revolves about *T* backward or forward; in ſuch manner that whilſt the firſt Body goes thro' any Arc, as *A L* in the Orbit *A L P*, which is at reſt, the laſt ſhall deſcribe a ſimilar and equal Arc *a l* in the revolving Orb *a l p*.

Let *T l*, keeping always equal to *T L*, move about the Center *T* in the Plain *A L T*; and always make the Angle *A T l* proportionable to the Angle *A T L*; that is, in ſuch manner that this Angle *A T l* may be to its correſpondent *A T L* as any other Angle *ATl* to any other correſpondent *ATL*. Therefore the Area, which *Tl* (thus carried in an angular Motion about *T*) deſcribes, will be proportional to that which *TL* deſcribes. Let *AlEGH* be a Curve which the Point *l*, mov'd as has been ſaid, touches: Now becauſe the Body which is urg'd by a Centripetal Force that tends towards *T* is revolv'd along with *L*, the Area *ATLA* (by *Prop.* 11. *B.* 1.) will be proportinable to the Time; but it is alſo in an unmoveable Plain, namely, that in which the Body *L* revolves about *T*; that is, the Motion of the Point *l* is ſuch, that it is always found in the unmoveable Plain drawn thro' *T*, and make the Areas *ATlA* (deſcrib'd by a Radius drawn thro' *T*) proportionable to the Times. Therefore (by *Prop.* 12. *B.* 1.) it is manifeſt that a Body being acted upon by a due quantity of Centripetal Force, may revolve together with the Point *l* in the Curve *AlEGH*, which the Point *l* deſcribes. But this Curve *AlEGH* is given, as being generated by the given Line *ALPA* in a conſtant and given manner; therefore let the Law of the Centripetal Force by which a Body may revolve in the ſaid Curve be found (by *Cor. Prop.* 37. *B.* 1.) Let the Angle *ATa* be made equal to the Angle *LTl*, and the Line *Ta* to *TA*; and then the Angle *aTl* will be equal to the Angle *ATL*. Now let the Figure *alT* be ſimilar and equal to *ALT*; therefore the Point *l* is in the Perimeter of a Figure ſimilar and equal to *ALPA* revolving about *T*. Therefore a Body carried along with *l*, and deſcribing the unmov'd Figure *AlEGH*, deſcribes alſo the Perimeter *alpa* of an Orb ſimilar to *ALPA* and revolving about *T*; ſo that the Arc *al* is deſcrib'd in the revolving Orb *alpa*, at the ſame time that its ſimilar and equal *A L* is deſcrib'd by another Body in the Orb *A L P A*, which is at reſt. Therefore the Law of the Centripetal Force requir'd is found. *Q. E. F.*

PROPOSITION IV. LEMMA.

THe difference of the Centripetal Forces, by which two Bodies may be equally mov'd, the one in a quieſcent, the other in a revolving Orb (which is ſhewn in the foregoing Propoſition) is in a Triplicate Proportion of the common Altitude inverſly.

All things being as laid down in the foregoing Prop. I ſay that the difference of the Centripetal Forces, by which the Bodies *L* and *l* [*fig.* 4.] are acted upon, are reciprocally in a Triplicate Ratio of the common Height *T L* or *T l*; or that the difference of the Forces in the Points *L* and *l* is to the difference of the Forces in any other Points of the quieſcent and revolving Orbs, where the Bodies will be found at the ſame moment, as the Cube of the Height or diſtance of thoſe Points from *T* the Center of the Forces, to the Cube of the Right-line *L T* or *l T*.

Let the very ſmall equal Arcs *L B*, and *l b* be taken, which therefore will be deſcrib'd by the Bodies *L* and *l* in very ſmall Times. From *B* and *b* draw *B E*, *b e* perpendicular to *T L*, *T l*, which will be equal as well as *L E* and *l e*. In the Line *b e* (produc'd towards *b* if occaſion require) take the Point *C*, ſo that *C e* may be to *b e* as the Angle *A T l* to the Angle *A T L*. Now becauſe each motion of the Bodies in the places *L* and *l* is diſtinguiſh'd into Two, of which One is along the Lines *L T*, *l T*, and the Other tranſverſe to the firſt, along Lines perpendicular to *L T*, *l T*; the motions towards the Center will be equal, becauſe, by ſuppoſition, *L T* : *l T* : :

BT : *bT*, and therefore their ſmall Increments and Decrements are equal. And the tranſverſe Motion of the Body *l* will be to the tranſverſe motion of the Body *L*, as the angular motion of the Line *Tl* to the angular motion of the Line *TL*; that is, as the Angle *ATl*; to the Angle *ATL*, or as *Ce* to *be* or *BE*. And therefore in the ſame time that the Body *L*, by its two Motions comes to *B*, the Body *l* will be found in the place *C*. And this will happen where the Bodies *L* and *l* are equally mov'd along the Directions *LT*, *lT*; that is, when they are acted upon by equal Centripetal Forces: But when the Point *D* (the place of the Body deſcribing the Revolving Orbit) is found (by the foregoing Propoſition) by taking the Angle *lTD* in that ſaid conſtant Ratio to *lTb* (*viz.* the Ratio of the Angle *ATl* to *ATL*) and the Right-line *TD* equal to *Tb* or *TB*, (the Figure *alp* and the Line *lT* being in this caſe put for the Figure *ALP* and *TA* in the other;) the difference of the Centripetal Forces of the Body in *L* or *l*, and that of the Body in *D*, is as its effect *CD*; namely, the Interval of the Places thro' which the Body carry'd in the revolving Orb, muſt by the action of the Orb be transfer'd in that given ſpace of a very ſmall time. Therefore we muſt compare this *CD*, or the difference of Centripetal Forces equal to it (by which the Body is urg'd in the Quieſcent Orb, and by which it is urg'd in the Revolving Orb, in that ſituation of a moveable Orbit, and that diſtance *Tl* of the Body from the Center) with the like difference in the other Caſe, where the ſame, or a like Conſtruction is ſuppos'd as here, and the ſame things are likewiſe ſuppos'd to have been done: namely, let a Circle cutting *Te* in *n*, *Ce* in *F*, and CD in G be drawn with the diſtance Tb or TD round the Center T; and let the new Triangle lbT be ſuch, that its Area may be ſwept thro' by the Radius Tb, in the ſame time that this Area lbT is gone thro' by this Radius Tb: Whence (foraſmuch as in equal Triangles the Baſes are reciprocally proportionable to their Heights) this Line eb is to the other eb as another Tl to this Tl, that is, eb is reciprocally as the Correſpondent Tl; wherefore alſo Ce (always having the ſame Ratio to be) will be reciprocally as Tl: therefore CF the aggregate of 'em, and the Interval cb will be reciprocally proportional to the ſame Tl; and therefore the Rectangle under them, *viz.* $FC \times Cb$ is reciprocally proportionable to the Square of the ſaid Tl. Wherefore (by *Prop.* 35 or 36. *El.* 3.)

$$CD \times CG = FC \times Cb, \text{ that is, } CD = \frac{FC \times Cb}{CG}.$$

Therefore CD, in that caſe is to the analogous CD, in any other caſe, directly as this $FC \times Cb$ to that other $FC \times Cb$, and reciprocally as this Line CG to any other CG at the ſame time. But it has been ſhewn above, that $FC \times Cb$ is reciprocally as the Square of the Right-line Tl; and CG is as DG its nearly equal, or as its half TD or Tb, or Tl very little different from it: Therefore CD in this ſituation of a revolving Orbit is to CD produc'd in the ſame manner in any other ſituation of the ſame Orbit (that is, the difference of the Centripetal Forces, by which one Body in a quieſcent Orbit, and another Body in the ſame Orbit revolving, may be equally mov'd in the Height TL or Tl, is to the like difference of Forces in any other Height) reciprocally as the Square of the firſt Height, to the Square of the laſt, and reciprocally as the firſt Height to the laſt Height at the ſame time;

that is, reciprocally as the Cube of the firſt Height, to the Cube of the laſt Height; that is, (ſpeaking in ſhort of one Height, or Altitude, when we are to be underſtood of two taken together) the ſaid difference of Centripetal Forces is in a Triplicate Ratio of the common Height inverſly. *Q. E. D.*

COROLLARY 1.

A Body moving from its upper to its lower Apſis, in an Orb revolving *in conſequentia*, is urg'd by a greater Centripetal Force than if its Orb was at reſt. For becauſe (in the foregoing Figure) in that caſe the Angle *ATl* is greater than *ATL*, *eC* alſo will be greater than *eb*; as being in the ſame Ratio as the foregoing Angle, therefore the Point C is out of the Periphery of the Circle *BbD*. But the Body mov'd in the revolving Orbit is found in the Point *D*; and it has been above ſhewn, if the Body *l* be mov'd with Centripetal Force equal to *L*, that it will come to the Point C; therefore not be ſo much urg'd towards the Center *T*. [*Fig.* 5.]

Again, the like will happen if (things being as before) the revolving Orb be mov'd *in antecedentia* with a greater Celerity than the double of that Celerity in which the Line *TL* is mov'd *in conſequentia*, as in *Fig.* 1. Becauſe in that caſe *aTA* is more than twice greater than *ATL* or *aTl*, *ATl* will be greater than *ATL*; therefore (as before) *eC* will be greater than *eb*, and the Point C will be without the Circle *bDB*; and for the ſame reaſon as before, the Centripetal Force of the Body moving in the revolving Orbit *alp* is even now greater than the Centripetal Force of the Body moving in the quieſcent Orb *ALP*.

But if this Orb *a l p* goes back with a celerity leſs than twice that, by which the Line *T L* goes forward, as in *Fig.* 3. the Body *l* deſcending towards its lower Apſis will be urg'd by a leſs Centripetal Force than *L* in the quieſcent Orb. For becauſe the Angle *A T a* is not double *A T L* or *a T l*, *A T l* will be leſs than *a T l*, (that is, *A T L*) and therefore *e c* leſs than *e b*; and for that reaſon the Point *C*, to which *l* wou'd come if it was urg'd with the ſame Centripetal Force as *L*, will be within the Circumference of the Circle *B b D*. But a Body moving in a revolving Orb has been before ſhewn to be found in the Point *D*, more remote from the Center than *C* is; therefore the Body *l* deſcribing the Orb *a l p* is urg'd with a leſs Centripetal Force, than the Body *L*, which is carried in the quieſcent Orb *A L P*. And all things happen contrary to a Body aſcending from its lower, to its upper Apſis; and ſo on every Time.

COROLLARY 2.

Hence alſo the difference of the Centripetal Forces in a movable and unmovable Orbit (as in *B* and *b*, *fig.* 6.) is to the Centripetal Force, by which a Body may be mov'd in a Circular Motion from *N* to *B* (in the ſame time that the Body *L* in the unmovable Orbit deſcribes the Arc *L B*) as $RC \times Cb$ to eb^{q}. For the ſaid Forces are as their Effects *C D* and *H b* produc'd in the ſame ſmall portion of Time; but the naſcent *H b* is equal to *n e*, and *C D* has been ſhewn equal to $\frac{RC \times Cb}{CG}$: wherefore the ſaid Forces are as $\frac{RC \times Cb}{CG}$ and *n e*, that is, as $RC \times Cb$ and $CG \times ne$. But the Right-line *C G* differs little from the Right-line *M e*, and (by *Prop.* 35. *El.* 3.)

the Rectangle under Me and en is equal to the Square of eb: therefore the ſaid Centripetal Forces are as $RC \times Cb$, and eb^q. Now if the known Quantities F and G are taken in that Ratio to one another as the Angle ATL has to ATl, that is of be to Ce; CR will be as $G+F$, and Cb as $G-F$: therefore the difference of the Centripetal Forces by which Bodies are urg'd in a movable and unmovable Orb in B and b, is to the Centripetal Force by which a Body may be mov'd in a Circular Motion from N to B in the ſame time as L deſcribes the Arc LB, as $G+F \times G-F$ to F^q; that is, as G^q-F^q to F^q.

LEMMA.

If a Body tending to a Focus or Center T, *deſcribes the Ellipſis* ABP, *whoſe Apſes are* A *and* P; *the Centripetal Force by which the Body is urg'd in the Apſis* A, *is to the Centripetal Force of another Body revolving in the Circle* AEH *at the ſame diſtance* TA *with the ſame Velocity, as the Diameter of the Circle to the* latus rectum *of the Ellipſis.* [*Fig.*7.]

Thro' A draw the Right-line AG perpendicular to AT, which will touch the Circle as well as the Ellipſis. Thro' the Point G near A draw GEB towards T, cutting the Circle in E and the Ellipſis in B. Becauſe the naſcent Arcs AB, AE are nearly equal, Bodies ſetting out together with equal Velocities from A will come at the ſame time to B and E; and their Central Forces are as their Effects GB, GE produc'd the ſame ſmall Time. Beſides, from the nature of the Circle, $EI^q = AI \times IH$, or $AI \times Diam.$ And by the nature of the Ellipſis, $BD^q =$ to the *latus rectum* $\times AD$ (for the defect, or quantity by which BD^q is leſs than that Rectangle, vaniſhes in this caſe as ſoon as BD is naſcent;)

but the ſmall naſcent Lines *EI* and *BD* are equal: Therefore *AI* × Diam. of the Circle = *AD* × *latus rectum* of the Ellipſis: Therefore *AD* is to *AI*, or *GB* to *GE*, or the Centripetal Force in the Ellipſis to the Centripetal Force in the Circle as the Diameter of the Circle to the *latus rectum* of the Ellipſis. *Q. E. D.*

PROPOSITION V. LEMMA.

IF *the Orb* ALP *be an Ellipſis, whoſe* latus rectum *is* 2 R, *having its Focus at* T, *its upper Apſis at* A, *and lower at* P; *and there be another Ellipſis* alp *equal and ſimilar to it, revolving in the manner aforeſaid round the Focus* T, *and the given quantities* G *and* F *have the ſame Ratio as the Angles* AT*l* *and* ATL; *if the Height of the revolving Body, or the Diſtance from the Center be* A: *the Centripetal Force by which the Body may revolve in the movable Ellipſis will be, as* $\frac{Fq}{Aq}+\frac{R\times Gq-R\times Fq}{A\ \text{cub.}}$; *and on the other hand, when ſuch a Centripetal Force acts, the Body will be carried in a movable Ellipſis.*

Since the Force by which the Body can revolve in the unmovable Ellipſis *ALP*, is (by *Prop.* 39. *B.* 1.) reciprocally as the Square of the diſtance from the Center, or of the Height *A*; let that Force be expreſs'd by $\frac{Fq}{Aq}$, and the Force in the Apſis *A* will be $\frac{Fq}{ATq}$. But the Force by which the Body may at the diſtance *TA* revolve in a Circle with the Velocity, that the Body revolving in an Ellipſis has in *A*, is (by the foregoing Lemma) to the Force by which the Body revolving in an Ellipſis is acted upon in *A*, as the half *latus rectum* of the Ellipſis to the Semidiameter

diameter of the Circle, or as R to AT; therefore its value is $\frac{R\times F^q}{AT\ cub.}$. Now becauſe (by *Corol.* 2. *Prop.*4.) the difference of the Forces in the Points A and a is to the aforeſaid Force in the Circle, (which is equal to $\frac{R\times F^q}{AT\ cub.}$) as $G^q - F^q$ to F^q: that Difference of Forces will be equal to $\frac{R\times G^q - R\times F^q}{AT\ cub.}$: And (by *Prop.*4.) the Difference of the Forces in any other Points of the ſame Height (namely, where the Height A is undetermin'd) is to the Difference of the Forces in the Apſis A (where the Height is TA) as the Cube of TA to $A\ cub.$; therefore in any Height its Value will be $\frac{R\times G^q - R\times F^q}{A\ cub.}$. If therefore to the Force $\frac{F^q}{A^q}$, by which a Body may be revolv'd in an unmovable Ellipſis ALP, be added the ſaid Difference $\frac{R\times G^q - R\times F^q}{A\ cub.}$; the whole compounded Force will be $\frac{F^q}{A^q} + \frac{R\times G^q - R\times F^q}{A\ cub.}$; by which a Body may in the movable Ellipſis alp be revolv'd in the ſame time. *Q. E. D.*

COROLLARY 1.

Hence if to the Centripetal Force, (which is in a Duplicate reciprocal Ratio of the Diſtance from the Center, by which a Body revolves in an unmov'd Ellipſis) be ſuperadded to any other Force which is reciprocally as the Cube of the Diſtance; the Body will deſcribe an Ellipſis whoſe Apſes are carried *in conſequentia.* For becauſe the Difference, which is as $\frac{R \times Gq - R \times Fq}{A\ cub.}$, is in this caſe poſitive, *G* will exceed *L*; that is, the Angle *ATl* does exceed the Angle *ATL*; whence (by *Corol.* 1. *Prop.*4.) the Apſes are either carried, *in conſequentia*, or *in antecedentia*, with a Celerity greater than twice that by which the Body *L* is carried, *in conſequentia*, in the unmovable Orb; but becauſe this laſt does not happen in any of the Celeſtial Bodies, we abſtract from it in this caſe. But if from the Force (which is reciprocally in a Duplicate Ratio of the Diſtance) be taken the Force which is reciprocally in a Triplicate Ratio of it, the Remainder will be the Force, by whoſe action a Body will deſcribe an Ellipſis whoſe Apſes go backward; for the Difference of the Forces $\frac{R \times Gq - R \times Fq}{A\ cub.}$ is in this caſe negative; and therefore *G* is leſs than *F*, or the Angle *ATl* than *ATL*.

COROLLARY. 2. [*Fig.* 9.]

Hence it follows alſo, that if in an Orb which differs very little from a Circle, the centripetal Force, as you recede from the Center, decreaſes more (or becomes leſs) than in a Duplicate Ratio of the encreas'd diſtance; the Apſes of the Orb will be mov'd, *in conſequentia*. For let the unmovable Ellipſis *ALP* be the Orb deſcrib'd by a Body, which is urg'd by a Force tending to the Center *T*, whoſe upper Apſis *A* is twice as far diſtant from the Center *T* as the lower *P*; not that ſuch an Orb comes near to a Circle, but we put this Proportion only by way of Example, and to make the caſe plainer. Let us ſuppoſe the Centripetal Force by which the Body *P* is urg'd, to conſiſt of 400 Parts, and the Force by which it is urg'd in *A* will be of 100, and ſo proportionably in the intermediate Places; namely, that Force by whoſe action the Body wou'd deſcribe the unmovable Ellipſis *ALP*, *viz.* in a duplicate reciprocal Ratio of the Diſtance from the Center. Now if beſides this Force, the Body in *P* is urg'd by another Force, which is for *Ex.* as 64, and in *A* by a Force which is as 8, and proportionably in the intermediate Places; namely, if that ſuperadded Force be inverſly as the Cube of the Diſtance from the Center; in that caſe (by the foregoing Corol.) the Apſes will be carried *in conſequentia.* But the whole Force in *P*, or that made up of the Two, is as 464, and the Force in *A* as 108, and proportionably in the intermediate Places; that is, as you recede from the Center the Centripetal Force decreaſes faſter than in a Duplicate Ratio of the encreas'd Diſtance; for if it was only diminiſh'd in that Duplicate Ratio of the Diſtance, the Force in *A* wou'd be as 116. And tho' it be requir'd that a new Force, ſuperadded to that Firſt, which is inverſly as the Square of the Diſtance, be every where preciſely as the Cube of the ſaid Diſtance inverſly, that an Elliptic Orbit may be revolv'd, *in conſequentia*, about the Center of the Forces: yet if this Ellipſis differs but little from a Circle, and ſuch a Force wholly acts upon the Body, as in receding from the Center is diminiſh'd more than the Square of the Diſtance from the Center is encreas'd; the Apſes of ſuch an Orb will be mov'd, *in conſequentia.* For becauſe in an Orb that is almoſt circular, the Diſtances from the Center differ but very little; that Force, (the adding of which occaſions the whole Force by which the Body is urg'd to be diminiſh'd more than the Square of the Diſtance from the Center is encreas'd) is diminiſh'd in a Ratio which differs not much from a Triplicate Inverſe Ratio of the Diſtance.

It follows alſo, that if in an Orb nearly circular the Centripetal Force decreaſes leſs than in a Duplicate Ratio of the encreas'd Diſtance, (that is, be greater than in the ſaid Ratio) the Apſes of the Orb muſt move *in antecedentia*. For the ſame Force remaining in *P* and *A*, by which a Body deſcribes an unmovable Ellipſis, if from it you take away a Force reciprocally proportionable to the Cube of the Diſtance from the Center; namely, if from the Force in *P* you take a Force that is as 64, and from the Force in *A* a Force as 8, and after the ſame manner in the intermediate places; there will every where remain a Force which (by the foregoing Coral.) will make a Body deſcribe an Ellipſis whoſe Apſes are mov'd *in antecedentia*. But the Force remaining in *P* is as 336, and the Force remaining in *A* as 92, and after the ſame manner in the intermediate places; therefore as you recede from the Center the Centripetal Force is leſs diminiſh'd (or is greater) than in a duplicate Ratio of the encreas'd Diſtance: for the Force diminiſh'd in a duplicate Ratio of the ſaid diſtance, wou'd in *A* conſiſt of 84 Parts. Whence, by the Reaſons before given, if a Body in an Orb nearly circular be urg'd by a Force, which, as you recede from the Center, is diminiſh'd leſs than in a Duplicate Ratio of the encreas'd Diſtance; the Apſes of the Orb will be carried *in antecedentia*. Which alſo appears by *Corol.* 1. *Prop.* 4.

PROPOSITION VI. LEMMA.

IN Elliptic Orbits, nearly circular, to determine the motion of the Apſes from the given Law of the Centripetal Force; and, on the contrary, from the given motion of the Apſes to find the Law of the Centripetal Force, ſuppoſing the Centripetal Force to be as ſome Power of the Altitude.

If from any given Law of Centripetal Force the Motion of the Apſes be requir'd, the Problem will be ſolv'd by making the Orb, which a Body revolving in a movable Ellipſis deſcribes in an unmov'd Plane, to come near the figure of the Orb whoſe Apſes are requir'd, and finding the Apſes of an Orb ſo deſcrib'd. Now the Orbs come neareſt to that figure, when the Centripetal Forces, by which they are deſcrib'd, are proportional at equal diſtances from the Center.

Let the Point A (in any of the foregoing Figures) be the upper Apſis, T the center of the Forces. Let AT the greateſt Height be call'd T, and any other undeterminate Altitude A, and their difference X. And the Force by which the Body is urg'd in an Ellipſis, which revolves about its Focus T, is, (by the foregoing) as $\frac{F^q}{A^q} + \frac{R \times G^q - R \times F^q}{A\ cub.}$, or as $\frac{A \times F^q + R \times G^q - R \times F^q}{A\ cub.}$; and laſtly, if inſtead of A you ſubſtitute $T-X$, as $\frac{T \times F^q - X \times F^q + R \times G^q - R \times F^q}{A\ cub.}$. Let the Value of the Centripetal Force, whoſe Law is given, be reduc'd to a ſimilar Fraction; that is, ſuch a one whoſe Denominator is the ſame *A cub.* and the Numerators (by comparing the homologous Terms) are to be taken analogous; from the compariſon of which will be made out the Ratio of G to F, or the requir'd Motion of the Apſes. Let the Law of the Centripetal Force be given, as for Example, let it be as a Power of the Diſtance from the Center, whoſe Exponent is the given p, or as A^p, that is as $\frac{A^{p+3}}{A^3}$. Inſtead of the given $p+3$ put n, and inſtead of A in the Numerator $T-X$; and the Centripetal Force will be as $\frac{T-X^n}{A^3}$. Let $T-X^n$ the Numerator of this Fraction be reduc'd to an Infinite Series, namely, to $T^n - n\times T^{n-1}X + n\times\frac{n-1}{2}\times T^{n-2}X^2$ — &c. Let the Terms of this Numerator be compar'd with the Terms of the foregoing Numerator, *viz.* $T\times F^q - X\times F^q + R\times G^q - R\times F^q$; that is, the given with the given, and the not given (or thoſe in which the undetermin'd X is found) with the not given; and $T\times F^q + R\times G^q - R\times F^q$ will be to T^n as $-F^q\times X$ to $-n\times T^{n-1}X + n\times\frac{n-1}{2}\times T^{n-2}X^2$ &c. that is, as $-F^q$ to $-n\times T^{n-1} + n\times\frac{n-1}{2}\times T^{n-2}X$ &c And becauſe the Ellipſis in queſtion is nearly circular, in which half the *latus tranſverſum*, and half the *latus rectum*, or T and R are equal, and all the Altitudes or Diſtances from the Center are nearly the ſame, therefore X, in reſpect of them, is almoſt equal to nothing; that is, the Terms affected by that vaniſhing; by the mutual deſtruction of the Terms, and the neglect of thoſe that vaniſh, the firſt Analogy will be chang'd into this, $R\times G^q$ is to T^n as $-F^q$ is to $n\times T^{n-1}$; and ſtill putting T in the room of R in the firſt

Term, $T \times G^q$ will be to T^n as F^q to $n \times T^{n-1}$; therefore G^q will be to T^{n-1} as F^q to $n \times T^{n-1}$, and again, G^q to F^q as T^{n-1} to $n\,T^{n-1}$; that is, as 1 to n: therefore G will be to F (that is, the Angle ATl to the Angle ATL) as 1 to $\sqrt{n}$. Wherefore ſince the Angle, which the Body in the unmovable Ellipſis deſcribes in its deſcent from the upper to the lower Apſis, is equal to Two Right; the Angle, which the Body (that is urg'd by a Centripetal Force proportional to a Power of the Altitude whoſe Exponent is p or $n-3$) moving in an Orbit nearly circular, deſcribes in its deſcent from the upper to the lower Apſis, will be equal to Two Right; the Angle, which a Body, (which is acted upon by a Centripetal Force proportional to a Power of the Altitude whoſe Exponent is p or $n-3$) deſcending from its upper to its lower Apſes, in an Orbit nearly circular, deſcribes, will be to Two Right as 1 to $\sqrt{n}$; therefore in an Arithmetical Phraſe equal to $\frac{180 \text{ Deg.}}{\sqrt{n}}$: and an equal Angle being repeated, *A* Body will return from its lower to its upper Apſis; and ſo on continually.

And, on the contrary, if the Centripetal Force be as any Power of the Height, that Power may be found from the Motion of the Apſes; namely, if the Angular Motion, by which a Body deſcends from the upper to the lower Apſis, or the whole Angular Motion by which a Body receding from an Apſis returns to it, be to the Angular Motion of a whole Revolution (or to 360 deg.) as Unity is to the Number $\sqrt{n}$; the Centripetal Force will be as a Power of the Altitude, whoſe Exponent is $n-3$. For if the Centripetal Force be as A^{n-3}, by the firſt Part of this Propoſition there will be the ſaid Ratio between the Angular Motions, or between *G* and *F*; and therefore this Law of the Centripetal Force will depend upon that proportion between *G* and *F*. Therefore the Centripetal Force requir'd is found. *Q. E. F.*

SCHOLIUM.

We will give ſome Examples of this Propoſition in ſome of the moſt remarkable Caſes. The Angle is requir'd, which a Body makes in its deſcent from its upper to its lower Apſis in an Orb nearly circular, whilſt it is urg'd by an uniform Centripetal Force, or a Force which is the ſame in all Altitudes. Becauſe in this caſe the Centripetal Force is conſtant, either as A^0, or as A^{3-3}, $n=3$, and $\surd n=\surd 3$; therefore the Angle deſcrib'd by a Body moving from its upper to its lower Apſis is of $\frac{180^{\text{deg.}}}{\surd 3}$ or of $103^{\text{deg.}}$ $55'$, and that Angle being again deſcrib'd, it returns to the upper Apſis, the Apſis being carried ſwiftly *in antecedentia*. But if the Centripetal Force be as the Altitude itſelf or A^1, that is, as A^{4-3}; n will be $=4$, and the Angle between the upper and the lower Apſis $\frac{180^{\text{deg.}}}{\surd 4}$ or $90^{\text{deg.}}$ therefore the Apſes of ſuch an Orbit are carried *in antecedentia* with the ſame angular Celerity, as the Body deſcribing the Orb is carried *in conſequentia*. Likewiſe if the Centripetal Force be as $A^{\frac{11}{4}}$ inverſly, or as $A^{-\frac{11}{4}}$, or as $A^{\frac{1}{4}-3}$; n will be $=4\frac{1}{4}$ and $\surd n=\frac{1}{2}$; therefore the Angle between the upper and the lower Apſis, which is of $\frac{180^{\text{deg.}}}{\frac{1}{2}}$, is of $360^{\text{deg.}}$ that is, a Body which is urg'd by ſuch a Force, as it recedes from its upper Apſis, and then continually deſcends, will come to its lower Apſis when it has made one whole Revolution; then by a conſtant aſcent, as it goes thro' another whole Revolution, it will return to the upper Apſis, and ſo on conſtantly.

Suppoſe, on the other hand, that the Motion of the Apſes is given, and the Law of the Centripetal Force be requir'd: for Example, let the Law of that Centripetal Force be requir'd by which that Body is acted upon, which in one Revolution and a half, by an alternate deſcent and aſcent, from the upper Apſis returns again to the upper Apſis. Becauſe in that caſe, the angular Motion by which a Body deſcends from the upper Apſis to the lower is of 270 deg. the fourth Proportional to 270 deg. 180 deg. and 1, or $\surd n$, will be equal to $\frac{2}{3}$, and therefore $n = \frac{4}{9}$: and ſo (by the ſecond part of this Propoſition) the Centripetal Force is as $A^{\frac{4}{9}-3}$; that is reciprocally as $A^{3-\frac{4}{9}}$ or $A^{2\frac{5}{9}}$. Again, let the Law of that Centripetal Force be requir'd by which that Body is acted upon, which from the upper Apſis to the upper Apſis again, makes 363 deg. that is, which is carried in an Orbit almoſt Circular, whoſe upper Apſis every Revolution of the Body goes thro' three Degrees *in conſequentia.* Becauſe the whole Angular Motion by which a Body receding from the upper Apſis goes back to it again, is to the angular Motion of 360 deg. as 363 to 360; $\surd n$, or the fourth Proportional to 363, 360 and 1 will be $\frac{360}{363}$; therefore n will be $\frac{129600}{131769}$: and the Centripetal Force as $A^{\frac{129600}{131769}-3}$ or as $A^{-\frac{265707}{131769}}$; that is, inverſly as $A^{\frac{265707}{131769}}$, or inverſly as $A^{2\frac{4}{242}}$. Therefore the Centripetal Force, by which a Body is urg'd, that deſcribes an Orbit whoſe Apſes in one Revolution of the Body are mov'd $3^{deg.}$ *in conſequentia*, as the Body recedes from the Center decreaſes in a Ratio ſomething greater than Duplicate, but 60 times nearer a Duplicate than a Triplicate.

COROLLARY.

Hence it is plain, that if the Centripetal Force, by which the Body is urg'd in its approach to the Center, encreaſes in a Triplicate or a leſs Ratio of the diminiſh'd Altitude, the Body if it begins to deſcend, will never come to the lower Apſis, but deſcend quite to the Center; but if from the Apſis it aſcends ever ſo little, it will never come to the upper Apſis, but continue to aſcend *in Infinitum.* For becauſe the Centripetal Force is reciprocally as A^3, that is as A^{-3}, that is $p = -3$; $n (= p + 3)$ will be $= o$, and $\surd n = o$; and the Angle by the deſcription of which a Body will go from the upper to the lower, or from the lower to the upper Apſis, will be equal to $\frac{180 \text{ deg.}}{0}$, or be infinite: therefore it will never come to the other Apſis, but after innumerable Revolutions it will either deſcend to the Center or aſcend *in Infinitum.* But if the Force in the receſs of the Body from the Center decreaſes in a Ratio of the Diſtance greater than Triplicate, n will be a negative quantity, and $\surd n$ will be a thing impoſſible; whence the Truth of the Propoſition is plain. And on the other hand it appears, that if a Body aſcending and deſcending alternately from Apſis to Apſis, never comes to the Center; the Force, as you recede from the Center, will either be encreas'd or diminiſh'd in a leſs Ratio of the Height than a Triplicate: and by how much leſs the Angle is, which deſcrib'd by the Body, whilſt it goes from Apſis to Apſis, by ſo much more will that Ratio of the Forces differ from a Triplicate reciprocal Ratio.

PROPOSITION VII. LEMMA.

IN Elliptical Orbits, which are nearly Circular, to determine the Motion of the Apſes from the given Law of the Centripetal Force, ſuppoſing the Centripetal Force to be as the Sum or Difference of any Two Powers of the Altitudes.

Things being as before, let *m b* and *C* any given quantities, of which *m* is the Exponent of the Power, and alſo *n* as in the foregoing; and let the given Law of the Centripetal Force be ſuch, that it may be every where as $\frac{bA^{m}+CA^{n}}{A^{3}}$; or elſe putting $T-X$ inſtead of A, as $\frac{b \times T-X^{m}+C \times T-X^{n}}{A^{3}}$. Reducing its Value to an Infinite Series, the Centripetal Force will be as

$$\frac{bT^{m}-bT^{m-1}X+bm \times \frac{m-1}{2}T^{m-2}X^{2}+CT^{n} -CnT^{n-1}X+Cn \times \frac{n-1}{2} \times T^{n-2}X^{2} \;\&c.}{A^{3}}$$

We muſt lay down as analogous this Value of the Centripetal Force, and the Value before found of that Force which makes a Body move in a movable Ellipſis, *viz.* $\frac{T \times F^{q}-X \times F^{q}+R \times G^{q}-R \times F^{q}}{A^{3}}$; that is, when theſe Values have hitherto the ſame Denominator, the given part of one Numerator is to have the ſame Ratio to its part not given, as the given part of the other Numerator to its part not given; that is, $R\times Gq - R\times Fq + T\times Fq$ muſt be to $-Fq\times X$, as $bT^m + CT^n$ is to $-b\times mT^{m-1}X + b\times m\times\frac{m-1}{2}T^{m-2}X^2 - C\times n T^{n-1}X + C\times n\times\frac{n-1}{2}\times T^{n-2}X^2$ &c. And as the Orbits are ſuppos'd almoſt circular, T and R will be equal. Beſides, by applying the Conſequents to $-X$, and neglecting the Terms affected by X as being very ſmall, (becauſe X is infinitely little) $T\times Gq$ will be to Fq as $bT^m + CT^n$ to $b\times m\times T^{m-1} + C\times nT^{n-1}$. Or, to make it more ſimple, that all the Powers of T may become equal, let T be put for Unity; then Gq will be to Fq as $b+C$ to $bm+Cn$. And for that reaſon G to F, or (as in the fig. of *Prop.* 4.) the Angle ATl to the Angle $\overline{ATL}$ as $\sqrt{b+C}$ to $\sqrt{bm+Cn}$, or as 1 to $\sqrt{\frac{bm+Cn}{b+C}}$. And therefore when the Angle between the upper and lower Apſis in an unmovable Ellipſis becomes equal to Two right; the Angle which the Body (by a Centripetal Force proportional to $\frac{bA^m+CA^n}{A^3}$) deſcribes in its deſcent from its upper to its lower Apſis, or contrary aſcent, is to Two Right; as 1 to $\sqrt{\frac{bm+Cn}{b+C}}$; and in an Arithmetical Phraſe that Angle is $\sqrt{\frac{b+C}{bm+Cn}}\times 180^{\text{deg}}$. If the Centripetal Force be as the Difference of Two Powers of the Altitude, (namely as $\frac{bA^m - CA^n}{A^3}$) putting $-C$ in the room of $+C$ in the value already found, the Angle between the upper and lower Apſis, of a movable Orbit nearly circular, is equal to $\sqrt{\frac{b-C}{bm-Cn}} \times 180^{\text{deg.}}$ The ſame Way will the Problem be ſolv'd in more difficult Caſes.

SCHOLIUM.

If a Body acted on by a Centripetal Force, which is reciprocally as the Square of the Altitude be revolv'd in a movable Ellipſis, that has its Focus in the Center of the Forces, and to this Centripetal Force be added, or from it be taken any other Force, whoſe Law and Quantity is given; the Motion of the Apſes ariſing from that given Force will be determin'd by this Propoſition; namely, if from the Force $\frac{1}{A2}$ be taken an extrinſecal Force, which is as the Altitude or Diſtance from the Center, namely, as CA; the remaining Force by which the Body is urg'd, will be $\frac{A - CA^4}{A^3}$, where $b=1$, $m=1$, $n=4$: Therefore the Angle of the Revolution between the Apſes is $\sqrt{\frac{1-C}{1-4C}} \times 180^{\text{deg.}}$ For Ex. Let the extrinſecal Force which is taken away be 357, 45 times leſs than the Primary Force, by which the Body is revolv'd in an Ellipſis, which, to make it eaſy we have call'd 1; and C will be $= \frac{1}{357{,}45}$ or $\frac{100}{35745}$, and $1 - C = \frac{35645}{35745}$ and $1 - 4C = \frac{35345}{35745}$; therefore $\frac{1-C}{1-4C} = \frac{35645}{35345}$ $= 1{,}008487754$, and $\sqrt{\frac{1-C}{1-4C}} = 1{,}004234$. And therefore $\sqrt{\frac{1-C}{1-4C}} \times 180^{\text{deg.}}$ or the Angle between the Apſes in a movable Orbit, will be $1{,}004234 \times 180^{\text{deg.}}$ or $180^{\text{deg.}}.76228$, that is,

$180^{deg.}\ 45'\ 44''$. Therefore the Motion of the Apſes is found in one half Revolution, or whilſt the Body deſcends from the upper to the lower Apſis, *viz.* 45′. 44″ *in conſequentia*, and ſo much does the Body go forward when it aſcends from the lower to the upper Apſis; that is, each Revolution of the Body the Apſis is carried *in conſequentia* 1°. 31′. 28″.

And converſely, if the Angle between the Apſes be given, you may find what Ratio the extrinſecal Force (whoſe Law is given) has to that Force by which a Body wou'd be mov'd in an unmovable Ellipſis: namely, if a Body return'd from the upper Apſis to the upper Apſis again, makes by its angular Motion an Angle of 315°, the Apſis at the ſame time moving 45° *in antecedentia*, and the Ratio of that ſuperadded Force (which is as the Altitude) to the Primary Force by which a Body wou'd move in an unmov'd Ellipſis be requir'd. For, becauſe it has by this Propoſition been demonſtrated, that the Angle between the upper and the lower Apſis is $\sqrt{\frac{b+C}{bm+Cn}} \times 180°$, or that the Angle between the upper and the upper Apſis is $\sqrt{\frac{b+C}{bm+Cn}} \times 360°$, that is (in the preſent caſe, where the ſuperadded Force is as the Altitude) $\sqrt{\frac{1+C}{1+4C}} \times 36°$, $315°$ will be $= \sqrt{\frac{1+C}{1+4C}} \times 360°$. And by reſolving that Equation its Value C will be found $\frac{5}{44}$. But as this Primary Force, by which a Body wou'd move in an unmovable Ellipſis, be ſuppos'd 1; this Force will be to the added Force (which is as the Altitude, and which makes the Apſes go back ſo ſwiftly) as 44 to 5.

COROLLARY.

From the foregoing Scholium, and the 2d Corollary of *Prop.* 5. it follows, that if from a Force which is in a Duplicate Ratio of the Altitude inverſly, be ſubtracted a Force which is directly as the Altitude, there will be left a Force, which as you recede from the Center decreaſes more (or is leſs) than in a Duplicate Ratio of the encreas'd Altitude, or a Force which as you approach to the Center encreaſes more (or is greater) than in a Duplicate Ratio of the diminiſh'd Altitude; for as well in that caſe as in this, the Apſes of the Orb go forward, and for that reaſon the two Caſes become the ſame. And it appears by the ſame reaſoning, that if to a Force which is in a Duplicate reciprocal Ratio of the Diſtance from the Center, be added an extrinſecal Force, which is directly as the ſaid diſtance, the Sum of 'em will be a Force, which, as you recede from the Center, decreaſes leſs (or is greater) than in a Duplicate Ratio of the encreas'd diſtance, or (what is the ſame) which as you come towards the Center, encreaſes leſs (or is leſs) than in a Duplicate Ratio of the diminiſh'd Diſtance. But theſe alſo are an immediate Conſequence of *Cor.* 1. *Prop.* 4.

PROPOSITION VIII.

IF a Secondary Planet L *deſcribes an Elliptic Orbit whoſe Focus is* T, *about the Primary Planet* T; *its greater Axis, or Line of the Apſes by its Angular Motion goes forward twice, and backward twice, every Revolution of the Satellite* L; *that is, it goes forward twice when* L *is in the Syzygies* G and B, *but it goes back when* L *is in* C *and* D *the Quadratures with the Sun.* [Fig. 10.]

Let *S* denote the Sun, *T* a Primary Planet moving in the Orbit *D C*, from *D* towards *C* about the Sun, *L* a Secondary Planet deſcribing an Elliptic Orbit, whoſe Focus is *T*, from *C* to *G* towards *D*. Now becauſe in theſe Three Bodies the accelerating Attractions of any Two on the Third are (by what has been ſhewn in *Book* 1.) to one another reciprocally in a Duplicate Ratio of the Diſtances from it, this brings us to the Caſe of the firſt Propoſition of this Book (whoſe Conſtruction we here ſuppoſe) in which it is ſhewn, that the Satellite *L* is urg'd by a double accelerating Force, the one from *L* to *T*, and the other from *L* to *S*. And as the firſt Force from *L* to *T*, ariſing from the mutual Attraction of the Bodies *L* and *T*, be reciprocally proportional to the Square of the Diſtance (by *Corol.* 2. *Prop.* 51. *Book* 1.) the Body *L* will by that Force alone deſcribe an Ellipſis, which has its Focus in *T*. But becauſe the laſt Force, by which *L* is acted upon, and which is directed from *L* to *S* (and is, by what has been before ſhewn in *Prop.* 1. as the Right line *A S*) is reſolv'd into Two Forces, one of which is as *A M*, and the other as *M S*, which is always reduc'd to *M N*, as has been ſhewn, and with the direction of theſe Right lines; of which that which is as *A M*, tho'

tho' it has the ſame direction with the firſt, by which *T* draws *L*, yet it does not obſerve the ſame Law; neither is *AM* reciprocally as the Square of the Right line *LT*, which is the Diſtance of the Body *L* from *T*, but almoſt directly as *LT*; for as *LT* encreaſes, things remaining as before, *AM* encreaſes almoſt in the ſame Ratio; for *SL*, by reaſon of the Diſtance of the Body *S*, is almoſt parallel to *ST*; the Force by which *L* when it is in the Quadrature *C* and *D* is urg'd toward *T*, being compounded of two (namely that firſt by which *T* draws *L*, and which is inverſly as the Square of the Diſtance *LT*, and that which being as *AM*, is almoſt directly as the very Diſtance *LT*, for the third Force *MN* in that caſe is nothing) is not as the Square of the Diſtance inverſly, but as you recede from *T*, decreaſes in a leſs ratio than a duplicate of the encreaſed Diſtance: When therefore to the Force, which is reciprocally proportional to the Square of the Diſtance from the Center or Altitude, namely, that by which *L* draws *T*, an extrinſecal Force, which is as the Altitude it ſelf, is added, there will (by Cor. of the foregoing Prop.) be compounded a Force by whoſe action the Body deſcribes an Ellipſis, whoſe Apſes are mov'd *in antecedentia*. Therefore when the Satellite is in the Quadratures, the Apſes of the Elliptic Orbit go backward.

But if the Satellite *L* (*fig.* 11.) be in *G* the Conjunction, or *B* the Oppoſition with the Sun *S*, in which caſe the Force *AM* vaniſhes, beſides the Force by which *T* draws *L*; and which is reciprocally as the Square of the Altitude *TL*; the Body *L* will be urg'd by the Force *MN*, puſhing from *M* to *N*, which is not as the Square of the Altitude *TL* inverſly; but encreaſing as the ſaid Altitude encreaſes, and almoſt as TL it ſelf directly. Therefore when from a Force reciprocally proportional to the Square of the Altitude, by which T draws L, is taken away an extrinſecal Force, (*viz.* a Force ariſing from the Sun's Action) which is almoſt as the Altitude itſelf; there will (by the Corollary of the foregoing Propoſition) be left a Force by whoſe action a Body is carried in an Ellipſis, whoſe Apſes are mov'd *in conſequentia.* Therefore when a Secondary Planet is in the Syzygies with the Sun, the Apſes of the Elliptic Orbit deſcrib'd by it go forward.

But when the Place of the Satellite is between the Syzygies and the Quadratures with the Sun, the Motion of the Apſes depends upon the two forementìon'd Cauſes jointly; namely, upon an extrinſecal Force, added as AM, and taken away as MN; in ſuch manner that the Apſes, according to the exceſs of the one or the other, go forward or backward: Therefore in each Revolution of the Satellite L about T, the Apſes by turns go forward twice, and backward twice.

PROPOSITION IX. LEMMA.

IN fig. 11. *the Force* MN *in the Syzygies is almoſt twice greater than the Force* AM *in the Quadratures.*

In the Syzygies SM is to SN in a Duplicate Ratio of SN to SL: but the Diſtance of the Points N and T, when compar'd with ST the Diſtance of a Planet from the Sun vaniſhes, therefore ST is equal to SN; and alſo SM is to ST in a Duplicate Ratio of ST to SL: But Quantities Arithmetically proportional as SB, ST, SG, *&c.* when their Interval BT or TG is very little in reſpect of any one of 'em as SG,

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are alſo as it were Geometrically Proportional, and ſo inverſly; wherefore if you ſuppoſe *B M* (in the firſt *fig.* 11.) and *G M* (in the ſecond *fig.* 11.) equal to *T B*, you will find that (in the firſt *fig.* 11.) *M S*, *B S*, *T S*, and *L S*, and (in the ſecond *fig.* 11.) *M S*, *G S*, *T S*, *L S*, will be nearly in a continu'd Geometrical Proportion: Therefore in each Figure *M S*, is to *T S* in a Duplicate Ratio of *T S* to *L S*. But *M T* is (by Conſtruction) twice greater than *T B*, which (by the foregoing) is equal to *A M* in the Quadratures; and *M N* is almoſt equal to *M T*. Therefore *M N* in the Syzygies is as it were twice greater than *A M* in the Quadratures. *Q. E. D.*

PROPOSITION X.

IN each Revolution of any Satellite the Apſes for the moſt part go forward more than they go backward, and are by the Exceſs of their going forward carried in conſequentia.

For the Force *M N* in the Syzygies, (which being taken away from the Force with which *T* draws *L*, is the cauſe that the Apſes go forward) is (by the foregoing Lemma) as it were twice greater than the Force *A M* in the Quadratures, by whoſe addition to the ſaid Force, by which *T* draws *L*, the Apſes go backward: And therefore the Exceſs in one whole Revolution of the Satellite will be within the Power of *M N*, and will transfer the Apſes *in conſequentia.* *Q. E. D.*

Theſe will happen ſo commonly, and *cæteris paribus*: But there are particular caſes, (as will be ſhewn in the next Prop.) in which, in one Revolution of the Satellite about its Primary Planet, the Apſes go backward more than they go forward.

PROPOSITION XL.

IN one Revolution of the Satellite L *about* T, *the Progreſs of the Apſes exceeds their Regreſs, when they are in the Syzygies with the Sun, more than if they were farther from it: But the Regreſs of the Apſes exceeds their Progreſs in one Revolution of the Satellite, when the Apſes are in the Quadratures with the Sun: Yet they go forward more and ſwifter in their Syzygies, but they recede leſs and ſlower in the Quadratures; and by the Exceſs of the Progreſs above the Regreſs, in one Revolution of the Apſes about the Sun, are carried* in conſequentia, [fig. 11. and 12.]

Becauſe (by *Cor.* 2. *Prop.* 5.) the Progreſs or Regreſs of the Apſes depends upon the Centripetal Force of the Satellite towards its Primary, which decreaſes more or leſs than in a Duplicate Ratio of the encreas'd Diſtance as you recede from the Center, or in the Paſſage from the lower to the upper Apſis, as alſo upon a contrary increment as you go towards the Center or return to the lower Apſis; it is plain that the Progreſs or Regreſs is the greateſt, when the Ratio between the Forces in the Apſes differs very much from an inverſe Duplicate Ratio of the Diſtances. Suppoſe the ſituation of the Orbit of a Satellite in reſpect to the Sun, to be ſuch, that its Apſes may be in the Syzygies with the Sun: Therefore the extrinſecal Force *MN*, which urges the Satellite when it is the Syzygies, being taken away from the Force by which the Primary draws the Satellite, leaves a Force in the Satellite at *B*, whoſe Ratio to the Force likewiſe left in the Satellite at *G*, differs more from a Duplicate Ratio of the diſtances inverſly, than if the Apſes were not in the Syzygies; becauſe the Ratio between the diſtances *TB*, *TG*, is more unequal, or differs more from the Ratio of Equality, than the Ratio between any other Two in this Orbit: Therefore the Progreſs of the Apſes in this caſe is the greateſt. But when a Satellite is come to *C*, the extrinſecal Force *AM* being added to the Force by which *T* draws the Satellite, does not ſo much make it recede from a Duplicate inverſe Ratio of the diſtances in the paſſage from *C* to *D*; becauſe the diſtances *TC*, *TD* become equal: Therefore the Effect of this added Force is leſs: that is, the Apſes of the Orb go back leſs in this caſe. Therefore, becauſe when the Apſes are in the Syzygies, they go forward very ſwiftly when the Satellite goes thro' the Syzygies, and go back very ſlowly when the Satellite is in the Quadratures with the Sun; it is plain, that in one whole Revolution of the Satellite about the Primary Planet, the Apſes in this caſe will be carried, *in conſequentia*, very ſwiftly.

Now let the Apſes of the Satellite's Orb be in the Quadratures: in which caſe the foreſaid extrinſecal Force *A M*, which acts upon the Satellite in the Quadrature *C*, being added to the Primary Force by which *T* draws the Satellite, makes up a Force compounded of both, by which the Satellite, when at *C*, tends towards *T*; (*fig.* 12.) whoſe Ratio to a Force compounded after the ſame manner, by which a Satellite, when at *D*, is urg'd towards the ſaid *T*, recedes more from a Duplicate reciprocal Ratio of the Diſtances, than if the Apſes were not in the Quadratures; becauſe the Ratio between *TC* and *TD* is more unequal than the Ratio between any other Two of this Orbit; therefore the Regreſs of the Apſes is the greateſt in this caſe. But when the Satellite is come to *G* or *B*, the ex-

extrinſecal Force *MN* now urging the Satellite, being taken from the Force by which *T* draws the Satellite, does not ſo much make it recede from a Duplicate inverſe Ratio of the Diſtances, whilſt the Planet goes from *G* to *B*, and ſo *è contra*. Becauſe the Diſtances *TG*, *TB*, become equal; whence the Effect of this Force taken off is leſs in making the Apſes go forward; that is, the Apſes of the Orb go forward ſlower. Therefore, from theſe two Cauſes, when the Orb is ſituated as here, in reſpect to the Sun, the Apſes of the Orb of a Satellite (whilſt it makes one entire Revolution about its Primary) are carried backward.

Beſides, if the Motion of the Apſes of the Satellite about *T*, and the Motion of the Primary *T* about the Sun *S* be ſuch, that the Apſes of the Satellite's Orbit are nearly in the ſame Aſpect with the Sun for a conſiderable Time, or whilſt the Satellite it ſelf changes all its Aſpects will the Sun often; by reaſon of the length of Time in which the Velocity of the Progreſs and the ſlowneſs of the Regreſs of the Apſes are continu'd, namely, when the Apſes ſtick about the Sun's Syzygies, they will go forward remarkably enough: and likewiſe, by reaſon of the length of time that the Apſes of the Satellite ſtick near the Sun's Quadratures, and conſequently by which the Velocity of the Regreſs and the ſlowneſs of the Progreſs are continu'd, they will recede ſenſibly enough.

But becauſe the extrinſecal Force *MN* (which we take off) that cauſes the Progreſs of the Apſes, has in *Prop.* 9. been ſhewn almoſt twice greater than the Force *AM*, which makes them go back; and beſides, the Apſes ſtick longer in the Syzygies than in the Quadratures, becauſe there being carried *in conſequentia*, they go forward with the Sun, but here being carried *in antecedentia*, they quickly paſs thro' the Sun's Quadrature, which is carried *in antecedentia*; it is plain that the Apſes go forward faſter, and for a longer time in their Syzygies, and go back ſlower and not ſo long in their Quadratures; and by the exceſs of the Progreſs above the Regreſs, in one whole Revolution of the Apſes to the Sun (that is, whilſt the Apſes ſeen from *T* go thro' the Aſpects with the Sun) they are carried *in conſequentia.*

All that has been ſaid will almoſt happen the ſame way, tho' the Species of the Secondary Orbit don't preciſely continue the ſame, if ſo be that the change of the Excentricity is not too great.

PROPOSITION XII.

IF a Satellite move about its Primary in an excentric Orbit, the Excentricity of it will be twice chang'd in every Revolution: And in one Revolution of the Satellite it will be the greateſt, when the Satellite is in the Syzygies with the Sun; but the leaſt, when in the Quadratures: And in the Paſſage of the Satellite from the Quadratures to the Syzygies, it is perpetually encreas'd, and on the other hand, in its Paſſage from the Syzygies to the Quadratures, it is perpetually diminiſh'd.

For if the Body which did deſcribe a given Ellipſis, (namely, by being acted upon by a Centripetal Force, which, tending to one of the Foci or to the Center, is reciprocally proportional to the Square of the Diſtance from it) in its deſcent from the upper to the lower Apſis, be urg'd by a Centripetal Force, which is encreas'd more than in a Duplicate Ratio of the diminiſh'd Altitude or Diſtance from the Center; it is manifeſt, that a Body thus urg'd towards the Center, wou'd be more ſtrongly puſh'd towards it, than if (as before) it was urg'd by a Force encreaſing only in a Duplicate Ratio of the diminiſh'd Diſtance; therefore it wou'd deſcribe an Orb interior to the Elliptic Orbit before deſcrib'd (except the upper Apſis) and in the lower Apſis it wou'd come nearer to the Center than before; that is, it wou'd deſcribe an Orb more excentric than the forementioned Orb, or ſuch an one in which the diſtance of a Body in the upper Apſis has a greater Ratio to its diſtance in the lower Apſis, than in the former Orb: Now if the Force, in the Aſcent of the Body from the lower Apſis ſhou'd decreaſe in the ſame Degrees by which it increas'd in the Deſcent; the Body wou'd return to the former Diſtance. But if the Centripetal Force ſhou'd decreaſe more than in the ſame Ratio, the Body then being leſs attracted, will aſcend to a greater Diſtance, and deſcribe an Orb, whoſe Excentricity is yet greater. Likewiſe, if a Body in its deſcent from the upper to the lower Apſis be urg'd by a Force, which is increas'd leſs than in a Duplicate Ratio of the diminiſh'd Diſtance; it is plain that that Body will deſcribe an Orb exterior to the Elliptic Orb before deſcrib'd, (when the Force was reciprocally as the Square of the Diſtance) except in the upper Apſis, whence it did ſet out; that is leſs Excentric; and that that Excentricity will yet be diminiſh'd, if in the Aſcent of the Body the Centripetal Force decreaſes leſs, or ſlower than before.

Let the Satellite *L* in the former figures and in *fig.* 14. be ſuppoſ'd near the Quadrature in *C*, in which caſe the Force *M N* vaniſhes, and beſides the Force by which the Primary *T* draws the Satellite *L*, which is reciprocally as the Square of the Diſtance, the Satellite will be urg'd by a Force as *A M*, which is as the Diſtance *L T* directly. And the Force made up of them (by *Cor. Prop.* 7.) as you recede from the Center, decreaſes leſs than in a Duplicate Ratio of the encreas'd Diſtance, and as you go to the Center, encreaſes leſs than in a Duplicate Ratio of the diminiſh'd Diſtance: Therefore if the Body, during one whole Revolution, is impell'd by this Force towards the Center, it wou'd deſcribe an Orb leſs excentric than if that extrinſecal Force *A M* was taken away, and it was urg'd only by the Force with which *T* draws *L*, which is inverſly as the Square of the Diſtance; and therefore, in that part of the Revolution where it is impell'd by that Force, it deſcribes a part of an Orbit leſs excentric, than if the ſaid Force *A M* was away.

If the Satellite be in the Syzygies as in *B* or *G*, when the Force *A M* vaniſhes, the Force *M N* is taken away from the Force by which *T* draws *L*; that is, a Force which is as the Diſtance is taken away from a Force reciprocally proportional to the Square of the Diſtance; therefore the remaining Force by which the Satellite *T* is now urg'd (by *Cor. Prop.* 7.) as you approach to the Center, encreaſes more than in a Ratio of the diminiſh'd Diſtance, and decreaſe in the ſame manner as you recede from the Center; and beſides, by what has been ſhewn, if the Satellite, during one whole Revolution, was impell'd towards the Center by that remaining Force, it wou'd deſcribe an Orb more excentric than that which it wou'd deſcribe, if it was urg'd only by the attracting

Force of *T*; and therefore much more excentric than it wou'd deſcribe if it was urg'd by that Centripetal Force which acted upon it when it was in the Quadrature at *C*: Whence in that part of its Revolution when the extrinſecal Force *M N* is the greateſt (that is, in the Syzygies) the Satellite deſcribes a part of an Orbit more excentric than when it is in the Quadratures; or, in other words, the Excentricity of an Orb deſcrib'd by a Satellite, is greater when the Satellite is in the Syzygies, and leſs when it is in the Quadratures.

Beſides, becauſe the extrinſecal additional Force *A M* is greateſt of all, and the Force *M N* is leaſt or none at all, when the Satellite is in the Quadratures, the Force made up of that and of the Force by which *T* draws *L*, is *(cæteris paribus)* greateſt, and very much differs from a Duplicate reciprocal Ratio of the Diſtances; therefore in that caſe, the part of the Orb which is deſcrib'd, will belong to an Orb very little excentric. When the Satellite is gone from *C* towards *B*, the extrinſecal Force *M N*, which is to be taken away, encreaſes alſo, and therefore exerts its effects more and more; that is, makes the Satellite deſcribe part of an Orbit belonging to a more excentric one: wherefore as a Satellite goes from the Quadratures to the Syzygies, its Orbit becomes more and more excentric, till the Satellite being come to the Oppoſition or Conjunction with the Sun, the Orb become moſt excentric of all, the Force *M N* (which at other times is taken off) then exiſting alone, and being greateſt and moſt direct. But afterward as the Satellite goes towards the Quadrature, the Excentricity begins again to diminiſh, becauſe the Force *A M*, which diminiſhes it is en-

encreas'd. Therefore the Excentricity of the Orb, as the Satellite goes from the Quadratures to the Syzygies, is continually encreas'd; and as it goes from the Syzygies to the Quadratures, is continually diminiſh'd: And therefore is the greateſt (*cæteris paribus*) in the Syzygies, and the leaſt in the Quadratures.

PROPOSITION XIII.

THE ſame things being laid down as in the foregoing Prop. *I ſay, that if ſeveral Revolutions of the Satellite be compar'd together, the excentricity of the Orbit of the Satellite is greateſt, when its Apſes are in the Syzygies; and then that it is continually diminiſh'd when the Apſes paſs from the Syzygies to the Quadratures with the Sun, where it becomes the leaſt; but when the Apſes go from the Quadratures to the Syzygies the Excentricity is continually encreas'd.*

This is prov'd the ſame way as the laſt Propoſition but one. For (by *Cor. Prop.* 7.) the Forces of the Satellite in the Quadratures, encreaſe or decreaſe leſs than in a duplicate Ratio of the diminiſh'd or encreas'd Diſtance: Wherefore if the Apſes of an Orb are in the Quadratures, the ſaid Ratio of the whole encreaſe or decreaſe is the leaſt in the paſſage between the Apſes. Likewiſe, the Forces of a Satellite in the Syzygies encreaſe or decreaſe more than in a Duplicate Ratio of the diminiſh'd or encreas'd Diſtance; and therefore if the Apſes of the Orbit happen to be in the Syzygies, the ſaid Ratio of the whole encreaſe or decreaſe of the Forces is greateſt of all between the Apſes; that is, the Ratio of the whole encreaſe or decreaſe of Forces is leaſt when the Apſes are in the Quadratures, and greateſt when they are in the Syzygies;

zygies; and therefore it is continually encreas'd in every Revolution of the Satellite about the Primary, whilſt the Apſes go from the Quadratures to the Syzygies, and continually diminiſh'd whilſt they go from the Syzygies to the Quadratures. But it has been ſhewn by the foregoing *Prop.* that if in each Revolution of a Satellite the Ratio of the encreaſe or decreaſe of the Centripetal Force be augmented, the Excentricity of the Orb will likewiſe be augmented; and on the contrary, the Excentricity will be diminiſh'd, if that Ratio decreaſes: And therefore whilſt the Apſes paſs from the Quadratures to the Syzygies, the Excentricity of the Orb is continually encreas'd, and in their paſſage from the Syzygies to the Quadratures, perpetually diminiſh'd; therefore the greateſt Excentricity of the Orb is when the Apſes are in the Syzygies, and the leaſt when they are in the Quadratures. And by reaſon of the length of the time that the Apſes ſtick near the Quadratures or the Syzygies, the encreſe or decreaſe of the Excentricity becomes very ſenſible, and the inequality between the Excentricity in one caſe, and the Excentricity in the other is the greateſt of all. *Q. E. D.*

SECTION III.

Of the Errors which the Sun produces in the Motion of a Satellite, when the Plane of the Orbit deſcrib'd by it is inclin'd to the Plane of the Orbs deſcrib'd about the Sun by the Primary.

WE have in the firſt Book, treated of the Errors of a Satellite in Latitude in general in the 61ſt *Prop.* and more particularly afterwards in the 64th *Prop.* when we ſhew'd the reaſon of the Preceſſion of the Equinoctitial Points ariſing from the ſame Cauſe. Yet we'll take the liberty to transfer ſome of thoſe things hither, as to their proper place, and explain other things more particular concerning theſe Errors.

PROPOSITION XIV.

IF a Satellite revolves about a Primary Planet, in an Orb whoſe Plane is inclin'd to the Plane of the Primary about the Ecliptic, the Line of the Nodes will, by an angular Motion be mov'd in antecedentia, *with an unequal Velocity; with the greateſt when the Nodes are in the Quadratures with the Sun, afterwards ſlower, when the Nodes being in the Syzygies it is altogether at reſt. In the places that are between the Quadratures and the Syzygies, the Nodes participating of each condition recede ſlower; therefore being always either Retrograde or Stationary in each Revolution of a Satellite about the Primary, they are carried* in antecedentia: *and in the ſame Revolution of a Satellite they go back faſter,* cæteris paribus, *when the Satellite is in the Syzygies.*

Let *S* (as often before) denote the Sun; (*fig.* 14.) *ETE* the Orbit of a Primary Planet *T*; *CBDG* the Orbit of a Secondary *L* about the Primary: Let the Right-lines *SL*, *ST*, be drawn, and if there be occaſion, let 'em be produc'd, and in them be taken the Points *K*, *A*, *M*, *N*, and let *AM* be drawn. It is plain from *Prop.* 60. *Book* 1. that all the Errors of the Satellite *L* ariſe from the Forces of it that are as *AM*, *MN*, and with the direction of thoſe Right lines. Now the Force acting on *L*, according to the Direction of the Right line *AM*, (that is, according to the Right line *LT*) does not diſturb the Motion of *L* in Latitude, when *LT* is in the Plane of the Orb *CBDG*. Beſides, if the direction of another Force as *MN*, lye alſo in the Plane of the Orb *CBDG* (that is, if a Right line drawn thro' *L* and parallel to *MN* lies in a Plane parallel to *CBDG*,) neither will it then diſturb the Motion of the Satellite in Latitude. But *MN* or *ST*, when it is in the Plane of the Orb *SET*, will alſo be in the Plane *CBDG*, if it be the common interſection of thoſe two Planes, namely, of the Primary and Secondary Orb; that is, if the Nodes be in the Syzygies. Therefore the Nodes being in the Syzygies, there will ariſe no Error of the Motion in Latitude, neither from the Force *AM*, nor from the Force *MN*, which are the only Forces that diſturb the Motion of a Satellite; therefore there will be no Error in Latitude. Whence the Line of Nodes and the Nodes alſo in that caſe are at reſt, when their Motion is an Error ariſing from ſome diſturbing Force.

But when the Nodes of the Orbit *CBDG* [*fig.* 15.] are in the Quadratures *C* and *D*, the Force *MN* very much diſturbs the Satellite's

Motion in Latitude; becauſe in that caſe the Line *MN* (and *LF* drawn thro' *L* parallel to it, along which the Force *MN* of the Satellite *L* exerts it ſelf) is moſt of all inclin'd to the Plane of the Orbit *CBDG*, *viz.* as much as the Planes of the Orbs *STE* and *CBDG* are inclin'd to one another. In that caſe let us ſuppoſe one part of the Plane *CBDG*, *viz.* *CGD*, to be rais'd above the Plane of this Scheme (which agrees with the Plane of the Primary Orbit *STE*) and the other *CBD* to be depreſs'd below it: And let the Orbit *CbDg* be ſuppos'd equal, ſimilar, and alike poſited, in reſpect of the Center *T* with the Orb *CBDG*: Therefore in this caſe, as ſoon as the Satellite *L* paſſes the Node *C*, that it may come to the Conjunction with the Sun in *G*, the Force *MN* with the direction of the Right-line *MN* or its parallel *LF*, that is acting from *L* towards *F*, draws it from the Orbit *CGD*, which otherwiſe it wou'd deſcribe, and receding continually towards *S* from the Plane *CGD* until the next Node, it does not now go thro' the Plane *STE* or *CgD* in the Point *D*, (thro' which it wou'd paſs, if that Force was away,) but a little on this ſide of it in reſpect to the Sun *S*; that is, the Node of an Orbit deſcrib'd by a Satellite (when urg'd by the Force *MN*) is nearer to the Sun than the Point *D*, in which it was found in the foregoing Revolution of the Satellite.

Likewiſe, whilſt the Satellite goes from the Node *D*, to its oppoſition with the Sun in *B*, the Force *MN* acting in the Direction *MN* from *M* towards *N*, or from *L* towards *F* makes the Satellite *L* continually recede from the Orbit *DBC*, and deſcribe an Orbit leſs inclin'd to the Plane *SET*, or *DbC*. And ſince this Force on the Satellite *L* exerts it ſelf till it vaniſhes, namely, when the Satellite is come to the Quadrature in *C*, it alſo makes the Orbit truly deſcrib'd by the Satellite ſooner to interſect the Plane *bCgD* than the Orbit *DBCG*, or a little nearer to *b* than the Point *C*, and there makes the Node of the Orbit of a Satellite with that of its Primary. And the ſame will again happen when the Satellite goes the ſecond time thro' the ſame Circle *CGD*; namely, the Node or Point in which the Satellite goes thro' *bCgD*, is *in antecedentiæ* in reſpect of the Point in which it had gone thro' it the time before, and ſo on; that is, this Force *MN* forces the Satellite to deſcribe an Orbit, whoſe Nodes go *in antecedentia* at every Revolution of *L* about *T*; but the Nodes go backward the faſteſt of all when they are in the Quadratures *C* and *D*, becauſe (as it has been before ſhewn) the Right line *MN* or *LF* according to which the Force *MN* acts, which cauſes the Regreſs of the Nodes, is moſt of all Right or Perpendicular in that caſe to the Orbit *CGDB*.

Beſides, what is aſſerted of the Situation of the Nodes between the Quadratures and Syzygies, is ſelf-evident, becauſe the Right line *MN* is in that caſe more oblique to the Plane *GBDG* and therefore has not ſo much Power to diſturb the Satellite from it. And ſince the Regreſs of the Nodes depends upon the Force *MN*, that Regreſs is (*cæteris paribus*) greateſt, when that Force is greateſt, that is, when the Satellite is in the Syzygies. *Q. E. D.*

If the Inclination of the Orb of a Satellite to the Plane of the Orb of the Primary about the Sun be chang'd, the Motion of the Nodes is not ſenſibly chang'd, unleſs the change of the Inclination be too great; for the Force that gives

Motion to the Nodes is not much chang'd in that caſe.

PROPOSITION XV.

THE ſame things being laid down, I ſay that the Inclination of the Plane of the Orb of a Satellite to the Plane of the Orb in which the Primary revolves about the Sun, is continually chang'd, and is the greateſt when the Nodes are in the Syzygies with the Sun; and the leaſt (cæteris paribus) *when they are in the Quadratures: And beſides, that in this caſe the ſaid Inclination is diminiſh'd whilſt the Satellite paſſes from the Quadratures to the Syzygies, and that it is again augmented whilſt the Satellite paſſes from the Syzygies to the Quadratures. Whence it happens, that when the Satellite is in the Syzygies, the Inclination of the Planes is the leaſt, and returns nearly to the former Quantity when the Satellite is come to the next Node. I ſay beſides, that when the Nodes paſs from the Syzygies to the Quadratures, that Inclination of the Planes diminiſhes, and becomes leaſt of all* (cæteris paribus) *when the Nodes are in the Quadratures; and then that it encreaſes by the ſame degrees that it decreas'd before, and the Nodes being again return'd to the Syzygies, it returns to the firſt Magnitude.*

Tho' all the Parts of this Propoſition follow eaſily enough from the 61ſt *Prop. B.* 1. yet we ſhall in this place explain them more fully, and demonſtrate each of them by a particular Scheme.

Firſt, let the Nodes of the Satellite *L* [*fig.* 16.] be in the Syzygies with the Sun *S*; that is, let the Line of the Nodes produc'd paſs thro' the Sun; in which caſe the Right line *M N* (which has been demonſtrated in *Prop.* 61. *B.* 1. to ſhew by its Poſition the Direction, and by its Magnitude the Quantity of that Force, which only af-

affects the Motion of a Satellite in Latitude) is in the Plane of the Orbit of the Satellite *L*: And therefore howſoever it diſturbs the Motion of the Satellite in the ſaid Plane, it does not impel it to any Motion out of the ſaid Plane *BCGD*; that is, it does not at all affect the Motion of the Satellite in Latitude, but leaves the Satellite undiſturb'd as to its Motion in Latitude, to deſcribe his Orbit about its proper Primary. And as that diſturbing Force *MN* by its nature always diminiſhes the Inclination of the Orb of the Satellite to the Orb of the Primary, and never encreaſes it but by accident, upon account of its making the Nodes to go *in antecedentia*; it is plain that the Inclination of the ſaid Orbs is greateſt of all, when the Nodes are in the Syzygies.

Now ſuppoſing the Nodes to be in Quadrature with the Sun, as ſoon as the Satellite has paſs'd the Node *C* or *D*, being impell'd by the Force *MN*, and according to the direction *LF*, from *L* to *F*, it is continually drawn from the Plane of the Orbit *CGDB* (which, without this, it wou'd deſcribe) and deſcribes an Orbit, whoſe Plane is leſs inclin'd to the Plane of the Orbit of the Primary *CgDb*. That this may appear the plainer, let *ST* repreſent (in *fig.* 17. the Eye being ſuppos'd in the Line of the Nodes) the Plane of the Primary's Orbit about the Sun, *TL* the Plane of the Secondary inclin'd to the Plane *ST* by the Angle *LTS*, or the Complement of it to two Right. The Satellite at *L* drawn by the Force *MN* from *L* towards *F*, that is, from the place *L* to the place *l*, deſcribes an Orbit, whoſe Plane *Tl* is inclin'd by a leſs Angle *lTS* to *ST* the Plane of the Orbit of the Primary about the Sun. And this change of the Inclination of the Orb of a Satellite to the Orb of the Primary is (*cæteris paribus*) by ſo much greater as the Force *M N* which produces it is greater: But the Force *M N* is greater the nearer the Satellite is to the Syzygies *G* and *B*. Therefore the Nodes ſticking at the Sun's Quadratures, the Inclination of the Orb of the Satellite is the leaſt, when the Satellite is in the Syzygies. Again, becauſe the Right line *M N* and its parallel *L F* (*viz.* the Direction of the Force which diſturbs and diminiſhes the Inclination) is nearer to a Perpendicular to the Plane of the Orbit of the Satellite when the Nodes are in the Quadratures, than in any other Situation of the Nodes; in that caſe, the change of the Inclination will be the greateſt, therefore the Inclination of the Planes will be the leaſt: For the nearer the Angle *F L T* comes to a Right, the greater Angle does the little Right line *L l* (of a given Length) ſubtend. Therefore the Inclination of the Orbit of a Satellite to the Orbit of its Primary Planet will be leaſt of all, when the Nodes are in the Quadratures and the Satellite at the ſame time in the Syzygies.

Moreover, let *S T b* (*fig.* 18. the Eye being ſuppos'd in the Line of the Nodes produc'd, as before) repreſent the Plane of the Orbit of the Primary *T* revolving about the Sun; *G T B* the Plane of the Orbit of a Satellite, whilſt it is inclin'd to the former Orb by this ſmall Angle, (*viz. G T g* or *B T b*) which happens (as has been ſhewn) when the Satellite in the Syzygies with the Sun, is ſeen from *T*. And ſince it has been ſhewn, in the foregoing *Prop.* that the Nodes of the Orbit of a Satellite go *in antecedentia*, the Orbit which the Satellite, going from *G*, (when ſeen from T, it was in Conjunction with the Sun) deſcribes, will meet with STB on this ſide T in reſpect of the Sun, and will be the Line Gd; and the Orbit which it deſcribes going from B, where it was in Oppoſition with the Sun, *viz.* the Line $B\delta$, will meet with the Plane STb at δ, *viz.* beyond T, in reſpect of the Sun. Now it is plain that the Line Gd or $B\delta$ contains a greater Angle with STb to d and δ, than that which GT or BT contains with the ſame Plane STb; *viz.* about equal to that which the Orbit of the Satellite contain'd with the Plane STb, when the Satellite was near a Node. Therefore when the Nodes are in the Quadratures, the Inclination of the Plane of the Orbit of the Satellite diminiſhes, as the Satellite paſſes from the Quadratures to the Syzygies; and on the other hand, encreaſes as the Satellite paſſes from the Syzygies to the Quadratures; and returns to the firſt Quantity when the Satellite comes to the next Node.

Now, ſince it has been ſhewn that the Force MN (diſturbing the Satellite's Motion in Latitude, and eſpecially diminiſhing the Inclination of the Plane of the Orbit of the Satellite to the Plane of the Orbit of the Primary) is the greateſt when the Satellite is in the Syzygies, and leaſt (becauſe none at all) when it is in the Quadratures, and that it does moſt powerfully produce this Effect, (*cæteris paribus*) when the Satellite is in the *Limit*; it is plain that by how much the *Limits* are neareſt to the Quadratures when the Satellite is in the Syzygies, by ſo much leſs does this Force MN diminiſh the Inclination of the ſaid Planes, becauſe its Quantity is none when the Satellite is in a ſituation which is the moſt convenient for the producing of this Effect, and its Quantity is greateſt when by reaſon of the Satellite's ſituation, this Effect ceaſes. And on the other hand, ſince it has been lately ſhewn that the Force *MN*, by bringing round the Nodes *in antecedentia* does again increaſe the ſaid Inclination, and very powerfully encreaſe it when the Satellite is near either of the Nodes, and that in the mean time the Force it ſelf is greateſt when the Satellite is in the Syzygies; it is plain that by how much they are nearer to the Syzygies with the Sun, by ſo much more does the Force *M N* encreaſe the Inclination of the Planes. And therefore ſince the *Limits* are equally near to or remov'd from the Quadratures, as the Nodes are to or from the Syzygies, it is plain that by how much the Nodes are nearer to the Syzygies, by ſo much is the Inclination of the Plane of the Orbit of the Satellite, to the Plane of the Orbit of the Primary greater, and ſo much leſs as the Nodes are nearer to the Quadratures. And becauſe (by the foregoing *Prop.*) the Nodes are carried *in antecedentia*, this inclination of the Planes is diminiſh'd, whilſt the Nodes paſs from the Syzygies to the Quadratures, and becomes leaſt of all (*cæteris paribus*) when the Nodes are in the Quadratures; then it encreaſes by the ſame Degrees by which it had before decreas'd, and the Nodes being again got to the Syzygies, it returns to its firſt Quantity. *Q. E. D.*

PROPOSITION XVI.

ALL the Inequalities before deſcrib'd, which happen in the Motion of a Satellite, as well thoſe that affect its Motion in length, as thoſe that affect it in Breadth or Latitude, as well thoſe which happen to a Concentric, as thoſe that happen to an Excentric Orbit, are ſomething greater in the Conjunction of a Satellite with the Sun, than in the Oppoſition.

Becauſe (by *Prop.* 25. *El.* 5.) the Sum of the greateſt and leaſt of Four Quantities Geometrically Proportional, always exceeds the Sum of the Two others; by taking away on either ſide the Sum of the Two leaſt, the Interval of the greateſt and next leaſt will exceed the Interval of the next greateſt and the leaſt. And becauſe (by the Conſtruction of *Prop.* 14.) the Satellite [*Fig.* 19.] being in Conjunction with the Sun at *G*, *S M* is to *S N*, (by reaſon of the ſmalneſs of *N T* in reſpect of *S N*) that is to *S T* in a Duplicate Ratio of *S T* to *S G*; the Right line *M T* exceeds the Double of *T G*: For that which is next to the greateſt exceeds *S B*. Again, ſince when the Satellite is in Oppoſition, as at *B*, *S m* is to *S T*, in a Duplicate Ratio of *S T* to *S B*; the Right line *T m* (for the ſame reaſon) wants of being equal to twice *T B*: therefore *T M* much more will want of being equal to *T m*, that is, *N M* exceeds *N m*. But *N M*, *N m* repreſent the Forces reſpectively diſturbing the Satellite in the Conjunction and Oppoſition; and therefore the Effects of the former are greater than the Effects of the latter. And ſince beſides the Forces repreſented by *N M*, *N m* (which are the only Forces that diſturb the Motion of the Satellite when it is in the very Conjunction and Oppoſition) and that another Force, as AM (which diſturbs the Satellite near the Syzygies) is greater in the Conjunction than in the Oppoſition, *viz.* in that greater than the Radius LT, and in this leſs; it is evident that all the Inequalities explain'd in the foregoing *Prop.* are (*cæteris paribus*) ſomething greater near the Conjunction of a Satellite with the Sun, than near the Oppoſition with it. *Q. E. D.*

SCHOLIUM.

The foregoing Propoſition wou'd always hold true, if the Orbit of the Satellite was concentric with the Primary; otherwiſe it wou'd only hold in an entire Revolution of the Apſes about the Sun. But the Motions of the Satellites when nicely obſerv'd, ſeem to ſhew that the Center of the Circular Orbit of a Satellite is ſomething farther from the Sun than the Center of the Primary Planet; to the intent, that the Satellite may all along accompany the Primary Planet. For if the Center of the Satellite's Orbit was the ſame as the Center of the Primary Planet, the Satellite leaving the Primary Planet wou'd fall into the Sun. For the Satellite deſcribing its Orbit, makes that Orbit to be look'd upon as an heavy Ring, and the gravity of each part of that Ring is to be look'd upon as the ſame with the gravity of the Satellite in it, and that place to be reckon'd heavy which is the Center of Gravity of it. And ſince by *Prop.* 16. the accelerating gravity of a Satellite towards the Sun, over and above the accelerating Gravity of the Primary towards it, is more encreas'd in the inferior part of its Orbit near the Conjunction, than it is diminiſh'd in the ſuperior part of it near the Oppoſition, and the Satellite dwells as long in the one as in the other; the

Center of Gravity of the ſaid heavy Ring or Orbit, and therefore the Orbit itſelf is urg'd by a greater accelerating gravity to the Sun than the Primary; therefore leaving the Primary it will come nearer to the Sun: which alſo will much more be true, if the Center of the Circular Orbit be nearer to the Sun than the Center of the Primary. But the Center of the Orbit is ſo plac'd by the All-wiſe CREATOR according to the Diſtance of each Primary from the Sun, that the Center of Gravity of the ſaid Ring (look'd upon as heavy) be as far diſtant from the Sun as the Center of the Primary Planet, leſt the one ſhou'd be left by the other: this alſo holds good in Excentric Orbs, the gravity of each ſmall part of the Orbit being eſtimated by the time that the Satellite ſtays in it.

SECTION IV.

Of the Errors which the Sun produces in the Motion of a Satellite, when the Primary Planet is mov'd in an Excentric Orbit about the Sun.

PROPOSITION XVII.

IF by reaſon of the Diſtance being encreas'd and diminiſh'd by turns between the Sun S *and a Primary Planet* T, *the action of* S *be encreas'd and diminiſh'd by turns;* L T *the Radius of the Orbit of the Satellite will at the ſame time be encreas'd and diminiſh'd, and the Periodical Time of the Satellite about the Primary will be encreas'd and diminiſh'd in a Ratio compounded of a Seſquiplicate Ratio of the Radius, and the Subduplicate Ratio by which the attracting Force* T *of the Primary Planet is diminiſh'd or augmented by the encreaſe or decreaſe of the Sun's action.* [Fig. 20.]

The Force with which the Primary *T* draws the Satellite *L* (namely that by which *L* is kept in its Orb *B C G D*) is encreas'd (as it has been often ſhewn above) when *L* is in the Quadratures *C* or *D* by the addition of an extrinſecal Force ariſing from the Sun *S*, which is as *A M*; and whilſt *L* is in the Syzygies, that attracting Force of the Body *T* is diminiſh'd, by taking away a Force which is as *M N*. And becauſe *M N* perpendicular in the Syzygies (by *Prop.* 9.) has been ſhewn as it were twice greater than *A M* in the Quadratures, by reaſon of theſe Extrinſecal Forces ariſing from the Sun, the attracting Force of the Primary is more leſs diminiſh'd than it is encreas'd; and therefore it muſt ſimply be look'd upon as diminiſh'd. And becauſe the attracting Force of the Body *T* is that by which *L* is kept in its Orbit; if it ſhou'd decreaſe by degrees, the Satellite *L* wou'd (being leſs and leſs attracted) continually recede from *T*; and on the contrary, if that Force was encreas'd in the Primary, then it wou'd approach nearer to *T*. Therefore the Action of *S*, the Sun's Body, being encreas'd as *T* comes nearer to it, the *Ablatitious* Force (or Force which is to be taken away) will be encreas'd, and conſequently the attracting Force of the Primary *T* will be diminiſh'd by degrees, and for that reaſon *TL* the diſtance of the Satellite will be encreas'd; and on the contrary, the Action of *S* it ſelf being diminiſh'd by the Receſs of the Primary *T* from it, *MN* will be diminiſh'd at the ſame time; and again, *T* its attracting Force will be encreas'd, and the Satellite will again approach the Primary. And theſe things will happen by turns: namely the Orbit of the Satellite will be dilated whilſt the Primary is in the *Perihelium*, and will be contracted again when it returns to the *Aphelium*. Beſides, the Force in the Primary which draws *L* poſited at different Diſtances, or the Centripetal Force of the Satellite *L* towards *T*, is (by *Prop.* 26. *B.* 1.) in a Ratio compounded of the ſimple Ratio of the Radii directly, and the Duplicate Ratio of the Periodical Time of *L* about *T* inverſly: therefore if the Radius remains the ſame, the inverſe Duplicate Ratio of the Periodical Time is the ſame with the Ratio of the Centripetal Force of the Satellite *L*; that is, the Square of the Periodic Time is encreas'd in the ſame Ratio in which that Centripetal Force is diminiſh'd, or the Periodical Time it ſelf is encreas'd in the Subduplicate Ratio in which the ſaid Centripetal Force is diminiſh'd; and on the contrary, the Periodical Time is diminiſh'd in the Subduplicate Ratio in which the Centripetal Force is encreas'd. And beſides, as the Law of Attraction in the Primary *T*, or of the Centripetal Force in the Satellite *L*, is ſuch, as to be reciprocally as the Square of the Radius *TL*, and the extrinſecal *Ablatitious* Force be in reſpect of it very ſmall; the remaining Force by which *L* does yet tend toward *T*, is almoſt in a Duplicate Ratio of the Radii inverſly; therefore (by *Prop.* 28. *B.* 1.) the ſame abſolute attractive Force remaining in *T*, (tho' the Radius be any way chang'd) the Squares of the Periodical Times are as the Cubes of the Radii, and therefore the Periodical Times are in a ſeſquiplicate Ratio of the Radii. Therefore if neither the attractive Force remains the ſame in the Primary *T*, but is by turns encreas'd and diminiſh'd by the Action of the Sun, nor the Radius *TL* continues the ſame, (which we have before ſhewn to be liable to change) the Periodical time will alſo be encreas'd and diminiſh'd in a Ratio compounded of the two former Ratio's: that is, if the Action of the Sun *S*, by which the attracting Force of the Primary *T* is diminiſh'd, be diminiſh'd and encreas'd by turns; likewiſe the Periodical Time will be encreas'd and diminiſh'd in a Ratio compounded of the Seſquiplicate Ratio of the Radii, and the Subduplicate Ratio in which the attracting Force of the Body *T* is diminiſh'd or encreas'd. *Q. E. D.*

SCHOLIUM.

If the Force attracting the Satellite *T* be augmented any other way than by an extrinſecal

Force ariſing from the Sun, as for Ex. if by a new acceſſion of Matter the Primary becomes bigger, and for that cauſe (by *Prop.* 49. *B.* 1.) its Attraction becomes great in that proportion; the Satellite *L* will both revolve in a leſs Orbit with the diminiſh'd Radius *TL*, and its Periodical Time will be diminiſh'd in a Ratio compounded of the Seſquiplicate Ratio of the Radii directly, and a ſubduplicate Ratio of the attracting Forces of the Body *T* inverſly; as before: likewiſe, if the Primary be diminiſh'd by the taking away ſome of its Matter, the periodical Time of the Satellite will be encreas'd. For the compound or the remaining Force, by which the Primary being encreas'd or diminiſh'd draws the Satellite, is preciſely in a Duplicate Ratio of the Diſtance inverſly, which only holds good in the caſe of the foregoing *Prop.* The ſame will happen in reſpect of a Primary, which has been here ſhewn of a Secondary, if the Sun ſhou'd in any wiſe be encreas'd or diminiſh'd.

PROPOSITION XVIII.

THE ſame things being laid down, in a dilated or contracted Syſtem of a Satellite, the Periodical Effects of the diſturbing Forces, or the Linear Errors of a Satellite in any Revolution, are as the Radii of the Orbits and the Squares of the Periodical Times joyntly: But the Angular Errors of it ſeen from the Center of the Primary, are as the ſaid Squares of the Periodical Times. [fig. 21.]

For the diſturbing Forces, or the Forces which cauſe Errors, *viz.* *AM*, *MN* (other things being as before) are as the Radius *TL*; and therefore the Periodical (Errors or) Effects of 'em, are (by *Cor. Prop.* 23. *B.* 1.) as the Forces and the Squares of the Periodic Times jointly.

But theſe Errors are Linear Errors, namely, meaſur'd by the Diſtances of Bodies from ſimilar Figures in thoſe places, to which the Bodies in the ſame proportional Times wou'd come without thoſe Forces. And for that reaſon the angular Errors ſeen from the Center *T*; that is, the Angles under which the ſaid Linear Errors appear from the ſaid Center, are in every Revolution of the Satellite *L* as the Squares of the Times of the Revolution nearly; the other of the compounding Ratio's (*viz.* the Ratio of the Radii) vaniſhing, becauſe the Linear Errors are ſeen at the diſtance of the Radii.

SCHOLIUM.

Tho' all the angular Errors in the Motion of a Satellite, ſeen from the Center of the Primary, are chang'd as the Satellite's Period is chang'd; namely, the Motion of the Apſes or Apſides, the Regreſs of the Nodes, and the other before-mention'd Deviations, as well in Longitude or Length, as in Latitude, ariſing from the extrinſecal Forces of the Sun; yet, as in one Period of a Satellite, ſome of thoſe Errors are ſcarce ſenſible, and not remarkable till after ſeveral Revolutions, and ſome Errors compenſate for others, therefore they have been the leſs known to Aſtronomers; at leaſt one Error deſcrib'd in *Prop.* 1. by which the Motion of a Satellite in the Quadrants of its Orbit before the Syzygies is accelerated, and retarded in the Quadrants after them, which happens every Revolution, or every half Revolution: for in the Quadratures of the Sun this Error ceaſes, the Cauſe ceaſing. Therefore this Error, as it is deduc'd from the ſaid *Prop.*. 1. or rather as it is corrected by *Cor. Prop.* 2. by this 18th *Prop.* (its Periodical Time being encreas'd or diminiſh'd) in one of its whole Periods is encreas'd or diminiſh'd in a Duplicate Ratio of the Time between the Quadratures. But the time between the Quadratures does not depend upon the Period alone, but is greater or leſs according as the Satellite's place is near to the upper or lower Apſis in its proper Orbit.

PROPOSITION XIX.

If a Primary Planet be mov'd about the Sun at a great diſtance from it in an excentric Orbit, the Forces of the Sun diſturbing the Motion of a Satellite and their Effects, viz. *all the above deſcrib'd Errors in Longitude, Latitude, and Altitude, are reciprocally as the Cubes of the Diſtances of the Primary from the Sun; that is, directly as the Cubes of the apparent Diameters of the Sun ſeen from the Primary.* [Fig. 22.]

Becauſe the Sun is ſuppos'd at a great diſtance, the diſtance between the Points *T* and *N* in compariſon of *ST*, vaniſhes, or *SN*, *ST* are equal. And ſince the Excentricity of the Orbit *CBDG* (if it has any) being compar'd with *TS* will alſo vaniſh, *SK* will likewiſe become equal to *ST*. Beſides, *TS* being encreas'd or diminiſh'd, the Orbit *CBDG* together with the Lines *TL*, *MN*, *AM*, &c. that depend upon it, are encreas'd and diminiſh'd by the foregoing *Prop.* And indeed, by reaſon of the great diſtance of the Sun *S*, they are all encreas'd or diminiſh'd in the ſame proportion; that is, the the Right line *AM* (which in its middle Quantity is equal to *TL*, becauſe when the Satellite is in Conjunction with the Sun it becomes greater, and about the Oppoſition leſs) is to *LT* as *SK* to *ST*; therefore a Force as *AM* is to a Force as *SK* in the ſame Ratio as *LT* is to *ST*. Likewiſe

wiſe, a Force as *MN* is to a Force as *SK* almoſt as *LT* to *ST*; that is, the Forces as *MN* *AM*, are nearly as the Force *SK* and the Ratio of *LT* to *ST* jointly; that is, as the Force $SK \times \frac{LT}{ST}$. And as the Force conſtantly accelerating towards the Sun's Body is (by *Prop.* 42. *B.* 1.) inverſly, as the Square of the diſtance from it; that is, in the Diſtance *SK* as $\frac{1}{SK^q}$; inſtead of the Force *SK* putting $\frac{1}{SK^q}$ the Forces *AM*, *MN*, will be as $\frac{1}{SK^q} \times \frac{LT}{ST}$; that is, as $\frac{LT}{SK^q \times ST}$. But it has been before ſhewn, that *SK* is in that caſe equal to *ST*: Whence the the Forces *AM*, *MN* are as $\frac{LT}{ST\ cub.}$. Now if *LT* be alſo look'd upon as conſtant and unmovable, the diſturbing Forces will be as *AM*, *MN*, therefore the Effects of thoſe Forces, namely, all the Errors (of which we have before made mention) will be as $\frac{1}{ST\ cub.}$, that is, reciprocally as the Cube of the Diſtance of the Primary from the Sun. And it is commonly known that the apparent Diameters of a Sphere of a given Magnitude are inverſly as the Diſtances; and therefore the ſaid diſturbing Forces, and the Errors of a Satellite ariſing from it, are as the Cubes of the apparent Diameters of the Sun ſeen from the Primary. *Q. E. D.*

PROPOSITION XX.

TO *find the Ratio between the Sun's Forces, by which the Motion of a Satellite is diſturb'd, and the Forces by which a Satellite is retain'd in its Orbit about the Primary.* [Fig. 23.]

It has been ſhewn in the foregoing Propoſitions, what Ratio the Forces of the Sun diſturbing the Motion of a Satellite have to the Force by which the Sun draws the Primary, or by which the Primary is retain'd in its Orbit about the Sun: namely, that which the Right lines *A M*, *M N*, have reſpectively to *S N*, or *A M*, *M T* (the Points *N* and *T* coming into one) to *S T*. What remains is to determine the Ratio between the ſaid diſturbing Forces, and the Force by which the Satellite *L* is retain'd in its Orbit *C B D G* about its Primary *T*. The Right line *A M* in its middle Quantity is equal to *T L*; wherefore the Mean Force as *A M*, is to the Force *S N* or *S T*, by which *T* is retain'd in its Orb, as *T L* to *T S*: And (by *Prop.* 26. *B.* 1.) the Force by which *T* is retain'd in its Orbit *E T E*, is to the Force, by which *L* is retain'd in its Orbit *C B D G*, in a Ratio compounded of the Ratio of *S T* to *T L* and a Duplicate Ratio of the Periodical Time of the Satellite about the Primary to the Periodical Time of the Primary about the Sun. And therefore, *ex æquo*, the foreſaid Mean Force *A M* is to the Force, by which *L* is detain'd in its Orbit, in a Ratio compounded of the Ratio of *T L* to *T S*, *S T* to *T L* and a Duplicate Ratio of the Periodical Time of the Satellite about the Primary to the Periodical Time of the Primary about the Sun; that is, (the two foregoing Ratios taking off each other) the Mean Force *A M* is to the

Force, by which *L* is retain'd in its Orbit about *T*, is in the foreſaid Duplicate Ratio of the Periodical Times. If the Ratio between *MN* and *AM* be found out, let the ratio of each Force diſturbing *AM* and *MN* to the Force by which *L* is retain'd in its Orbit will be alſo found. Nay, you may find the Force *AM* not only in its Mean Quantity, but very exactly according to the various poſition of the Satellite *L*, and from it nicely determine the ratio between the diſturbing Forces and the Force, by which *L* is return'd in its Orbit.

COROLLARY.

If the Sun be ſuppos'd at a great Diſtance, as it happens in the Circumſolar Planets, at leaſt in thoſe that have Satellites; *AM* will always be equal to *TL*, the Right lines *MT*, *AL*, being parallel; and *MN* or *MT* equal to *AL*. But becauſe *SA* is to *SK* in a Duplicate Ratio of *SK* to *SL*, and *SL* infinite in reſpect of *KL*; *AK* (the Difference of *AS* and *SK*) will be double of *KL* (the Difference of *SK* and *SL*) and *AL* the triple of it: But *KL* is the Right Sine of the Arc *CL*, namely of the Diſtance of the Satellite from the next Quadrature *C*, ſuppoſing *TL* the Radius. Therefore in that caſe you have always the Ratio of the Force *AM* to *MN* given, *viz.* that of the Radius to the triple of the Right Sine of the Satellite's Diſtance from the Quadrature.

SCHOLIUM.

Much after the ſame manner that the Sun, plac'd without the Orbit of a Satellite, diſturbs its Motion, and that ſeveral ways, as it appears from ſeveral Propoſitions of this Book; the Superior Planets will diſturb the Motion of the Inferior, and the Comets will diſturb the Motion of all the Planets. For the afore deſcrib'd Diſturbance of the Satellites ariſing from the Sun, is owing to the Gravitation of each Satellite towards the Sun, by which (amongſt other things) it happens that the Gravitation of a Satellite towards its Primary is no longer in a Duplicate reciprocal Ratio of the Diſtance from it. And the Actions of the Planets on one another, will produce the like Effects, tho' far leſs ſenſible, by reaſon of the ſmallneſs of their Bodies when compar'd with the Sun, and their immenſe Diſtances. Yet theſe Effects (how ſmall ſoever) will be produc'd, and if they continue, and are always directed the ſame way, they will at length become ſenſible: for Example, it is found that the Apſes of the Earth's Orbit do after ſeveral years, ſenſibly appear to have been carried *in conſequentia*, tho' this Motion be very ſmall, if it be compar'd with that of the Apſes of the Moon *in conſequentia*; becauſe in twenty years they don't move quite four minutes and a half, whilſt the Apſes of the Moon in the ſame time have made more than two whole Revolutions and a quarter.

Beſides, the Diſturbance of the Motion of the Planets, ariſing from other Planets and Comets will be very unequal, becauſe the Diſtances of the diſturbing Bodies are very unequal. But as the Effects of all this Diſturbance are ſmall, their inequalities will be almoſt inſenſible; and the Mean or middle Quantity of each will ſuffice for Aſtronomical Calculations.

SECTION V.

Concerning the Motion of the Moon, ſeen from the Earth.

IT has been ſhewn at large, in the foregoing Propoſitions, what the Errors produc'd by the Sun in the Motion of the Secondary Planets are; of all which Errors we will give an Example in the Moon, the Earth's Satellite, and ſummarily ſhew by what Inequalites it is affected; how they are obſerv'd and diſtinguiſh'd from each other, and alſo how they may by Computation be trac'd from their Springs; namely, the Sun's Forces which cauſe them; and laſt of all we ſhall give a Method for the calculating and making uſe of Tables of the Moon's Motions, where it will appear, by the by, what Errors the Conſtructors of Tables have hitherto taken notice of, and what Errors they have paſs'd by.

POPOSITION XXI.

TO deſcribe and explain the Motion of the Moon and its Inequalities hitherto obſerv'd, whether diſtinguiſh'd or mix'd with others, from what has been ſaid and prov'd.

That the Earth, which is a great Body and a Primary Planet, as it is carried about the Sun, can (whilſt it ſelf is carried about the Sun) carry the leſſer Body of the Moon, revolving about it under the form of a Satellite, in an Orb any way inclin'd to the Plane of the Ecliptic in which the Earth is carried about the Sun; and that the Moon can revolve in an Ellipſis having a Focus nearly in the Earth's Center; is plain from the 59th *Prop.* of *B.* 1. and its Corollaries: And theſe are *Phænomena* which every Body plainly ſees; for from them the Phaſes of the Moon and the Eclipſes of the Luminaries are explain'd in the 16th, 18th, and 19th *Prop.* of *B.* 1. But the Motion will be very much diſturb'd by the Action of the Sun and the Moon, and affected with theſe Inequalities which a conſtant and more careful Obſervation has diſcover'd, whoſe Effect will nevertheleſs almoſt vaniſh, if they be compar'd with the firſt Inequalities; whence it happens that in the forementioned Propoſitions the Motion of a Satellite about its Primary, is ſuppos'd very like the Motion of a Primary about the Sun, tho' upon a nice Obſervation it differs a little in ſome things.

For firſt of all, if it was not for the Sun's Action, the Moon wou'd be ſo mov'd about the Earth, as to deſcribe Areas proportionable to the times, by a Radius drawn to the Earth: But by the ſaid action it happens (by *Prop.* 1.) that it deſcribes an Area greater for the time (that is, that it is carried ſwifter) in the Syzygies with the Sun, than in its Quadratures. And therefore this Error is alſo chang'd, and in every Period of it in a Duplicate Ratio of the time between the Quadratures, by *Schol.* of *Prop.* 18.

Secondly, if the Sun's Action did not diſturb the Moon's Motion, the Moon would deſcribe an Ellipſis having a Focus in the Earth's Center: But now it is plain by *Prop.* 2. that the Orbit of the Moon will (*cæteris paribus*) become leſs curve in the Syzygies, and more in the Quadratures; and therefore that it is nearer the Earth in the firſt than in laſt.

Thirdly, the Moon wou'd conſtantly deſcribe an unmov'd Ellipſis about the Earth, if the Sun did not diſturb it; but now from its diſturbance it happens, that this Orbit is very far from being an Ellipſis; neither can it be reduc'd to one but by the artful Induſtry of the Aſtronomers. For its Orb is a Curve, which is generated of an Ellipſis mov'd about an unmov'd Focus or Center of the Earth, its greater Axis in the mean time vibrating with angular Motion ſometimes forward, ſometimes backward; as it is plain from *Prop.* 8. But the unmovable Curve *AEGH* (in *fig.* 3.) is truly the Moon's Orbit, which is no more to be reckon'd an Ellipſis than an Helix is a Right line, becauſe it is generated of a Point moving in a Right line, whilſt the Right-line is carried about by an angular Motion. But if out of reſpect to the old Aſtronomers (who are to be valu'd) we call the Orbit of a Planet an Ellipſis, the Line of its Apſides (by *Prop.* 8.) does in each Lunation go forward twice, namely, when the Moon is in the Syzygies, and goes back twice when the Moon is in the Quadratures; and this Progreſs (by *Prop.* 11.) is greater than the Regreſs, when the Moon's *Apogæum* and *Perigæum* is in the Syzygies, and on the other hand the Regreſs does exceed the Progreſs when it is in the Quadratures. Therefore in the former Caſe, if you obſerve one whole Revolution of the Moon, it will ſimply appear that the *Apogæum* goes forward; but in the latter it goes back: but as the firſt Caſe is moſt laſting in one whole Revolution of the Apſes about the Sun, they go forward ſimply.

Fourthly, If the Sun's action had no power to diſturb the Moon's Motion, the Moon wou'd always deſcribe the ſame Ellipſis about the

Earth; whereas now it continually changes its ſhape; for the Excentricity of the Orbit is every Day chang'd, and (by *Prop.* 12.) it is greateſt in every Revolution when the Moon is in the Syzygies, and leaſt when the Moon is in the Quadratures. But if the Moon's *Apogæum* and *Perigæum* be in the Syzygies, and the Moon be at the ſame time in the *Apogæum*, that is, if the Excentricity be greateſt of all; the Moon in the Syzygies will be both moſt remote and moſt ſlow, contrary to what was ſaid in the firſt and ſecond Paragr. of this *Prop.* But as there we abſtracted from that Excentricity, and we only ſay that, *cæteris paribus*, what was there ſaid holds good, what we ſay now does not contradict it. But even in this caſe the Moon does not move ſo ſlowly neither does the Excentricity of the Orbit become ſo great as it would be, if the Errors explain'd in the ſaid Paragraphs were away.

Fifthly, if the Sun did not diſturb the Moon's Motion, the Moon wou'd deſcribe an Orbit about the Earth in an unmov'd Plane, as a Primary Planet does about the Sun; which appears from *Prop.* 11. *B.* 1. to be the property of the Way of a Projectile: But the Sun cauſes the Poſition of this Plane to vary differently, either by the change of its common Section or Line of the Nodes with the Plane of the Ecliptic, or of the Inclination it ſelf with the fix'd Plane of the Ecliptic. And by *Prop.* 14. in one of the Moon's Revolutions about the Earth, the Nodes go back very faſt when the Moon is in the Syzygies, and very ſlowly when it is in the Quadratures; and if ſeveral Revolutions be compar'd, the Regreſs of the Nodes is the ſwifteſt when they are in Quadrature the Sun, and ſloweſt (becauſe none) when in the Syzygies.

Sixthly, the Inclination of the Moon's Orbit to the Plane of the Ecliptic is alſo changed. For (by *Prop.* 15.) in the ſame Revolution of the Moon this Inclination is the leaſt when the Moon is in the Syzygies, and greateſt when it is in the Quadratures; but if ſeveral Revolutions be compared together, it is leaſt when the Nodes are in the Quadratures, and greateſt when they are in the Syzygies.

Seventhly, all the afore deſcrib'd Errors ariſing from the Sun's Action produc'd in any ſmall time; that is, the Velocities or Horary Motions of the *Apogæum*, Nodes, *&c.* and of the Moon are chang'd according to the Earth's Diſtance from the Sun, which cauſes thoſe Errors, and that (by *Prop.* 19.) in a Triplicate Ratio of the Diſtance of the Earth inverſly, or in a Triplicate Ratio of the apparent Diameter of the Sun ſeen from the Earth.

Eighthly, the Periodical Time of the Moon varies alſo according to the Earth's different Diſtance from the Sun: For it is leaſt (by *Prop.* 17.) when the Earth is in the *Aphelium*; and greateſt when the Earth is in its *Perihelium*. And therefore, beſides all the Inequalities above mention'd in the Moon's Motion, which happen at thoſe times, the Moon's Mean Motion it ſelf is ſwifter when the Earth is in the *Aphelium*, than when it is in the *Perihelium*, and proportionably ſlower or ſwifter as the Earth is nearer or farther from it.

Perhaps there are ſome other Inequalities not yet obſerv'd (or at leaſt not yet diſtinguiſh'd from the others) in the Moon's Motion by which it is ſo diſturb'd, that it can no way be reduc'd to a certain Rule; *viz.* thoſe of which mention is made in the *Schol.* of *Prop.* 18. *&c.* After theſe which are enough known, the reſt will more eaſily be obſerv'd, and their cauſes deduc'd from the Principles before laid down.

PROPOSITION XXII.

TO find the Diſtance of the Moon from the Center of the Earth, by one Eclipſe of the Moon, without any previous Obſervation of the Moon.

The common Inveſtigation of the Moon from the Earth (built upon the Methods laid down in the 2d and 3d Books) preſuppoſes the Theory of the Moon to be in ſome meaſure eſtabliſh'd, becauſe this Theory depends upon it, and cannot be obtain'd without it; as *Ptolemy* obſerves, in the 11 *Chapter* of the 5*th Book of Almageſt*. He, by an obſervation of the Moon in the Zenith, when at the ſame time it has its greateſt Latitude, performs the matter. Few places are fit for ſuch an Obſervation, certainly not our Latitude. Therefore we muſt have recourſe to Geometry, and ſee if it will afford us any help, and find the Parallax of the Moon without any previous Theory or Obſervation of the Moon.

At the Moment that the Horns of the Moon (in its Immerſion into the Earth's Shadow, or its Emerſion from it) are parallel to the Æquinoctial, let the apparent Altitude of the Moon, and (by help of the neighbouring Fix'd Stars) its apparent Right Aſcenſion and Declenſion be obſerv'd. Now the Phaſis of the Eclips'd Moon is very well diſtinguiſh'd, if the Horns appear to be in a Right line perpendicular to the Gnomon of a Sun-Dial, which is parallel to the Axe of diurnal Rotation, which Line repreſents the

Æquator; in which caſe it is plain that the Moon and the Shadow of the Earth have the ſame Right Aſcenſion. In *fig*. 24. let the Circle *ZPHO* repreſent the Meridian, in which the Pole is *P*, *Z* the Zenith. Let *E*·*Q* be the Æquator, in which the place from which you begin to count the Right Aſcenſion is *V*; let *HO* be the Obſerver's Horizon, and *A* the Moon's apparent Place, and *V* the time of the Obſervation; which Places will be in the ſame Vertical Circle, becauſe the Parallax does ſomething depreſs the Phenomenon. Let the Circles of Declination *PAB*, *PVD* meeting with the Æquator in *B* and *D*, be underſtood to be drawn thro' *A* and *V*. From the given Place of the Sun will be given its Right Aſcenſion, and conſequently the Right Aſcenſion of the Point oppoſite to it, *viz*. the Center of the Shadow; that is, *VD*: therefore *BD* the difference of *VD* the true and *VB* the apparent Right Aſcenſion will be given, or the Angle *BPD*, which is meaſur'd by this Arc. In the Spherical Triangle *APZ* three ſides are given; namely, *PZ* the Complement of the Pole's Height, *ZA* the Complement of the obſerv'd apparent Altitude of the Moon; and *AP* the Complement of the obſerv'd apparent Declination of the Moon; therefore the Angle *ZAP* will be given. Again, in the Triangle *VAP* are given the two lately found Angles, *VAP*, and *VPA*, with the intercepted ſide *AP*; therefore the ſide *AV*, the Moon's Parallax will be found, which being given, you may, by *Prop*. 46. of *B*. 2. find the requir'd Diſtance of the Moon from the Center of the Earth.

PRO-

PROPOSITION XXIII.

TO *determine the Diſtance of the Moon from the Center of the Earth, at a given Time, by obſervation.* [*Fig.* 25.]

Let *OR* denote the Earth, whoſe Center is *T*; O the Place of the Obſerver; and let *L* be the Moon in the former Obſervation, when you have determin'd its Diſtance as well from the Earth's Center as from the Obſerver's Eye, (namely, the Right lines *TL*, *OL*) by the foregoing; and immediately before the Eclipſe of the Moon, on which the ſaid Obſervation has been made, let the Moon's apparent Diameter in *L* be obſerv'd. Now let the Moon be ſuppos'd in any other place of its Orbit, as at *l*: its Diſtance *Tl* from the Earth's Center is requir'd. Let the Moon's apparent Diameter in *l* be obſerv'd; now (as it is commonly known) O*l* is to O*L* as the apparent Diameter of the Moon at *L* is to its apparent Diameter at *l*; that is in a given ratio; therefore alſo O*l* is given: But by the apparent Altitude of the Moon in *l*, the Angle *lOT* is given; if therefore in the Triangle *lOT* the ſides O *l* and *OT*, as well as the Angle *lOT* be given, you will find *Tl* the Diſtance of the Moon from the Earth's Center. *Q. E. F.*

There is no need of expreſſing the apparent Diameters exactly in parts of a Circle; it is ſufficient if their ratio to one another be known: For theſe Arcs or Angles are not compar'd with other Angles or Arcs expreſs'd by parts of a Circle; but only one with another; for their inverſe Ratio is the Ratio of the Diſtances of the Moon from the Eye at the times of the Obſervations. Therefore it is enough if they be obſerv'd ſerv'd with a Micrometer fitted to a Teleſcope, and the Diſtances of the Threads being divided into equal parts, they will be mark'd by the number of thoſe parts.

PROPOSITION XXIV.

TO determine the Inequalities and Errors of the Moon by Obſervation and diſtinguiſh them one from another.

The Moon's apparent Places are determin'd by help of the neighbouring Fix'd Stars, whoſe apparent Places are before exactly known, by *Prop.* 29. *B.* 2. and true places from the apparent by the 68th *Prop.* of the ſame *Book*: But the Diſtance from the Earth's Center is known at any time, by the foregoing *Prop.* Therefore all that regards the Moon's Motion is known.

Since the Inequalities in the Moon's Motion, which are taken notice of in the 21ſt *Prop.* are known, and we know how to look for 'em, by comparing ſeveral of theſe kind of Obſervations made conſtantly and for ſeveral Years, their Quantities will be determin'd: as for Examp. we may this way determine whether (*cæteris paribus*) the Moon moves ſwifter in the Syzygies with the Sun than in the Quadratures; whether it be farther diſtant from the Earth in theſe than in thoſe; whether the Moon's mean Motion be ſwifter in the Earth's *Aphelium* than in its *Perihelium*, *&c.* ſome of 'em being determin'd (*viz.* thoſe which depend upon the Syzygies with the Sun, and perhaps thoſe which end and begin again twice in a Synodical Month) the reſt will more eaſily be diſtinguiſh'd from one another, which either depend upon the Earth's Annual Motion, or upon the Revolution of the Apſes or Nodes of the Moon in reſpect of the Sun.

Theſe being ſeparated from one another, there is left that Inequality of the Moon, of which we have ſpoken in general in the *Scholium* of *Prop.* 16. whoſe Quantity being known by its Period will be gather'd from Obſervations. And at length by comparing the Period of the Moon which is now obſerv'd, with that that was known ſeveral Ages ago by their Obſervations, whether the Globe of the Earth is in this proceſs of Time encreas'd or diminiſh'd, (as I ſuſpect) and Geometrically determine, by *Schol. Prop.* 17. what Increment or Decrement has happen'd in the whole time. But if other Inequalities, hitherto not obſerv'd, affect the Moon's Motion, taking away the forementioned ones, they will appear, and being made plain will encreaſe our Aſtronomy, by making up the Moon's Theory.

PROPOSITION XXV.

To find the Forces of the Sun which diſturb the Moon's Motions, or determine the Ratio between thoſe diſturbing Forces, and the Force of our Gravity. [*Fig.* 26.]

The ſame things being laid down as before, the Forces which diſturb the Moon's Motion are repreſented by the Right lines *AM*, *MN*, and by reaſon of the Sun's Diſtance, the Line *MN* does not differ from *MT*, and is the Triple of the Right Sine of the Arc *CL* of the Moon's Diſtance from the Quadrature, ſuppoſing *TL* the Radius; and *AM* is equal to *TL*, as it has been ſhewn in *Cor. Prop.* 20. Alſo ſimilar, but leſs Forces diſturb the Motion of the Earth, becauſe it is not the Center of the Earth it ſelf, but the common Center of Gravity of the Earth and Moon, that deſcribes the *Magnus Orbis* (or great Orbit) about the Sun, as it has been ſhewn in the 63d *Prop. B.* 1. ſo that the Earth and Moon are as it were Satellite's revolving about the ſaid Center of Gravity. But we refer both the Forces to the Moon, by reaſon of what has been demonſtrated in the 50th and 51ſt *Prop.* of *B.* 1. Moreover, by *Prop.* 20. the Force of the Sun which diſturbs the Moon, being as *AM* is to the Force, by which the Moon is retain'd in its Orbit, in a Duplicate ratio of the Moon's Periodical Time about the Earth, to the Earth's Periodical Time about the Sun: And the Force by which the Moon wou'd revolve about the Earth, if it was unmov'd, at its preſent Diſtance from it, (which, by *Prop.* 51. *B.* 1. is equal to the Force, by which it is now retain'd in its Orbit about the foreſaid Center of Gravity,) is, by *Cor.* 1. *Prop.* 26. *B.* 1. to the Force by which it might in the ſame time revolve about the Earth, at its Diſtance from the ſaid Center of Gravity, as the Diſtance of the Moon from the Earth, to the Diſtance of the Moon from that Center of Gravity; for the Periodical Times of the Earth and the Moon about their common Center of Gravity are the ſame: And the foreſaid Force by which the Moon wou'd be retain'd in its Orbit about the unmov'd Earth, at its diſtance from that common Center of Gravity, is to the Force which we call Gravity (which is indeed the ſame Force, but propagated to a leſs diſtance from the Center, namely, to the Earth's Surface only) in a Duplicate ratio of the Semi-Diameter of the Earth to the foreſaid Diſtance of the Moon from the common Center of Gravity of the Moon and Earth: And therefore the Force of the Sun as *AM* is to the Force of Gravity in the Earth's Surface in a Ratio compounded of theſe three Ratio's; namely, of the Duplicate Ratio of the Moon's Periodical Time about the Earth, to the Periodical Time of the Earth about the Sun, of the Ratio of the Moon's Diſtance from the Earth, to the Diſtance of the Moon from the common Center of Gravity of the Moon and Earth, and of the Duplicate ratio of the Semi-Diameter of the Earth to the diſtance of the Moon from the common Center of Gravity of the Earth and Moon. Now the three compounding Ratio's are given; therefore we have the Ratio compounded of them, *viz.* the Ratio between the Sun's Force as *A M* and the Force of Gravity. But the Ratio of the other diſturbing Force as *MN* or *MT* to *A M* or *LT* is known, namely, that of the Triple of the Sine of the Arc *CL* of the Moon's Diſtance from the Quadrature to the Radius: Therefore the Ratio of each Force of the Sun which diſturbs the Moon's Motion to the Force of our Gravity here is found.

The Mean compounding Force may without any ſenſible Error be neglected, as coming near to the ratio of Equality: which makes the Solution ſo much the eaſier.

SECTION VI.

Of the Moon's Tables, and their uſe.

PROPOSITION XXVI.

TO *explain the Order of the Lunary Tables, deſign'd for the readily finding of the Plane of the Moon ſeen from the Earth.*

Firſt of all, the *Epocha* or Root of the Moon's Motion is ſettled: that is, (as has been above explain'd in *B.* 3.) by taking a remarkable Point of the Mean Time, number'd in the place for which the Tables are deſign'd; on the ſide you muſt ſet down the Mean Longitudes of the Moon, and its *Apogæums*, and aſcending Nodes agreeing with the ſaid Time. Likewiſe, for readily finding the Moon's Aſpects with the Sun, the Root of the Moon's Mean Motion from the Sun is ſettled; that is, the Mean Diſtance of the Moon from the Sun is put down agreeable to the ſaid *Epocha.*

Secondly, the Mean Motions of the Moon it ſelf, and its *Apogæums* and Nodes are to be ſet down in the Tables for Years, Tens of Years, and Hundreds of Years, *&c.* and alſo for Months, Days, Hours, and parts of Hours. Likewiſe the Mean Motions of the Moon from the Sun, for the ſame numbers of Years, Months, Days, *&c.* muſt be ſet down; which are eaſily found, by taking away the Sun's Mean Motion from the Moon's Mean Motion, agreeable to the ſame Time.

Thirdly, you muſt ſet down the Correction of the Moon's Mean Motion, according to the

Earth's Diſtance from the Sun. For the Moon's Mean Motion being taken from the foregoing Table, is the Mean Motion in a larger Sence; that is, in reſpect of a Courſe of ſeveral Years. Now (by *Prop.* 17.) the Moon's Periodical Time is ſhorter when the Earth is in its *Aphelium*; but as it recedes from it, the Moon's Periodical Time is encreas'd, till it be the greateſt of all, when the Earth is come to the *Perihelium.* Therefore its Mean Place before found is to be corrected by the addition or ſubſtraction of an Arc or Angle depending upon the Mean Anomaly of the Earth: Yet the Place of the Moon ſo corrected is ſtill the Mean Place. This Table of the Correction of the Moon's Mean Motion depending upon the Earth's Anomaly, is ſettled by means of the 17, 20, and 25 *Prop.*

Fourthly, the Corrections (call'd *Æquations* in the Aſtronomic Phraſe) of the Mean Place of the Moon's *Apogæum.* And theſe Æquations are Three; the Firſt depends upon the Aſpect of the *Apogæum* and the Sun; for it has been ſhewn in *Prop.* 11. that the Apogæum goes forward when it is in the Syzygies, and goes backward when it is in the Quadratures with the Sun; therefore to the Mean Apogæum is to be added, or from it is to be ſubſtracted the Æquation depending upon the ſituation of the Apogæum in reſpect of the Sun: The quantity of this Æquation is proportional to the quantity of the Forces that produce thoſe Effects, which are to be determin'd by the 11, 20, and 25 *Propoſitions.* The ſecond Æquation of the Moon's Apogæum is Monthly, and depends upon the Sun and Moon's Aſpect. For tho' near the Conjunction or Oppoſition of the Moon's Apogæum with the Sun, the Apogæum goes forward in one whole Lunation, yet that Progreſs of the Apogæum is not equable, but when the Moon is in the Quadratures, it goes forward ſlower, or by chance it may alſo go back; when the Moon is in the Syzygies, its Apogæum goes forward much faſter. Likewiſe, tho' near the Quadrature of the Apogæum of the Moon with the Sun, the Apogæum of the Moon goes back in reſpect of one whole Lunation, yet it does not equably go back, but the Moon in the Syzygies either makes it go forward, or at leaſt go back more ſlowly. Theſe things are plain from *Prop.* 10. whence the Place of the Moon's Apogæum being firſt (by the foregoing Correction) brought to an Equality, muſt alſo be corrected by this monthly Æquation. The Quantity of this Æquation is to be gather'd from the different Aſpects of the Moon to the Sun (by *Prop.* 8, 20, and 25.) But there is alſo need of a third Æquation fully to rectify the Place of the Apogæum: For the Motion of the Apogæum, whether progreſſive or regreſſive, is greater or leſs according to the leſs or greater Diſtance of the Earth from the Sun; and its Velocity (by *Prop.* 19.) is in a Triplicate Ratio of the Sun's apparent Diameter: Therefore this depends upon the mean Anomaly of the Earth, as alſo the Correction of the mean Motion, of which we have ſpoken above. Its Quantity may be drawn from the knowledge of the Degree of the Anomaly in *Prop.* 19, 20, and 25.

Fifthly, the Excentricity of the Lunar Orbit is liable to a continual change, and the Times of that change are the ſame with thoſe of the progreſſive and regreſſive Motion of the Apogæum. Becauſe (by *Prop.* 13.) if you compare one whole Lunation with another whole one, the greateſt Excentricity is when the Apſes are in the Syzygies, and the leaſt when they are in the Quadratures. And in one Lunation (by *Prop.* 12.) it is greateſt when the Moon is in the Syzygies, and leaſt when the Moon is in the Quadratures: for Example, in that Month that the Apſes are in the Syzygies, the Excentricity is diminiſh'd, when the Moon itſelf is come to the Quadrature, at which time it is leaſt of all in that Lunation. And likewiſe, when the Apogæum is in the Quadrature with the Sun, the Excentricity, which already was little, is diminiſh'd, whilſt the Moon goes from the Syzygies to the Quadratures. Therefore the Excentricities of the Orbit, which agrees with the Aſpect of the Apogæum of the Moon are put down in the Tables, and that very conveniently over-againſt the Æquations of the Apogæum, which depend upon the ſaid Aſpect, and as it were make the ſame Table with them. Moreover, theſe Excentricities are again to be corrected by the Monthly Equation depending upon the Moon's Diſtance from the Syzygies: For when the Moon is in the Syzygies, the Excentricity (*cæteris paribus*) is greateſt. But the Excentricity thus equated muſt again be (by *Prop.* 18 and 19.) corrected according as the Sun's Diſtance from the Earth is encreas'd or diminiſh'd; and this Æquation depends upon the Earth's Anomaly: But theſe three Tables are calculated by help of the reſpective aforecited Propoſitions, together with the 20th and 25th. And as the Mutations of the ſaid Excentricity have the ſame Terms and Times with the aforeſaid Motions of the Apogæum, the two laſt Æquations of it, namely, the Monthly and Annual, will alſo very conveniently in the ſame Table be joyn'd

Join'd to the two laſt Æquations of the Excentricity. Theſe Corrections are made very well and eaſily by the Addition or Subtraction of the Particulars ſet down in the Table, according to the various Diſtance of the Moon from the Syzygies with the Sun, or the various Diſtance of the Earth from the Aphelium; that is, under the ſame Titles as you had before ſet down the Annual and Monthly Corrections of the Apogæum.

Sixthly, you muſt add the ſaid *Table of the Æquation of the Orbit*, by which, from the given Time of the Moon's being in the Apogæum, you may find the Angle by which the Moon, ſeen from the Earth, is diſtant from the Apogæum. For as the Moon in its temporary Elliptic Orbit moves after the ſame manner about the Earth in the Focus of an Ellipſis, as any Primary Planet moves about the Sun; that is, deſcribes (by a Radius drawn to the Earth) an Area proportionable to the Time; (at leaſt we now abſtract from all other Inequalities, having their proportion in the other Tables of Æquation of the Moon's Motion;) if the Orbit be given, its true Place (when it is ſeen from the Earth) will after the ſame manner be found for a given Time, as the Place of the Primary ſeen from the Sun is found; that is, the Excentricity or Ratio of the Diſtance of the Foci to the greater Axis being given, from the given mean Anomaly of the Moon (which conſiſts in an Area proportionable to the Time) the *coæquated* Anomaly will be found in this Place after the ſame manner as in that; namely, by help of *Prop.* 3. *B.*3. or, if we may be allow'd to uſe approximation, we may make uſe of ſome of the Propoſitions that follow in the 3*d Book. Ward's*

Approximation is ſufficient, in which caſe the Æquation of the Orbit, or the Æquation of the Excentric (which is alſo call'd the *Æquation of the Moon's Center*) is (by *Prop. 6. B. 3.*) found according to the Degree of the Moon's mean Anomaly, and the Æquations being found for every Degree, are ſet down in the Tables, to be added or ſubtracted according to the Semi-Orbit, from the Apogæum to the Perigæum, or from the Perigæum to the Apogæum. But becauſe the Excentricity of the Moon's Orbit is continually chang'd, and it would be impoſſible to conſtruct a Table for every Excentricity; therefore Aſtronomers commonly conſtruct Æquation Tables of this kind for every Degree of the Mean Anomaly for the greateſt and leaſt Excentricity of the Orbit, and ſometimes alſo for the Mean; and they alſo take a proportionable Æquation between the greateſt and the leaſt, and the Excentricity belonging to the Lunar Orbit is between the greateſt and the leaſt.

Seventhly, we muſt add a Table ſhewing that Inequality of the Moon, which we have (in the 1ſt *Prop.*) ſhew'd to belong to every Satellite, according to the Moon's various Diſtance from the Syzygies. For the Moon's Motion in the firſt Quadrant of the Month (or as the Moon goes from the Conjunction to the next Quadrature) is retarded, accelerated in the ſecond, retarded again in the third, and then accelerated again in the fourth.

Tycho was the firſt that diſcover'd this inequality in the Moon's Motion, and gave it properly enough the Name of *Variation.* But a Table of Variation is to be made from what has been ſaid in *Cor. 2. Prop. 2.* according as the Moon's Orbit is broader between the Quadratures, and narrower between the Syzygies, which has been ſhewn to be the true Track of the Moon (by *Prop.* 2. and its 1ſt *Corol.*) The greateſt Quantity of that Variation is drawn from the 20th and 25th *Prop.* and the Quantity belonging to every Diſtance of the Moon from the Syzygies (from *Prop.* 1. and 2. and the *Cor.* of the 2d.) Beſides, the Variation is again to be corrected, by help of the Table made by *Prop.* 18. and 19. For when the Earth is in the Perihelium, the Moon's Periodical Time is encreas'd, and among all the other encreas'd Errors, this Variation (which we now ſpeak of) is encreas'd every Revolution in a Duplicate Ratio of the Time between the Moon's Quadratures (by *Schol. Prop.* 18.) This Correction depends upon the Earth's Mean Anomaly, and alſo the other by which the Variation is encreas'd or diminiſh'd in a Triplicate Ratio of the Sun's apparent Diameter, ſeen from the Earth.

Eighthly, you muſt ſet down three Æquations of the Moon's Nodes in Tables. The firſt of 'em depends upon the Aſpect of the Nodes with the Sun: For (by *Prop.* 14.) the Nodes are altogether at reſt in the Syzygies; but when they ſtick about the Sun's Quadratures, they are carried very ſwiftly *in antecedentia*; and eſpecially ſo, if you compare one whole Lunation with another whole one. For if you obſerve one by itſelf, the Nodes go back ſwifteſt (by *Prop.* 14.) when the Moon is in the Syzygies: And from hence the Monthly Æquation is neceſſary, which depends upon the Aſpects of the Sun and Moon. The third Æquation is Annual; for the Nodes (*cæteris paribus*) go back ſo much the faſter, as the Earth is nearer to the Perihelium, by *Prop.* 19. The Quantities of thoſe Æquations (whether they be the greateſt, or thoſe that agree with any Aſpect of the Nodes, or of the Moon with the Sun, and the Anomaly of the Earth) are determin'd by the 20th and 25th *Prop.* together with the 14th and 19th.

Ninthly, there is need of the ſame Corrections to *equate* the Inclination of the Moon's Orbit to the Plane of the Ecliptic: For it is (by *Prop.* 15.) leaſt when the Nodes are in the Quadratures with the Sun, and greateſt when they are in the Syzygies; and in the ſame Lunation, leaſt when the Moon is in the Syzygies, and greateſt when it is in the Quadratures. Since therefore theſe have the ſame Times as the Æquations of the Nodes (as we have ſaid before) in reſpect of the Aſpect of the Nodes, as well with the Moon as with the Sun, they eaſily fall in together in the ſame Table. And the Correction of the Inclination of the Planes is made eaſily, by the *Particles* to be added in the leaſt Inclination (as we have ſaid before, concerning the correcting of the Excentricity) which vary according to the ſaid Aſpects of the Nodes and of the Moon to the Sun (or, in the Aſtronomers Phraſe, according to the Diſtances of the Nodes and of the Moon from the Sun) or perhaps more fitly by the Particles to be deducted from the greateſt Inclination: For this ſeems natural to the Moon's Orbit, becauſe it wou'd belong to it if the Sun did not diſturb its Motion. But this Inclination alſo is to be *equated* by the Annual Correction, which depends upon the Earth's Diſtance from the Sun, or the Earth's Anomaly, and which will fall into one Table with the Annual Æquation of the Nodes. The greateſt Quantities of thoſe three Æquations, and thoſe which belong to every Aſpect or Diſtance, are drawn from the 15, 18, 19, 20, and 25 Propoſitions.

Tenthly, a Table is to be made to ſhew the Proportion of all the foregoing Monthly Corrections, or Increments or Decrements made in the Syzygies, in the Conjunction and Oppoſition with the Sun. For (by *Prop.* 16.) all the Effects that are produc'd in the Syzygies of the Luminaries, are greater in the Conjunction and leſs in the Oppoſition. The Proportion may be drawn from the Ratio of the generating Forces, by the foregoing Propoſitions.

Eleventhly, there is added a Table of the Moon's Latitude according to every Degree of the Diſtance from the ſaid Node. Its Conſtruction depends upon the Doctrine of Spherics. For in a Spherical Rectangular Triangle beſides the Right line, the Hypothenuſe is given, namely the Moon's Diſtance from the Node, and the Angle at the Node, namely, the Inclination of the Plane of the Moon's Orbit to the Plane of the Ecliptic, (which is alſo call'd the *Inclination of the Limits* ;) therefore the Side oppoſite to that Angle will be found; namely, the Moon's Latitude requir'd, according to the ſaid Diſtance from the Node. But becauſe the Inclination of the Plane of the Moon's Orbit to the Plane of the Ecliptic is different at different Times, therefore commonly one Table only of the Moon's Latitude is made for the leaſt Inclination of the ſaid Planes, thoſe Particles being joyn'd which are to be added to the Latitude, when the ſaid Inclination is the greateſt. In the Mean Inclinations, ſuch a part of the annex'd Particle is to be added, as is to the whole, as the Exceſs of the given Inclination above the leaſt, to the Exceſs of the greateſt above the leaſt; as it is uſual in the like caſes.

Twelfthly, you muſt alſo ſubjoin a Table, ſhewing the Moon's Place reduc'd to the Ecliptic for every Degree of the Moon's diſtance from the Node. Its Conſtruction alſo depends upon the Doctrine of Spherics. For in the ſame Triangle in which the Latitude is firſt found out, the third Side, or the diſtance between the Node and the Point where the Circle of Latitude paſſing thro' the Moon's Place, cuts the Ecliptic, is the Moon's diſtance from the Node reduc'd to the Ecliptic: and the Interval between that and the Moon's diſtance from the Node, computed in its Orbit, is call'd the *Reduction*, and is diſpos'd into a Table according to Art. And as it varies according to the various Inclination of the Lunar Orbit to the Ecliptic, (after the ſame manner as before) the Exceſs of the foreſaid Reduction is mark'd when the Inclination of the Limits is the greateſt, that its proportional part may be taken when the Inclination of the Limits is Mean between the greateſt and the leaſt: For the Reduction itſelf is fitted to the leaſt Inclination, as the Latitude was before.

But alſo when other Errors are diſcover'd, other Tables muſt be conſtructed, fully and perfectly to rectify the Moon's Motion, which is far the hardeſt Work in Aſtronomy. Let what we have ſaid of the Order of theſe Tables, ſuffice in the mean time.

PRO-

PROPOSITION XXVII.

TO explain the Order of Lunar Tables, in order to find readily the Place of the Moon ſeen from a given Point in the Earth's Surface.

Becauſe the Moon's Place ſeen from a Point in the Earth's Surface differs ſenſibly from its Place when ſeen from the Center, which laſt Place alone is to be found by the foregoing Tables; therefore Aſtronomers make Tables alſo to diſcover the firſt from the knowledge of the laſt. And as the Parallax is all the difference between the ſaid Places, the knowledge of it is ſufficient. And becauſe when you know the Horizontal Parallax, you may find the Parallax of any other Elevation above the Horizon, by *Prop.* 47. *B.* 2. therefore we have need only of the Moon's Horizontal Parallax. And as the Moon's horizontal Parallax varies according to the Moon's greater or leſs diſtance from the Earth, there muſt be made a Table of thoſe Parallaxes. Now the Parallax for one Diſtance is known by *Prop.* 22. and the Moon's Diſtances from the Earth in the other Points of its Orbit in reſpect of the Diſtance found by the ſaid 22^d are determin'd by the Moon's Orbit, which has been before deſcrib'd; therefore the Moon's horizontal Parallaxes (which, by *Prop.* 48. *B.* 2. are reciprocally as its Diſtances from the Center of the Earth) according to its Situation in reſpect of the Apogæum and Perigæum of its Orbit, that is, according to its mean Anomaly, are known and reduc'd to a Table.

But becauſe ſuch a Table of the Moon's horizontal Parallaxes ſerves only for one Figure of the Orbit; therefore they conſtruct one for the greateſt Excentricity and another for the leaſt, and take the Moon's horizontal Parallax for a given Time in the ſame Mean Ratio between the Parallaxes agreeable to the greateſt and leaſt Excentricity, becauſe the Excentricity agreeable to that Time, is a Mean between the greateſt and the leaſt Excentricity.

The Moon's horizontal Parallax thus found, is to be rectified by a certain Æquation to be drawn from another Table, whoſe Foundation is as follows. It is plain from *Prop.* 2. that the Moon *(cæteris paribus)* is farther diſtant from the Earth in the Quadratures than in the Syzygies. One may find the Proportion between thoſe Diſtances of the Moon from the Earth, from the Proportion of the Forces of the Sun which produce that Effect by help of *Prop.* 20. and 25. Therefore a Table for *æquating* the Moon's Parallax conſiſts of Particles to be ſubſtracted from the Parallax which it has in the Syzygies, as tthe Moon comes nearer to the Quadrature, where *(cæteris paribus)* the Parallax is the leaſt, becauſe the Moon is the fartheſt diſtant from the Earth: Therefore this Æquation is Monthly, depending upon the Syzygies of the Sun and Moon.

But this Parallax of the Moon is alſo affected by the Annual Æquation; as being leſs when the Earth is in the Perihelium, greater when it is in the Aphelium, *(cæteris paribus)* becauſe the whole Lunar Syſtem is in this caſe contracted, but dilated in the other, by *Prop.* 17. This Æquation depends upon the Earth's Mean Anomaly, and is conſtructed by help of the 17th, 20th, and 25th Propoſitions.

PROPOSITION XXVIII.

TO find the Moon's Place, ſeen from any Place of the Earth's Surface, for a given Time, by help of the Tables above deſcrib'd.

Let the Time, reckon'd at the Habitation or Place given, be (by *Prop.* 32. *B.* 2.) reduc'd to the Time which is reckon'd under the Meridian of the Place for which the Tables are calculated; and when (by *Prop.* 17. *B.* 3.) it is equated to it, let the Sun's Place be found.

At the Epocha mark'd in the Table, which goes before the given Time, mark the Mean Places of the Moon, of the Moon's Apogæum, and of the aſcending Nodes, and collecting the Mean Motion of thoſe three made from that Epocha to the Time given, add the two firſt to the aforeſaid Places of the Moon and the Apogæum, that they may become the Mean of them for the Time given; but let the Motion of the Node then made, be ſubſtracted from the radical Place (becauſe they are mov'd *in antecedentia*) that it may become its Mean Place; and let the Moon's Mean Place be corrected by that Æquation depending upon the Earth's Mean Anomaly. Of which ſee *Sect.*III.*Prop.*26.

Moreover, let the Mean Place of the Apogæum be corrected by three Æquations, of which we have ſpoken in the 4th *Sect.* of the ſaid 26th *Prop.* For the Sun's Place being given, and the nearly true Place of the Apogæum, the Aſpect of the Lunar Apogæum with the Sun is given, and from thence the Æquation that depends upon it. Likewiſe, becauſe as well the Sun's true Place, as the Moon's nearly true Place is known, the Moon's Aſpect to the Sun is alſo known, and from thence the Monthly

Æquation of the Apogæum that depends upon it. And an annual Æquation of the Apogæum agreeable to the Earth's Mean Anomaly (firſt neceſſary to determine the Sun's Place) will be found in the third Table of Æquation. And this Æquation is the truly annual one depending upon the Earth's Period about the Sun; for that firſt, which depends upon the Aſpect of the Moon's Apogæum with the Sun, is by ſome improperly call'd Annual, ſince its Period is not finiſh'd in leſs than 400 Days; the Sun requiring that Time, that being gone from the Lunar Apogæum it may again follow the Apogæum which is carried *in conſequentia.* So alſo the Excentricity of the Moon's Orbit, taken out of the Table from the Sun's Apogæum, is corrected by the Monthly and the Annual Æquation.

Having therefore found the true Place of the Moon's Apogæum, and the Moon's Mean Place being firſt corrected, by taking the firſt from the laſt, the Remainder is the Moon's Mean Diſtance from its Apogæum reckon'd or number'd *in conſequentia*, or the Moon's Mean Anomaly; agreeable to which you will (in the Table of *Sect. 6. Prop.* 26.) find the *Proſtapherefis* for the Excentricity of the Orbit before found, by which the Moon's Mean Place, before corrected, being again rectified, will give the Moon's Place more correct, or *Equated*, as ſome call it.

From the Moon's Equated Place lately found, and the Sun's Place firſt of all determin'd, the Aſpect of the Sun and Moon is now enough known, and the Variation that depends upon it, is known by the Table, of which we ſpoke in *Sect.* 7. *Prop.* 26. but even this is to be corrected by the double Annual Æquation, of which we ſpoke in the ſame Place. The Moon's Equated Place is to be corrected by the Variation corrected in the abovemention'd manner; and thus you will to the greateſt nicety have the Moon's Place in its Orbit, or in reſpect of its Apſis.

Now in order to reduce it to the Ecliptic, firſt of all we muſt determine the Places of the Nodes. And as the Sun's true Place, and the nearly true Place of the Nodes, or their Mean Place will be given; ſo likewiſe will be given the Aſpect of the Nodes with the Sun, and conſequently its correſpondent Equation from the Table, of which we made mention in the 8th *Sect. Prop.* 26. by which the Node's Mean Place being corrected is again to be twice corrected by the Monthly and Annual Æquation of which we alſo ſpoke there.

At the ſame time, from the ſame Aſpect of the Nodes to the Sun, may be drawn the Exceſs of the Inclination of the Limits above their own leaſt Inclination (or their Defect from the greateſt) by which the leaſt Inclination (or the greateſt) is to be corrected, that it may become more exact: but it is again twice to be corrected; namely, according to the Moon's Aſpect to the Sun, and the Earth's Mean Anomaly; of which Tables of Equation we have ſpoken in *Sect.* 9. *Prop.* 26. If the moſt correct Place of the Moon in the Syzygies be requir'd, you may alſo have the Proportion of that Correction, which we have ſpoken of in *Sect.* 10. *Prop.* 26.

But becauſe the true Places of the Node and the Moon's Apogæum in the Ecliptic, and the Inclination of the Moon's Orbit to it are known by the reſolution of the Rectangular Spherical

Triangle, in which, beſides the Right-Angle, one of the Angles, and one of the Sides next to the Right-Angle are given, the Diſtance of the Apogæum from the Node, computed in the Moon's Orbit, is found; *viz.* the Hypothenuſe. And the Moon's Place in its Orbit reckon'd from the Apogæum is found: And therefore the Diſtance of the Moon from the Node, reckon'd in the Lunar Orbit will be alſo known. And as the Inclination of the ſaid Lunar Orb to the Ecliptic is alſo given; there will be found both the Moon's Latitude, and its Place in the Ecliptic, either by the Reſolution of the rectangular Spherical Triangle, comprehended between the Lunar Orbit, the Ecliptic and the Circle of Latitude, or ſooner by the Tables of Latitude and Reduction, (of which ſee *Sect.* 11. and 12. *Prop.* 26. where the ſaid Triangle is reſolv'd) whoſe uſe is enough known, and this Diſtance of the Moon from the Node computed in the Lunar Orbit, is call'd the *Argument of the Latitude.*

Therefore the true place of the Moon is found, (that is, ſeen from the Center of the Earth) as well in Longitude as in Latitude. But to find what Situation a *Phænomenon* of given Longitude and Latitude has in a given Horizon, that is, what Altitude and what Vertical it has at a given Time, is a Problem eaſy to be ſolv'd by the Doctrine of Spherics: for it is the inverſe of *Prop.* 26. and 27. *B.* 2. Now if the Moon's apparent Place, or that in which it is ſeen, be deſir'd, you muſt in the firſt Table, (ſee *Prop.* 27.) note the Parallax agreeable to the Excentricity of the Moon's Orbit and Mean Anomaly before known, which you muſt correct by the Monthly and Annual Æquations, of which we ſpoke in the ſaid *Prop.* 27. From the horizontal Parallaxes find (by *Prop.* 47. *B.* 2.) the Parallax agreeable to the Altitude before found, which being ſubſtracted from the firſt will have the Moon's apparent Altitude: And there is no other difference between the Moon's true and apparent place, beſides this decreaſe of Altitude or Parallax, and the common one ariſing from the Refraction of the Rays thro' the Atmoſphere, of which ſee *Prop.* 64. and 65. *B.* 2. As for the mutations of Longitude and Latitude which are owing to the Parallaxes and Refraction, you may gather them from *Prop.* 63. and 68. *B.* 2. Therefore the Moon's Place is found, as well the true one, as that ſeen from any Point of the Earth's Surface, both in Longitude and Latitude. *Q. E. F.*

PROPOSITION XXXIX.

TO *give a ſhort account of ſuch of the forementiond Corrections concerning the Moon's Motion as have been hitherto taken notice of by the Conſtructors of Tables, and ſuch of them as have been neglected.*

Firſt, they neglect all the Annual Æquations or Corrections depending upon the various Diſtance of the Earth from the Sun, which have been explain'd in *Prop.* 18. and 19. and conſider only that which ariſes from the encreas'd and diminiſh'd Periodical Time of the Moon about the Earth, from the diminiſh'd or encreas'd Diſtance of the Earth from the Sun; that is, which depends upon the Earth's Mean Anomaly. Nay, tho' this Inequality was obſerv'd by *Tycho*, *Kepler*, and *Horoxius*, yet they confounded it with the others: for by reaſon of it they were compell'd (by the nature of things) to reduce the

Apparent to the Mean Time in reckoning the Moon's Places, another way than they us'd to do in reckoning the Places of the Sun and other fix'd Stars; and eſpecially *Tycho* quite left out that part of the Æquation of Time which depends upon the Earth's Mean Anomaly, upon which alſo this Inequality of the Moon depends. But *Kepler* and *Horoxius* having carried the Matter farther for their equating of Time, make uſe of an additional Phyſical Part, when the true and Aſtronomical one may be taken away, and ſo on the contrary. The moſt ingenious Dr. *Halley* was the firſt that ſeparated this Inequality (which Mr. *James Gregory* ſuſpected to belong to the Moon, in the *Schol.* of *Prop*. 86. of his *Optics*) from the others, in a Treatiſe ſubjoin'd to his *Catalogue of the Southern Fix'd Stars.* But it is plain from the foremention'd *Prop*. 18. and 19. that all the Inequalities of the Moon want a double Annual Correction, to make them true and exact.

Laſtly, all the Monthly Æquations ariſing from the Aſpect of the Sun and Moon are neglected, except the Variation only. 'Tis true indeed that ſome of them are almoſt compenſated by others: as for Example, the Monthly Variation of the Limits, and the Monthly Æquation of the Nodes ſo take off and correct one another, that they may be both neglected in determining the Moon's Latitude; but the Monthly Æquations of the Motion of the Apogæum, and of the difference between the leaſt and greateſt Excentricity, and thoſe of the Nodes and of the Inclination of the Moon's Orbit to the Ecliptic, are the Baſis and Foundation of the other Inequalities, which depend upon the Aſpect of the Sun and the Apogæum, or of the Sun and the Nodes; as may be fully gather'd from the 10, 11, 12, 13, 14 and 15th Propoſitions. Likewiſe the Monthly Æquation between the New Moon and Full Moon (of which we ſpoke in *Prop.* 16.) is neglected.

And thoſe Æquations which affect the Moon's Motion, whether real or apparent, (of which we ſpoke in *Prop.* 2.) ariſing from the greater Diſtance of the Moon from the Earth, in the Quadratures than in the Syzygies, are altogether neglected: neither is the difference of the Moon's diſtance from the Earth conſider'd otherwiſe than as ariſing from the Excentricity of the Orbit; and depending upon the mean Anomaly; whence alſo the Monthly Æquation of the horizontal Parallaxes (*Prop.* 27.) is alſo neglected.

But thoſe Æquations which depend upon the Aſpect of the Sun and the Lunar Apogæum (mention'd in *Prop.* 11. and 13.) and alſo thoſe which depend upon the Aſpect of the Sun and the Nodes (treated of in *Prop.* 14. and 15.) are conſider'd by Aſtronomers. For they are far the greateſt, and make up the greateſt part of the Inequalities of the Moon's Motion.

For the Conſtruction of Tables therefore, beſides the foregoing Æquations depending upon the Aſpect of the Sun with the Moon's Apogæum or Nodes, an Annual and a Monthly Æquation (which we ſpoke of before) is to be conſider'd. But the Æquations of the Moon's motion, in ſeveral Lunar Tables, have Titles ſo different from the true Periods of theſe Inequalities, that it is difficult enough to aſſert which of them the Conſtructors conſider'd and which of them they neglected. And ſometimes alſo ſome Inequalities are mix'd and confounded with others: As for Example, they do not purely deduce from the Moon's Orbit given, the Æquation of the Excentric (or of the Moon's Center) but as it were confuſedly mix it with the others; as with that part of the Inequality of the Variation that depends upon the time between the Quadratures, which ariſes from the Situation of the Moon in reſpect of its Apogæum; ſee *Schol. Prop.* 18. But as they made their Tables not from known Phyſical Cauſes and their Periods, but only by attending to Obſervations; it is no wonder if they did not rightly diſtinguiſh the Inequalities from one another and diſpos'd them under foreign or leſs proper Titles.

SCHOLIUM.

I have thought fit to ſubjoyn the Theory of the Moon made uſe of by Sir *Iſaac Newton*, by which this incomparable Philoſopher has compaſs'd this extremely difficult Matter, hitherto deſpair'd of by Aſtronomers; namely, by Calculation to define the Moon's Place even out of the Syzygies, nay, in the Quadratures themſelves ſo nicely agreeable to its Place in the Heavens (as he has experienc'd it by ſeveral of the Moon's Places obſerv'd by the ingenious Mr. *Flamſtead*) as to differ from it (when the difference is the greateſt) ſcarce above two Minutes in her Syzygies, or above three in her Quadratures; but commonly ſo little that it may well enough be reckon'd only as a Defect of the Obſervation. In this Calculation, which we give in the Words of the Author, he does not wholly mention all the Inequalities, whoſe Cauſes are above explain'd, nor thoſe which are as yet only ſuſpected; but omitting thoſe which he knew wou'd take off one another, and others of leſs Moment, he only confines thoſe to Æquations and Tables, that have the greateſt Force and produce the moſt ſenſible Effects.

Sir Iſaac Newton's *Theory of the Moon.*

The Royal Obſervatory at *Greenwich* is to the Weſt of the Meridian of *Paris* 2°. 19′, of *Uraniburg* 12°. 51′. 30″, and of *Dantzick* 18°. 48′.

I put down the Sun and Moon's mean motions from the Vernal Æquinox at the Meridian of *Greenwich* as follows: *viz.* the laſt Day of *December* 1680 *Old Style*, at Noon, the Sun's mean Motion was 9$^{\text{ſign}}$. 20°. 34′. 46″. That of the Sun's Apogæum 3$^{\text{ſig}}$. 7°. 23′. 30″. The Moon's mean motion 6$^{\text{ſig}}$. 1°. 45′. 45″ That of the Moon's Apogæum 8$^{\text{ſig}}$. 4°. 28′. 5″. That of the aſcending Node of the Moon's Orbit 5$^{\text{ſig}}$. 24°. 14′. 35″. And *December* the laſt 1700 *Old Style* at Noon, the Sun's mean motion was 9$^{\text{ſig}}$. 20°. 43′. 50″. That of the Sun's Apogæum 3$^{\text{ſig}}$. 7°. 44′. 30″. The mean motion of the Moon 10$^{\text{ſig}}$. 15°. 19′. 50″. Of the Moon's Apogæum 11$^{\text{ſig}}$. 8°. 18′. 20″. And of the aſcending Node 4$^{\text{ſig}}$. 27°. 24′. 20″. For in Twenty *Julian* Years, or in 7305 Days, the Sun goes thro' 20$^{\text{rev}}$. 0$^{\text{ſig}}$. 0°. 9′. 4″. The motion of the Sun's Apogæum 21′. 00″. The Moon's motion 267$^{\text{rev}}$. 4$^{\text{ſig}}$. 13°. 34′. 5″. The motion of the Moon's Apogæum 2$^{\text{rev}}$. 3$^{\text{ſig}}$. 3°. 50′. 15″. Of the Node 1$^{\text{rev}}$. 0$^{\text{ſig}}$. 26°. 50′. 15″. All the foreſaid motions are from a Point of the Vernal Æquinox. And if from them be ſubſtracted the Preceſſion, or Retrograde Motion of the Æquinoctial Point it ſelf, which has mov'd in the mean time *in antecedentia*, *viz.* 16′. 40″: The Motions will remain in reſpect of the Fix'd

Stars in 20 *Julian* Years, namely the motion of the Sun, $19^{rev}. 11^{ſig}. 29°. 52'. 24''$. That of the Sun's Apogæum $4' 20''$. Of the Moon $267^{rev}. 4^{ſig}. 13°. 17'. 25''$. Of the Moon's Apogæum $2^{rev}. 3^{ſig}. 3°. 33'. 35''$. Of the Moon's Node $1^{rev}. 0^{ſig}. 27°. 6'. 55''$.

According to this Computation the *Tropical Year* is of $365^{days}. 5^{hours}. 48'. 57''$. And the *Sydereal Year*, of $365^{days}. 6^{hours}. 9'. 14\frac{1}{2}''$.

The mean motions of the Luminaries abovementioned have ſeveral Inequalities.

Firſt there are the Annual Æquations of the ſaid mean motions of the Sun and Moon, and of the Apogæum and Node of the Moon. The Annual Æquation of the mean motion of the Sun depends upon the Excentricity of the Earth's Orbit about the Sun, which is of $16\frac{1}{2}$ of ſuch parts of which the mean Diſtance of the Sun from the Earth is 1000; and therefore is call'd the *Æquation of the Center*: And when greateſt, it is $1°. 56'. 20''$. The greateſt Annual Æquation of the Moon's mean motion is $11'. 49''$. of its Apogæum $20'$. and of the Node $9'. 30''$. And thoſe four Annual Æquations are proportionable to each other: Therefore when any one of them is the greateſt, the three others are the greateſt; and when any one is diminiſh'd the others are alſo diminiſh'd in the ſame proportion: Whence if the Annual Æquation of the Sun's Center be given, you have the other 3 Æquations agreeable to it; therefore the Table of that alone is ſufficient. For if the Annual Æquation of the Sun's Center, agreeable to any time taken out of it, be call'd P, and $\frac{1}{10} P = Q$, $Q + \frac{1}{60} Q = R$, $\frac{1}{6} P = D$, $D + \frac{1}{30} D = E$, and $D - \frac{1}{60} D = 2F$; the Annual Æquation of the Moon agreeable to that Time will be *R*, that of the Moon's Apogæum *E*, and that of the Node *F*. It is to be noted, that if the Æquation of the Sun's Center is to be added, the foreſaid Æquation of the Moon is to be ſubtracted, that of the Lunar Apogæum to be added, and the Æquation of the Node to be ſubtracted: And on the contrary, if the Æquation of the Sun's Center is to be ſubtracted, the Moon's Æquation is to be added, that of the Lunar Apogæum to be ſubtracted, and that of the Node to be added.

There is another *Æquation of the Moon's mean Motion*, depending upon the ſituation of the Moon's Apogæum in reſpect of the Sun, which is the greateſt when the Apogæum of the Moon is in the *Octant* (or at half Right-angles) with the Sun, and none at all when it is come to the Syzygies or Quadratures. This Æquation, when greateſt, riſes to 3′ 56″ the Sun being in the Perigæum; but if the Sun be in the Apogæum, not above 3′ 34″. In other Diſtances of the Sun from the Earth, this greateſt Æquation is reciprocally as the Cube of that Diſtance. But when the Moon's Apogæum is out of the Octants, the ſaid Æquation becomes leſs, and is to the greateſt Æquation (ſuppoſing the diſtance of the Sun from the Earth to remain the ſame,) as the Sine of twice the diſtance of the Moon's Apogæum from the next Syzygy or Quadrature, to the Radius. This is added to the Moon's motion, whilſt the Moon's Apogæum paſſes from the Sun's Quadrature to the Syzygy; but is taken from it, as the Apogæum goes from the Syzygy to the Quadrature.

Moreover, there is another *Æquation of the Moon's motion* depending upon the Aſpect of the Nodes of the Moon's Orbit with the Sun;

and it is the greateſt when the Nodes are in the Octants with the Sun, and vaniſhes when they come to the Syzygies or Quadratures. This Æquation is proportional to the Sine of twice the diſtance of the Node from the next Syzygy or Quadrature, and when greateſt riſes to 47″. This is added to the Moon's motion, when the Nodes paſs from the Sun's Syzygies to his Quadratures; but is ſubſtracted, when they paſs from the Quadratures to the Syzygies.

From the Sun's true Place take away the mean Motion of the Moon's Apogæum equated as was before ſhewn; the remainder will be the Annual Argument of the ſaid Apogæum. Then reckon the Moon's Excentricity and the ſecond Æquation of its Apogæum in the following manner,[*which alſo obtains in the Computation of any other intermediate Æquations*:] Let *T* (*Fig.* 27.) repreſent the Earth; *TS*, a Right line joyning to Earth and Sun; *TACB* a Right line drawn from the Earth to the mean Place of the Moon's Apogæum equated as before; the Angle *STA*, the Annual Argumentof the ſaid Apogæum; *TA* the leaſt Excentricity of the Lunar Orbit; *TB* its greateſt Excentricity. Biſſect *AB* in *C*, and with the Center *C*, and Interval *CA*, deſcribe the Circle *AFB*; let the Angle *BCF* be drawn equal to twice the annual Argument: *TF* being drawn will be the Excentricity of the Lunar Orbit; and the Angle *BTF* the ſecond Æquation of the Lunar Æpogæum. In order to determine them, let the mean diſtance of the Moon from the Earth, or the Semidiameter of the Lunar Orbit conſiſt of 1000000 Parts; its greateſt Excentricity *TB*, will conſiſt of 66782 of thoſe Parts, and the leaſt *TA*, or 43319 ſuch Parts; ſo that the greateſt Æquation of its Orbit,

bit, namely when the Apogæum is in the Syzygies, ſhall be of 7°. 39′. 30″, or perhaps of 7° 40′. 00″; (for there is reaſon to ſuſpect that that Equation varies according as the Apogæum is in ♋ or ♑;) but when it ſticks in the Sun's Quadrature, the ſaid greateſt Æquation muſt be 4°. 57′. 56″. and that the greateſt Æquation of the Apogæum be of 12°. 15′ 4″.

Having upon theſe Principles conſtructed a Table of the Æquations of the Moon's Apogæum, and the Excentricity of its Orbit for every Degree of the annual Argument, whence the Excentricity *TF* and the Angle *BTF* (namely the ſecond and chief Æquation of the Apogæum) agreeable to a given Time, may be eaſily taken; to the Place of the Moon's Apogæum firſt equated as before, add the lately found Æquation, if the annual Argument be leſs than 90°, or greater than 180°, but leſs than 270°; but if it be otherwiſe, let it be ſubtracted from it. The Sum or Difference will be the Place of the Moon's Apogæum equated the ſecond time; which being ſubſtracted from the Moon's thirdly equated Place, the remainder will be the Moon's mean Anomaly agreeable to a given Time. Moreover, from that mean Anomaly of the Moon, and the late found Excentricity of the Orb, you will have (by help of the Table of Æquations of the Center of the Moon calculated for every Degree of the mean Anomaly, and ſome Excentricities, *viz.* 45000, 50000, 55000, 60000, and 65000) the *Proſtaphæreſis* or Æquation of the Center of the Moon, as commonly; which being ſubtracted in the firſt Semicircle or 180° of the mean Anomaly, and added in the laſt, to the Moon's Place, hitherto three times equated,

you

you will have the Moon's Place equated a fourth time.

The Moon's greateſt Variation, namely that which happens when the Moon is in the Octants with the Sun, is almoſt reciprocally as the Cube of the diſtance of the Sun from the Earth. Let it be taken of 37′. 25″. when the Sun is in the Perigæum, and of 33′. 4″. when it is in the Apogæum: And let the Difference of this Variation in the Octants be reciprocally as the Difference of the Cubes of the Diſtances of the Sun from the Earth, and from thence conſtruct a Table of the aforeſaid variation of the Moon in the Sun's Octants (or of its Logarithm) for every ten, or ſix, or five Degrees of the mean Anomaly; and for the variation out of the Octants, as the Radius is to the Sine of twice the Diſtance of the Moon from the next Syzygy or Quadrature; ſo let the above-found Variation in the Octant be to the Variation agreeable to the given Aſpect, which being added to the Moon's Place above-found in the firſt and third Quadrant (reckoning from the Sun) or ſubtracted from it in the ſecond aud fourth, will give the Moon's Place equated a fifth time.

Again, as the Radius is to the Sine of the Sum of the Diſtances of the Moon from the Sun and of the Moon's Apogæum to the Sun's Apogæum, (or to the Sine of the exceſs of that Sum above 360°) ſo 2′. 10″. to the ſixth æquation of the Moon's Place, which is to be ſubtracted if the foreſaid Sum or the ſaid exceſs be leſs than a Semicircle, to be added if greater.

As the Radius is to the Sine of the Moon's Diſtance from the Sun, ſo alſo let 2′. 20″. be to a ſeventh Æquation. Take that away when the Moon's Light is encreaſed, and (on the contrary) add it when the Light is diminiſhed; and you'll have the Moon's Place ſeventhly equated, which is alſo its Place in its proper Orbit. It is to be obſerv'd, that the Æquation which is here expreſs'd by the Mean Quantity 2′. 20″. is not always of the ſame magnitude, but is encreas'd and diminiſh'd according to the ſituation of the Lunar Apogæum. For if the Lunar Apogæum be joyn'd with the Sun's Apogæum, the foreſaid Æquation becomes greater by 54″; if it be oppoſite to it, it is as much leſs: and it varies between the greateſt Quantity 3′. 14″; and the leaſt 1′. 26″. And theſe things hold good when the Lunar Apogæum is in the Sun's Syzygies; but when it ſticks in the Sun's Quadrature, the foreſaid Æquation is to be diminiſh'd about 50″ or one whole Minute, when the Moon's Apogæum and the Sun's are in Conjunction; but if they are in Oppoſition, I can't determine (for want of Obſervations) whether it is to be encreas'd or diminiſh'd. Neither can I affirm any thing certain concerning the aforeſaid laid down encreaſe or decreaſe of the Æquation by 2′. 20″, for want of Obſervations exact enough.

If the ſixth and ſeventh Æquation are encreas'd or diminiſh'd in a reciprocal Ratio of the diſtance of the Moon from the Earth, that is, in a direct Ratio of the Moon's horizontal Parallaxis; they will become more exact. And that may be readily done, if Tables be firſt conſtructed for every Minute of the ſaid Parallaxis, and every ſixth or fifth Degree as well of the Argument of the ſixth Æquation for the ſixth Æquation, as of the diſtance of the Moon from the Sun for the ſeventh.

From the Sun's true Place take away the mean Motion of the Moon's aſcending Node equated as before; the remainder will be the Annual Argument of the Node; whence its ſecond Æquation will be computed in the following manner: Let *T* (*Fig.* 27.) repreſent the Earth as before; *TS* a Right-line joyning the Earth and the Sun: Moreover, let *TACB* repreſent a Line drawn to the Place of the Moon's aſcending Node equated as before, and *STA* the Annual Argument of the Node. Let *TA* be taken in the ſame Ratio to *AB* as 56 to 3, or $18\frac{1}{3}$ to 1. Biſſect *BA* in *C*, and with the Center *C* and Interval *CA*, or *CB*, draw the Circle *AFB*, and let the Angle *BCF*, be equal to twice the Argument of the Annual Node found as before; and the Angle *BTF*, will be the ſecond Æquation of the aſcending Node, to be added whilſt the Node goes from the Sun's Quadrature to the Syzygy, and ſubtracted as the Node goes from the Syzygy to the Quadrature. And ſo you have the true Place of the Node of the Moon's Orbit: Whence from Tables conſtrued after the common manner, may be computed the *Moon's Latitude* and the *Reduction of the Moon from its Orbit to the Ecliptic*, ſuppoſing the Inclination of the Moon's Orbit to the Plane of the Ecliptic to be of 4°. 59′. 35″. when the Nodes are in the Sun's Quadrature; and of 5°. 17′ 20″. when they are in the Syzygies. From the Longitude and Latitude lately found, and the given Obliquity of the Ecliptic of 23°. 29′, the Moon's Right Aſcenſion and Declination will be found.

I put down 57′. 30″. for the Parallax of the Moon in the Syzygies, when it is in its mean Diſtance from the Earth; 33′. 32″. 32‴. for the horary Motion; and 31′. 30″. for the apparent Diameter. But in the Moon's Quadratures at a mean diſtance from the Earth, I make the horizontal Parallax of the Moon to be 59′. 40″. the Horary Motion 32′. 12″. 2‴. and the apparent Diameter 31′. 3″. The mean Diſtance of the Moon's Center when it is in Octant with the Sun, is diſtant from the Earth's Center about 60$\frac{2}{9}$ Semi-diameters of the Earth.

I ſet down 10″ for the Sun's horizontal Parallax; and 32′. 15″. for its apparent Diameter in its mean diſtance from the Earth.

The Earth's Atmoſphere, by refracting and diſſipating the Sun's Light, caſts a Shadow juſt as if it was an opake Body, as far as the height of 40 or 50 Geographical Miles: (I call a Geographical Mile the 60th part of a Degree of a great Circle on the Earth's Surface:) This Shadow in an Eclipſe of the Moon falling upon the Moon, encreaſes the Earth's Shadow. And Seconds in the Moon's Diſc anſwers to Miles in the Earth's Atmoſphere: Therefore the Semi-diameter of the Earth's Shadow projected in the Moon's Diſc, is to be encreas'd of about 50 Seconds, (or what comes to the ſame) in a Lunar Eclipſe the horizontal Parallax is to be augmented in a Ratio of about 70 to 69.

If ſeveral Places of the Moon nicely obſerv'd (chiefly about the Quadratures) be compar'd with the Places of it calculated for the ſame Time in the Theory above; it will appear whether or no there are any ſenſible Æquations wanting to make it more perfect.

PROPOSITION XXX.

TO *determine the time of a Mean Conjunction or Oppoſition of the Sun and Moon next following any given Time.*

A mean Conjunction is, when the mean Place of the Sun is the ſame with the mean Place of the Moon in the Ecliptic: And a mean Oppoſition is when that is oppos'd to this laſt. Look in the Tables of the mean Motion of the Moon from the Sun, the mean Diſtance of the Moon from the Sun for the given Time (reduc'd to a Mean) which if it be none, or of ſix Signs, then there will be either a mean Conjunction, or a mean Oppoſition; if it be leſs, note the defect, (or the Arc which the Moon will run thro' in order to overtake the Sun again if you want a mean Conjunction, or in order to be ſix Signs diſtant from it if you want a mean Oppoſition,) and from the Table of the mean motion of the Moon from the Sun, in Days, Hours, and parts of Hours, gather the Time requiſite to run thro' that Arc; namely, by always ſubtracting from Arc, yet to be run thro' the next leſſer Arc found in the Table, by marking the Time required to run thro' the ſubtracted Arc, and gathering thoſe Times into one Sum: This being added to the given time, will make up the Time of the mean Syzygy requir'd.

If any Times on each ſide the Syzygies be requir'd, to the afore found Time muſt be added the Time of one whole Revolution of the Moon about the Sun; and you will find the Time of the next mean Syzygy. And this may be repeated as often as you will, and ſo any number of Syzygies may be found.

POPOSITION XXXI.

TO determine the Time of the next true Conjunction or Oppoſition of the Sun and Moon for any given Time.

A true Conjunction of the Sun and Moon happens when the Sun's true Place is the ſame with the Moon's true Place corrected and reduc'd to the Ecliptic; and a true Oppoſition when that is oppos'd to this. Having found (by the foregoing) the Time of the next mean Syzygy (which need not be exactly determin'd, it being ſufficient nearly to do it) find (by *Prop.* 18. *B.* 3. and *Prop.* 28. *B.* 4.) the true agreeable Places of the Sun and Moon, which, if either they fall in together or are oppoſite, the mean and true Syzygies will alſo coincide; but if they don't, mark the diſtance of the Moon from the Sun, namely, that by which the Moon either has not yet reach'd or has paſs'd the Sun or its Oppoſition. But to find out the ſpace of Time, in which it will go thro' or has gone thro' ſuch an Arc, you muſt uſe the following Method: Find out the Places of the Sun and Moon for the hour before or after that Point of Time for which you already have them; by which means you will have the Sun and Moon's horary motions, and conſequently their Difference, or the horary motion of the Moon from the Sun for a given Time; that is, the ſpace of Time (*viz.* an hour) requir'd for the Moon to move from the Sun thro' a known Arc in the manner before found. And therefore, ſuppoſing the Sun and Moon to move equably in that ſmall Time that the true Syzygy may be diſtant from the mean Syzygy, you will have the time requir'd for the Moon to move from the Sun the length of the Arc of its diſtance from the Sun at the time of the mean Syzygy. That Time added to the Time of the mean Syzygy if the Moon has not yet reach'd the Sun or its oppoſite, or taken from it if it has already pas'd it, will give the requir'd Time of the true Syzygy.

If the greateſt exactneſs be deſir'd (as is done in the Ecliptical Syzygies) for the lately found Time of the true Syzygies find again the Sun and Moon's Places, and if they happen to be juſt together, or preciſely oppoſite, the thing is done; if not, let that time be again corrected as before; namely, from the almoſt true Time of the true Syzygy already found, find the exactly true Time after the ſame manner as you found the almoſt true Time, from the before found Time of the Mean Syzygy.

Thoſe that make Calculations don't endeavour to find the Horary Motion of the Moon from the Sun, agreeable to the mean Syzygy, after the manner that we have taught; but take it out of the Tables, where the horary Motions of the Sun and Moon are ſet down according to the Degree of Anomaly which they reſpectively have; and that not amiſs in the Sun, but in the Moon, which is ſubject to ſo many Inequalities, we ought to proceed more cautiouſly: But a farther Correction takes off all Error, that may ariſe from hence. After the ſame manner, the Time of any other Aſpects of the Sun and Moon may be determin'd.

SECTION VII.

Of the Moon's Eclipſe.

HAving in the third Book ſpoken of the Earth's true motion about the Sun, to which the apparent motion of the Sun about the Earth anſwers in all things; and in this Fourth Book, of the various Motions of the Moon ſeen from the Earth; and treated of all thoſe things which appear in the Heavens upon the account of the Diurnal Rotation of the Earth about its own Axis, in the Second Book; it is eaſy, from what has been before laid down, to deduce whatſoever happens to the Sun and Moon. But becauſe the *Eclipſes* of the *Luminaries* are ſome of the moſt remarkable Appearances in the Heavens, and Aſtronomy owes a great deal to their Contemplation and Calculation and to the finding out of the Cauſes of them; after the manner of others we will treat of them by themſelves, eſpecially ſince here occur ſeveral Speculations worthy to be fully diſcuſs'd. To which end we muſt lay down ſome Lemma's for each of the Eclipſes concerning the apparent Semidiameters of the Luminaries, the Shadows of the Earth and Moon, *&c.* which are commonly call'd the *Diagram of Hipparchus*, becauſe by the compariſon of them (which is eaſy in a Diagram) *Hipparchus* attempted to define the Magnitudes and Diſtances of the Luminaries; as it was before done by *Ariſtarchus*, in the *Book of the Magnitudes and Diſtances of the Sun and Moon*, and afterwards by *Ptolemy*, in the 15th and 16th Chapters of the *Fifth Book of the great Conſtruction.*

PROPOSITION XXXII.

TO *find the Sun and Moon's apparent Semidiameters agreeable to a given Time.*

Let the Sun and Moon's apparent Semidiameters be obſerv'd at any Time; and ſince, from the Theories of the Sun and Moon above given, their Diſtances from the Earth, either at the Time of Obſervation, or at any other Time, are given; their apparent Semidiameters will be alſo given: For the apparent Semidiameters of a Spherical Body are reciprocally as its Diſtances from the Eye.

Aſtronomers make Tables of the Sun and Moon's apparent Semidiameters, for every Degree (or every fifth Degree) of a Luminary's mean Anomaly. And moreover, as the Semidiameter of a Sphere, in the ſame Diſtance from the Center of the Earth, is greater as it is nearer the Zenith, becauſe it is nearer to the Spectator; and the Difference of it in the Horizon and in the Zenith be ſenſible in the Moon, but not in the Sun: Therefore they alſo make Tables of the encreaſe of the Moon's apparent Semidiameter for every (or every fifth) Degree of the Elevation of the Moon above the Horizon.

PRO-

PROPOSITION XXXIII.

IF an opake Sphere be expos'd to a lucid one, the whole or full Shadow (that is, the ſpace of the ambient Medium to which no Ray of the lucid Sphere can come at) will be caſt towards the parts oppoſite to the lucid Sphere included in the Superficies of a Right Cone, whoſe Axis is a Right line joining the Centers of the Spheres, whoſe Baſe is a Circle in the opake Sphere, dividing its enlighten'd from its darken'd part: And the vertical Angle of the Triangle thro' the Axis is nearly equal to the difference of the Angles under which the lucid Sphere is ſeen from the Center of the opake one, and the opake one from the Center of the lucid one. And if the Cone of Shadow be cut any how by a Plane parallel to the Plane of the Baſe, the Diameter of the Circle of Shadow ſo generated or made will appear from the Center of the opake Sphere, under an Angle nearly equal to the Interval of two Angles; one of which is the aggregate of the Angles under which the opake Sphere is ſeen under from the Center of the lucid Sphere, and from the Circle of Shadow; and the other, the Angle under which the lucid Sphere is ſeen from the Center of the opake one. [*Fig.* 28.]

Let there be two Spheres, whoſe Centers are *A* and *B*, one of which is lucid and the other opake; draw the Right line *A B*, thro' which a Plane is to paſs, whoſe common Sections with the Spheres are the Circles *D F*, *E G*; draw *D E*, *F G* Tangents to the two Circles on the ſame ſide, at the Points *D* and *E*, *F* and *G*, and let them be produc'd till they meet at *C*, with *A B* alſo produc'd. It is plain that the whole or full Shadow, which either of the Spheres, ſuppos'd opake, being enlightned by the lucid one, caſts, is contain'd under a Conic Surface deſcrib'd by the

Rotation of the Right line *DC* about the Axis *AB*; that is, by the Surface of a Right Cone whoſe Axis is the Right line *ABC* and Baſe, a Circle whoſe Diameter is the Right line *DF* or *EG*. To the Conic Surface I add a Cylindric one made when the Spheres happen to be equal.

From *A* draw Right Lines touching the Circle *GE* in *H* and *I*, and let *AH* be produc'd till it meets with *CD* at *K*, and from *B* draw Tangents to the Circle *DF* in *L* and *M*. By *Prop.* 32. *El.* 1. the Angle *ACD* is equal to *AKD*, taking away *KAB*: But (by reaſon of the great diſtance of the Spheres that we ſuppoſe) the Angle *AKD* is nearly equal to *ABL*; and therefore *ACD* is equal to *ABL*, taking away *KAB*. And taking the Double of them, the Angle *FCD* is equal to the Angle *MBL*, taking away the Angle *IAH*.

Let us ſuppoſe either of the Spheres, as for Example, *EG*, to be opake, and let the Cone of Shadow *ECG*, be cut any how by a Plane parallel to the Plane of the Baſe, and the Section will (by *Prop.* 4. of *B.* 1. *El. Conic.*) be a Circle, whoſe Diameter *NO* is parallel to the Right line *DF*, or *EG*, and whoſe Center *P* is in the Axis of the Cone; from which draw *PQ*, *PR* Tangents to the Circle *EG*, and join the Right lines *BO*, *BN*. By *Prop.* 32. *El.* 1. the Angle *PBN* is equal to *BNE*, having taken away *BCN*: But (if *BN* be almoſt equal to *BP* which I ſuppoſe) *BNE* is nearly equal to *BPQ*, and it has lately been ſhewn, that *BCN* is equal to *ABL*, taking away *BAH*; and therefore *BPN* is equal to the aggregate of the Angles *BPQ* and *BAH*, taking away the Angle *ABL*. And taking the double of them, the Angle *OBN* is equal to the aggregate of the Angles

QPR, *HAI*, taking away the Angle *LBM*. *Q. E. D.*

If *EG* be ſuppos'd a lucid Sphere, and *DF* an opake one, the ſame things would hold good, if *ON* and all the other Lines depending upon it be drawn on the other ſide of the Sphere *DF*, in reſpect of *B*.

SCHOLIUM.

If the Sphere *DF* repreſents the Sun; *EG* the Earth, reckoning as part of it, whatever being circumfus'd about it hinders the paſſage of the Rays; *ECG* the Shadow of the Earth; and *NPO* part of the Moon's Orbit: The Angle *ABL* will be the Sun's apparent Semidiameter, ſeen from the Center of the Earth; *BAH* the Sun's horizontal Parallax: *BPQ* the Moon's horizontal Parallax at the Diſtance *BP*; therefore *BCE* the Vertical Semiangle of the Cone of Shadow is equal to the difference of the apparent Semidiameter and the Sun's horizontal Parallax; and *PBN* the apparent Semidiameter of the Earth's Shadow, (thro' which the Moon is to paſs) ſeen from the Center of the Earth, equal to the Sum of the horizontal Parallaxes of the Sun and Moon, when the Sun's apparent Semidiameter is taken from it. And therefore, three of theſe four being given, the fourth will alſo be known: As for Example, the Sun's Parallax, and conſequently (by *Prop.* 46. *B.* 2.) its diſtance will from hence be known: For the difference of the Moon's horizontal Parallax and the Sun's Semidiameter, by Obſervation of the Notes, being ſubtracted from the Note (by Obſervation of the Moon's Eclipſe) of the Earth's Shadow, leaves the Sun's Parallax for the diſtance of the Moon, known by the apparent Semidiameter.

But if *D F* be the Sun, *E G* the Moon, and *NPO* part of the Diſc of the Earth (that is, of the Plane of a great Circle of the Earth, to which a Right line joining the Centers of the Sun and Earth is perpendicnlar) in which the Shadow falls; the Semidiameter of the Shadow is from *B*, the Center of the Moon, ſeen under the Angle *P B N* equal to the Sum of the apparent Semidiameters of the Moon ſeen from the Centers of the Sun and Earth, ſubtracting the Sun's apparent Semidiameter ſeen from the Moon's Center. And becauſe, by reaſon of the immenſe diſtance of the Sun from the Earth, in reſpect of the diſtance between the Moon and the Earth, the Sun's Diameter ſeen from the Moon appears almoſt under the ſame Angle as when ſeen from the Earth, and the Moon's Semidiameter ſeen from the Sun is to be look'd upon as of no magnitude, as being almoſt ſubquadruple of the Sun's horizontal Parallax; the Semidiameter of the Moon's Shadow, receiv'd upon the Diſc of the Earth, is ſeen from the Center of the Moon, under an Angle almoſt equal to the difference of the Angles, under which the Semidiameters of the Sun and Moon appear from the Center of the Earth.

But if the Semidiameter of the Sun, ſeen from the Earth, be greater than the Semidiameter of the Moon ſeen from it, the apparent Semidiameter of the Shadow (ſince it is equal to the leaſt, after the greateſt is taken away) becomes negative; that is, after the Shadow is terminated and ended in the Vertex of the Cone, the little Circle, made by the Section of a Cone, whoſe Vertex is plac'd againſt the Vertex of the Cone of Shadow, is ſeen from the Moon, under an Angle equal to the difference of the Angles, under which the Sun and Moon are ſeen from the Center of the Earth. But this little Circle is inſtead of a Circle of Shadow, for it is the moſt obſcure part of the Circle of *Penumbra*, and (like a full Shadow) equally dark; namely, whoſe Points, one by one, become enlighten'd as the Diſc of the Sun exceeds that of the Moon from the apparent Earth: For all the foregoing things are true of the Right line *ON* drawn beyond *C*.

PROPOSITION XXXIV.

IF an opake Sphere be expos'd to a lucid one, beſides the total or full Shadow, there will be a Penumbra *(that is, a ſpace of the ambient Medium, from all whoſe Points ſome Rays are kept off by the opake Sphere) towards the Parts turn'd from the lucid Sphere, included by the ſurface of a Right Cone, whoſe Axis is the ſame with the Axis of the full Shadow, and the Vertical Angle of the Triangle thro' the Axis is equal to the ſum of the Angles under which the lucid Sphere is ſeen from the Center of the opake one, and the opake one from the Center of the lucid one. And if this penumbrous Cone be cut any how by a Plane parallel to the Plane of the Baſe, the Diameter of the penumbrous Circle ſo generated (taking in the ſmall Circle of full Shadow at its Center, if there be any ſuch) ſeen from the Center of the opake Sphere, appears under an Angle equal to the aggregate of three Angles; namely, thoſe Two under which the opake Sphere is ſeen from the Center of the lucid one, and from the Center of the penumbrous Circle, and the Third under which the lucid Sphere is ſeen from the Center of the opake one.* (fig. 29.)

Things remaining as in the foregoing *Prop.* draw the Right lines *DE*, *FG*, touching the Circles on the alternate ſides, and croſſing each other in the Point C of the Right line *AB*. It appears, that beſides the whole Shadow, there is alſo projected about it a Penumbra on the ſide turn'd from the lucid Sphere, becauſe all the ſpace contain'd under the Conic ſurface, generated by the Right line *DE*, produc'd beyond the opake Sphere, and turn'd round the Axis *AB*, is ſuch, that from any Point of it ſome Rays of the lucid Sphere are kept off by the interpoſition of the opake one, and therefore it is not ſo much enlighten'd as the Parts round about this Conic Space, from which no Rays are kept off. And thoſe parts of this Space which are neareſt to the full Shadow, (namely, thoſe about its middle, from which all the Rays of the lucid Sphere are kept off) are more obſcure than thoſe which are more remote; as being enlighten'd by Rays flowing from a leſs part of the lucid Sphere: And the Surface that circumſcribes the Penumbra, is that of a Right Cone, whoſe Axis is the Right line *AB*, extended beyond the opake Sphere, and Baſe, a Circle deſcrib'd on the Diameter *DF*, or *EG*.

After the ſame manner as before, draw from *A* Right lines touching the Circle *GE* in *H* and *I*, of which *AH* may meet with *DE* produc'd at *K*: And from *B* let Right lines be drawn touching the Circle *DF* in *L* and *M*. By *Prop.* 32. *Elem.* 1. the Angle *ACD* is equal to *AKD* and *BAK* together; but *AKD* is nearly equal to *ABL*; and therefore *ACD* is equal to *ABL* and *BAH* together. And taking the double of them, the Angle *FCD* is equal to the Angles *MBL* and *IAH* taken together.

Let us ſuppoſe either of the Spheres, as for Example, *E G* to be opake, and let the penumbrous Cone be cut by a Plane parallel to the Plane of the Baſe, and the Section will be a Circle, whoſe Diameter *N O* is parallel to the Right line *D F* or *E G*, and whoſe Center *P* is in the Axis of the Cone; from which, let *P Q*, *P R* be drawn touching the Circle *E G*, and let the Right lines *B O*, *B N* be drawn. By *Prop.* 32. *Elem.* 1. the Angle *P B N* is equal to *B N C* and *BCN* together: But (if *B N* be almoſt equal to *B P*, which I ſuppoſe) *B N C* is almoſt equal to *B P Q*; and it has been lately ſhewn, that *BCN* is equal to *A B L* and *B A H* together; and therefore *P B N* is equal to the aggregate of the three Angles, *B P Q*, *B AH* and *A B L*. And taking the double of them, the Angle *O B N* is equal to the aggregate of the three Angles, *Q P R*, *H A I*, and *L B M*.

SCHOLIUM.

If the Sphere *D F* repreſents the Sun, *E G* the Earth, and *N P O* part of the Lunar Orbit; the Semidiameter of the Penumbra ſeen from the Center of the Earth, is equal to the ſum of the Horizontal Parallaxes of the Sun and Moon, and the Sun's apparent Semidiameter.

But if the Sphere *D F* repreſents the Sun, *E G* the Moon, *NPO* that part of the Diſc of the Earth in which the Penumbra falls; the Semidiameter of the Penumbra ſeen from the Center of the Moon, appears under an Angle equal to the aggregate of the apparent Semidiameters of the Moon, ſeen from the Centers of the Earth and Sun, and the apparent Semidiameter of the Sun ſeen from the Center of the Moon; and therefore (for the Reaſons alledg'd in the

Schol. of the foregoing) under an Angle nearly equal to the aggregate of the Semidiameters of the Luminaries, ſeen from the Center of the the Earth.

PROPOSITION XXXV.

TO *determine whether a Full-Moon to come, will be Ecliptical; and if there will be an Eclipſe, to define its Species; that is, ſhew whether it will be Partial or Total; whether it be* cum morâ *or* ſine morâ; *that is, whether the Moon makes any ſtay in the Earth's Shadow, or not.* (Fig. 30.)

By *Prop.* 31. find the time of the true Full-Moon upon a given Month, for which, (by *Prop.* 18. *B* 2.) find the Sun's Place, and you will conſequently have the Point oppoſite to it: Let it be expreſs'd by O, (*Fig.* 30.) where C ☊ repreſents the Ecliptic. Find the Places of the Moon's Nodes for the ſame Time, (of which let ☊ be neareſt to the Point O) and alſo the Inclination of the Moon's Orbit to the Ecliptic. Let the Circle of Latitude OP meeting the Moon's Orbit ☊ P in the Point P, where the Moon is at the time of the true Full-Moon, be ſuppos'd to be drawn. In the Heaven of the Moon the circle of Shadow expreſs'd by RG, is conſtantly about the Center O, being a Circle made by the Section of the Conic Shadow of the Earth, by a Poſition of the ſaid Heaven parallel to the Baſe of the Cone, which if the Moon MH, does in part, or wholly enter, there will be an Eclipſe; otherwiſe none. But the Moon comes neareſt of all to O, and therefore, if it is immers'd in the Shadow, it is moſt of all immers'd, when its Center is in the Point L, which the Arc of the great Circle $O L$, perpendicular to the Moon's Way, ſhews: For the

Point *L* comes neareſt to O than any other of the Way of the Moon. But it is plain, that if the Arc *O L* exceeds *O R* and *L M*, the Semidiameters of the Shadow and of the Moon together, (as in the 1ſt *Fig.* 30.) the Moon will go on it its Way ☊ *P*, quite clear of the Shadow. If *OL* be equal to the ſum of the ſaid Semidiameter, the Moon will indeed touch the Shadow, but neither enter it nor be Eclipſed. If the Arc *OL* be leſs than the aggregate of the Semidiameters of the Moon and the Shadow, and yet greater than their Interval, (as in the 2d *fig.* 30.) the Moon will be Eclipſed; but the Eclipſe will not be Total, but Partial of ſo many Digits as there are 12th Parts of the Diameter of the Moon's Diſc immers'd in the Shadow. But if the foreſaid Arc O *L*, be equal to the ſaid Interval of the ſaid Semidiameter, there will indeed be a Total Eclipſe, but one *ſine mora*: For as ſoon as the whole Moon is immers'd into the Shadow, the Moon's Diſc touching the Circle of Shadow within, will immediately begin to emerge as the Moon goes ever ſo little forward. But if the Arc *O L* be leſs than the foreſaid Semidiameter of the ſhadow and interval of the Moon, then the whole Moon will be immers'd into the Shadow, ſome part of the Shadow being extended beyond the Moon, (as in the 3d *fig.* 30.) whence the Moon's Eclipſe will not only be Total, but laſt ſome time; that is, will be Total *cum mora*, and that more or leſs, according to the Quantity of the Arc O *L*, the thickneſs of the Shadow and Velocity of the Moon.

Now the way to find the quantity of the Arc O *L*, is as follows; In the ſpherical Triangle ☊ *O P* (*fig.* 31.) Rectangular at O, ☊ O, the diſtance of the Node from the Point oppoſite to the Sun, and the Angle at ☊ of the Inclination of the Moon's Track to the Ecliptic at the time of Conjunction are given, from which may be found the Angle at P, and the Side OP. Again, in the Triangle POL, beſides the Right Angle at L, you have given the lately found Side and Angle, from which may be diſcover'd OL and PL. But you may readily find out OL from the Tables of the Moon's Latitude, by taking it equal to the Latitude for the Argument of the Latitude equal to ☊O: For if you ſuppoſe ☊ω equal to ☊O, it is plain, that the Arc $\omega\lambda$ perpendicular to the Ecliptic, is equal to the Arc OL; for in the Triangle OL☊, $\omega\lambda$☊, having a common Angle at ☊ all things are equal. Moreover, for the Time of the Full-Moon before found, the apparent Semidiameter is (by *Prop.* 32.) given, and (by *Schol. Prop.* 33.) the apparent Semidiameter of the ſhadow of the Earth, where the Moon goes thro' it; and therefore the Sum or Difference of the Semidiameters of the Shadow and of the Moon, may be compar'd with the before-found Arc OL, and from it find out of what kind the Moon's Eclipſe will be, according to what has been above demonſtrated.

Commonly by the ſhadow of the Earth, the total ſhadow is only meant; for that alone takes away all the Light of the Moon, becauſe no rays of the Sun can come at it: Neither does the Moon appear to us, Inhabitants of the Earth, to be Eclips'd, unleſs that happens. But the Moon does truly begin to be Eclips'd as ſoon as it enters into the Earth's Penumbra, becauſe in that caſe it is leſs enlighten'd, the opake Earth intercepting ſeveral of the Sun's Rays. But the effects of the Earth's Penumbra itſelf upon the Moon, are manifeſt to us Inhabitants of the Earth. Hence it happens, that when the Moon is about to be Eclips'd, it firſt grows dull and pale, eſpecially in that part of it which is near the Shadow. For (as it has been explain'd in the 18th *Prop. B* 1.) before the whole Sun be cover'd from any part of the Moon; that is, before the Moon comes to the whole Shadow, part of the Sun is intercepted from it by the Earth, and the greater part of the Sun, by its Limb, which is the neareſt to the total Shadow: But the whole Sun does illuminate the Moon more clearly than part of it, and the greater part of it more than the leſſer. If therefore we ſpeak of ſuch an Eclipſe of the Moon, inſtead of the whole ſhadow of the Earth (of which alone we ſpoke above) you muſt take the Penumbra, whoſe Semidiameters will be found by *Schol. Prop.* 34.

COROLLARY.

Hence will the Moon's *Ecliptic Terms* (as they call 'em) be defin'd. For if OL be leſs than the ſum of the Semidiameters of the Shadow and of the Moon, there will at leaſt be ſome Eclipſe. Therefore in the Triangle ☊LO, having beſides the Right Angle at L, the Angle L☊O, and the Side OL equal to the ſaid ſum of the Semidiameters, the ſide ☊O will be found, namely the diſtance of the Moon from the Node reckon'd in the Ecliptic; which if it be ſuch, at the time of the true Full-Moon, the Moon will paſs by the Shadow without any decreaſe of its Light: But if at the moment of the ſaid Full-moon the Moon's diſtance from the Node be leſs than the above-found Arc ☊O; the Moon will ſuffer ſome Eclipſe: But by reaſon of the ſmallneſs of the Angle at ☊ and of the ſides of the Triangle ☊*OP*, *OP* differs very little from *OL*, and therefore the Ecliptical Terms may ſafely enough be plac'd in it, when at the time of the true Full-Moon, the Latitude is equal to the ſum of the Semidiameters of the Moon and of the Shadow.

LEMMA.

If there be given in the Surface of a Sphere, BC, DE, *Arcs of great Circles, which in given Angles, ſtand upon* ABD, *which is alſo a great Circle, and alſo the Arc* BD, *to find the Angle* A, *which a great Circle drawn thro'* C *and* E, *comprehends with* BD. (fig. 32.)

Let the great Circles *BE*, *CD*, be drawn. In the Triangle *CBD*, the ſides *CB*, *BD*, together with the Angle *CBD*, which they contain, being given, you have alſo the ſide *CD*, and the Angle *CDB*: But the Angle *BDE* is given, therefore *CDE* will alſo be known. Moreover, in the Triangle *CDE*, the ſides *CD*, *DE*, and the Angle *CDE* being given, the Angle *CED* will alſo be known. Laſtly, if in the Triangle *ADE*, the Angles *ADE* and *AED* together with the ſide *DE* be known, the Angle *A* will be found.

PROPOSITION XXXVI.

THE Inclination of the Moon's Orbit to the Ecliptic, and the Horary Motion of the Sun and Moon being given, to find the Inclination of the Moon's Way from the Sun to the Ecliptic, or to the Circle of Latitude, agreeable to a given diſtance of the Moon from the Node. [Fig. 33.]

It has been ſhewn, in the foregoing Propoſition, how the Inclination of the Moon's abſolute Way, or of its Orbit to the Ecliptic may be found for a given Time. But for calculating the Eclipſes of the Luminaries, you muſt know how the Moon's Way from the Sun, or from a Point oppoſite to it, is at that time inclin'd to the Ecliptic; and the motion of the Moon from the Sun is to be conſider'd as if the Sun ſtood unmov'd, or rather as if the Sun, after having mov'd a little way, ſhou'd be drawn back to its old Place, and the Moon alſo as much in lines parallel to each other. In ſuch a caſe it appears, that the Way which the Moon going from the Sun enters into in the forementioned ſenſe, is different from the Moon's abſolute Way, and differently inclin'd to the Ecliptic.

In order to find this Inclination, let SC repreſent the Ecliptic, LB the Moon's abſolute Way, ☊ the Node. Let S be the Sun, or any other point of the Ecliptic, having a given diſtance from the Sun, L the Moon's Place, and its Latitude LA. In a ſmall part of Time (as for Example, an hour or two) let the Sun be ſuppos'd to have mov'd from S to σ, and the Moon from L to λ. Now ſuppoſe that the Sun be drawn back from σ to S, and the Moon be as much drawn back, *viz* from L to λ, along the Right line λl, equal and parallel to σS;

draw the Arc Ll, which will be the Way which the Moon, mov'd from the Sun, goes thro', tho' its abſolute Way be the Circle LB. To determine that Way Ll, there are given LA the Latitude of the Point L given in the Moon's true Way, and la the Latitude of the Point l, which is the ſame as $\lambda\alpha$, the Latitude of the Point λ, alſo given in LB, the true and abſolute Way of the Moon; becauſe its Diſtance λL, from the given Point L, is given; namely, the Way of the Moon, gone thro' in a given Time; for Aa, the difference of Longitude is given, namely, the Exceſs of the given $A\alpha$ above $a\alpha$ or $S\sigma$; and therefore (by the foregoing Lemma) you will have given the Angle between Ll, the Moon's Way from the Sun, and the Ecliptic; and the Angle Lla, comprehended between the ſaid Way and the Circle of Latitude, al. *Q.E.F.*

COROLLARY.

By how much leſs the motion of the Moon from the Sun is, by ſo much (*cæteris paribus*) the Angle between the Ecliptic and lL is greater; for, λl remaining the ſame, it is plain that by how much $L\lambda$ is leſs, by ſo much the Angle λLl is greater: And by how much the Point L is nearer to the Node, by ſo much (*cæteris paribus*) will the Angle between lL and the Ecliptic be greater; for by how much L is nearer to ☊, by ſo much will λL, a little part, conſider'd as a Right line, contain a greater Angle with ☊C, and therefore it will hold much more in lL: But the Angle $l\lambda L$ is alſo greater, becauſe λl is every where parallel to ☊C.

SCHOLIUM.

Aſtronomers conſtruct Tables of thoſe Angles, which the Moon's Way from the Sun comprehends with the Ecliptic, according both to the various horary motion of the Moon from the Sun, and to the Moon's diſtance from the Node, that the Way of the Moon through the Shadow of the Earth, or that of the Shadow of the Moon, over the Diſc of the Earth, may be readily and eaſily determin'd. They calculate theſe Tables, for every minute of a Degree of the Exceſs of the Moon's motion, above the Solar, in one hour: But as they ſerve chiefly for Eclipſes, the Tables of them are only carried on ſo far as ſuch a diſtance from the Node, in which the Moon may run into the Earth's Shadow, or project its own Shadow on any part of the Earth.

PROPOSITION XXXVII.

TO *determine the Mean or Middle Time of a Lunar Eclipſe, happening in a given Month.* [Fig. 34.]

Let ☊ *K* repreſent the Ecliptic, as before; ☊ *E* the Way of the Moon from the Sun, found by the foregoing *Prop.* for the mean Eclipſe, (if you will have it as it wou'd be for the whole Time of the Eclipſe, which may be done ſafe enough; otherwiſe you may work more exactly by often finding that Way of the Moon from the Sun;) ☊ the Node next to it; *DRCG* the circular Section of the Shadow of the Earth, made by the Heaven of the Moon, which the Moon in its Eclipſe runs into; whoſe Center or rather Pole is O. Round the ſame Pole or Center O, deſcribe another Circle *AFB* exterior to the Circle of Shadow, in ſuch manner that *AC*, the diſtance between their Circumferences may be equal to the Moon's apparent Semidiameter. Let theſe Circles meet with the Moon's Way in *M*, *N*, *A*, *B*. If ☊*E* does not cut the Circle *AFB*, there will be no Eclipſe, by the foregoing *Prop.* Let *OA*, *OB*, Arcs of great Circles, be ſuppos'd to be drawn, meeting with the Circle *DCG*, at *C* and *D*; and *OP*, a Circle of Latitude, meeting with the Moon's Way at *P*, as before. If thro' *O* be drawn *ORF*, an Arc of a great Circle, perpendicular to the Circle ☊*E*, meeting it at *L*; the Arcs *LB*, *LA*, and the Arcs *LM*, *LN* will (by the *El. of Spherics* be equal. Therefore the Moon (which we ſuppoſe to move equably at the time of the Eclipſe) takes up as much time in going from *A* to *L*, as from *L* to *B*. Let Circles be ſuppos'd drawn round the Centers or Poles *AB*, with the diſtances *AC*, *AD*, which will rightly repreſent the Moon (becauſe *AC* is to *OC* as the Semidiameter of the Moon to the Semidiameter of the Shadow) and will touch the Circle *DCG*, namely, at *C* and *D*. When the Moon's Center is in the Point *A*, the Moon's Eclipſe begins, (becauſe the Limb of the Moon touches the Limb of the Shadow at *C*,) and that Center being come on to *B*, the Eclipſe ends for the ſame reaſon; and therefore the Mean of that Eclipſe happens whilſt the Moon's Center is in the Point *L*, namely, nearly that in which when the Center is, the Moon will be moſt immers'd in the Shadow, as has been ſhewn in the foregoing *Prop.* We ſhall likewiſe ſhew, that at the time in which the Moon's Center comes to the Point *L*, in Total Eclipſes, will be the middle of the whole Obſcurity, when the whole Moon is immers'd in the Shadow; or, in a word, it is the middle of the paſſage of the Center of the Moon from *N* to *M*, becauſe *N L* is equal to *L M*. The place of the Node and the inclination of the Limits, are look'd upon as unchangable, not only by reaſon of the ſmall time of the duration of the Eclipſe, but becauſe when the Nodes are in the Syzygies with the Sun, theſe are truly unmov'd, by *Prop.* 14 and 15. Therefore the Problem returns to this, *viz.* That the Time is to be determin'd when the Moon's Center is found at *L*. For doing of which, you muſt (by *Prop.* 31.) find the Time of thetrue Oppoſition; that is, the time when the center of the Shadow will be at *O* and the Moon at *P*; unleſs theſe two coincide, which happens when in the time of the true Oppoſition the Sun is in one of the Moon's Nodes, in which caſe *OL*, *PL*, and *OP*, vaniſh: But alſo the magnitude of the Arc *L P*, has been found in the foregoing *Prop.* by the reſolution of the Triangles ☊*OP*, *OLP*. Therefore the time requir'd for the Moon to run thro' *P L* muſt be found, and not only *P L* ſimply, but *P L* beyond the Sun's motion in the Ecliptic; (for whilſt the Moon paſſes from the true Oppoſition to a point of its Orbit, next to the center of the Shadow, or from this Point to that true Oppoſition, the Sun, and conſequently its oppoſite Point, which is the center of the Shadow, goes on in the Ecliptic;) which is done by finding the Horary Motions of the Sun and Moon agreeable to the time of the Eclipſes of the Sun and Moon, and from thence their differences agreeable to the time of the Moon's Horary Motion from the Sun, as in the 31ſt *Prop.* when by Analogy you will find the Time requir'd for the Moon to move from the Sun thro' the known Arc *LP*. By the inſpection of the Figures adapted to the reſpective Caſes, it is plain, that the Time in which the Moon's Center is in the Point *L*, near to the Center of the Shadow, goes before the Time of the true Oppoſition, when the Moon is come to *P*, in the preſent caſe; that is, when the Moon has juſt paſs'd the aſcending Node, which obtains alſo when it has juſt paſs'd the deſcending Node: But when the Moon is not yet come to the Nodes, the Time of the mean Eclipſe is after the Time of the true Oppoſition: Whence it is plain, when the Time between the true Oppoſition and the mean Eclipſe is to be added to the Time of the true Oppoſition, and when it is to be ſubſtracted from it, to make up the Time of the mean Eclipſe.

Aſtronomers commonly make Tables of the Time in which the Arc *LP* is run thro', to be added or ſubtracted from the Time of the true Syzygy, to make the Time of the mean Eclipſe, or of the greateſt Obſcurity. That Time varies according to the different Velocity of the motion of the Moon from the Sun, and the different diſtance from the Node. They continue theſe Tables, as well as thoſe of which we ſpoke before in the *Schol. Prop.* 36. for every degree of the Ecliptic Bounds.

PROPOSITION XXXVIII.

TO determine the Time of the Beginning and End of the Eclipſe of the Moon that happens in a given Month, and alſo of the whole immerſion of the Moon into the Shadow; and from thence alſo determine the Duration, as well of the whole Eclipſe as of the whole Darkneſs. [Fig. 35.]

Things being as in the foregoing, except that in the firſt *Fig.* 35. the Moon in its Eclipſe is ſuppoſed to tend towards the deſcending Node, and it appears that the Eclipſe begins when the Center of the Moon is in *A*, the Moon's Limb touching the Limb of the Shadow at *C*, by *Prop.* 4. of the 2d *B.* of *Theod.* Therefore in the Triangle *ALO*, Rectangular at *L*, you have, beſides the Right Angle, the Hypothenuſe *OA*, namely, the ſum of the Semidiameters of the Moon and of the Shadow, and *OL* the leaſt diſtance of the Center of the Shadow from the Center of the Moon, found by *Prop.* 35. Therefore the Arc *AL* will be found, and conſequently the Time requir'd for the Moon (by the exceſs of its Motion above the Sun's Motion) to go thro' it; that is, half the duration of the whole Eclipſe. But the Time of the mean Eclipſe is known by the foregoing; therefore its Beginning and End is known.

For determining the beginning of the whole immerſion of the Moon into the ſhadow of the Earth, things remaining as before, obſerve the 2d *Fig.* 35. where the Moon's Eclipſe happens whilſt the Moon goes towards the aſcending Node, and there let the Circle *AFB* be within the Circle of Shadow *CDG*; a Circle drawn with the Diſtance *AC* about the Center *A*, will (by *Prop.* 3. *B.* 2 of *Theod.*) touch *CGD* within, and therefore will rightly repreſent the Moon going on in its proper Way, as ſoon as it is all immers'd into the Shadow *GCD*. To find the Point of Time when the Center of the Moon is in *A*, you have given in the Rectangular Triangle *ALO*, the ſide *OL* (by *Prop.* 35.) and *OA*, which by conſtruction is equal to the difference of the Semidiameters of the Moon and of the Shadow; therefore the Arc *AL* of the Moon's

Way to be run thro' by the Moon's motion above that of the Sun, (from the time that the whole Moon is immers'd into the Shadow, until the middle of the Eclipſe) will be known. Therefore from knowing the motion of the Moon from the Sun agreeable to a certain Time, (*viz.* an Hour,) at the time of the Eclipſe, you will know what Time muſt be taken up that the Moon (by the exceſs of its motion above that of the Sun) may run thro' the Arc *A L*; that is half the duration of the whole Darkneſs. But by the foregoing *Prop.* the moment of the middle Eclipſe is known, whence the Beginning and End of the total immerſion of the Moon into the ſhadow of the Earth, will be known. *Q. E. F.*

SCHOLIUM.

In the trigonometrical reſolution of the Triangles *O L P*, *A L O*, *B L O*, as alſo of the Figures *L l λ*, *A L λ α*, in the foregoing *Prop.* they may be look'd upon as Rectilineal, (which alſo often happens) by reaſon of the ſhortneſs of the ſides: For ſo ſmall a Portion of a Spherical ſurface may ſafely be look'd upon as Plane.

PROPOSITION XXXIX.

TO *define the Quantity of an Eclipſe happening at a given Month, as well in its middle as in any other Time.* [Fig. 36.]

Things being as before, the Figure repreſenting the Moon's Eclipſe, which happens a little after it has paſs'd the deſcending Node, we muſt find the quantity of a middle or mean partial Eclipſe. Let the Diſc of the Moon in the middle of the Eclipſe drawn about the Center *C*, (already often determin'd) cut the Right line

O R at the Point *M*. Having then, (in parts of a great Circle in the Celeſtial Sphere) *O R*, the Semidiameter of the Shadow, and by *Prop.* 35. found the Arc *O L*, which is the leaſt diſtance of the Center of the Moon from the Center of the Shadow in the ſame Parts; you will have (by *Euc. Dat.* 4.) *L R*, the difference of thoſe Arcs: But *L M*, the Moon's Semidiameter is is given; whence (by *Dat.* 3.) *M R*, the ſum of them is alſo given in the ſame parts; and therefore (by *Dat.* 1.) the ratio to the Moon's whole Diameter *M H*, alſo known in parts of a Circle: And *M R* is part of the Diameter of the Moon darkned in a mean Eclipſe; therefore the ratio of the darkned Parts Diameter to the whole Diameter is given, and conſequently the number of Twelfths and parts of Twelfths of the darkned Diameter.

After the ſame manner may be found the Quantity of an Eclipſe for a given Time, ſeen either before or after the true Full-Moon. Having by the foregoing method found the Moon's horary Motion from the Sun, the Arc of the Moon's diſtance from the Point *P* will be determin'd, in which Point the true Full-Moon is. Let it be *P S*; therefore the place of the Moon's Center for the given Time is *S*. Draw the Arc O *S*, meeting with the Moon's Diſc at *M* and *H*, and the Circle C *D G* at *R*. In the Triangle ☋ O *P*, you have beſides the Right Angle at *O*, the ☋ *O*, and the Angle *O* ☋ *P*; whence you will find the ſide O *P* (already defin'd) and the Angle O *P* ☋. Again, in the Triangle *O P S*, you have the two ſides *P O*, *P S*, and *O P S*, the Angle which they contain; whence you will alſo have the Arc *O S*, and conſequently the difference between it and the known

Arc *S M*, namely *O M*: But *O R* is given, and conſequently *MR* the difference of *O R*, *O M*; and all theſe in parts of a Circle as before; and therefore, as in the foregoing caſe, the Quantity of the Eclipſe in Twelfths and parts of Twelfths.

If *MR* exceeds the Moon's whole Diameter (*Fig.* 37.) the Eclipſe ſaid to be of ſo many *Digits* as *MR* contains 12ths of the Moon's Diameter; but improperly, becauſe ſtrictly ſpeaking, the Moon cannot be Eclipſed more Digits than its Diameter contains; but *MR* is made up of the Moon's Semidiameter *MS* and *SR*, the leaſt diſtance of the Center *S* from *CDG*, the margin of the Shadow, which (by *Elem. Spheric.*) is an Arc of a great Circle joining the Centers O and *S*.

COROLLARY 1.

Hence may eaſily be determin'd the Places of the Earth, from which a given Phaſis of the Moon's Eclipſe may be ſeen. For ſince by this *Prop.* the Place of the Eclips'd Moon for that Phaſis is given, and the Hour is a given Place of the Earth; you will (by *Corol.* 36. *B.* 2.) have that Place of the Earth, to which the Mooon is Vertical, and all the Places, which are diſtant from it leſs than a quarter of a Circle, will ſee it; thoſe which are farther diſtant, will not ſee it; thoſe which are juſt at the diſtance of a Quadrant, will ſee it in the Horizon.

COROLLARY 2.

By help of the three laſt Propoſitions you may exactly draw a Scheme of the Moon's Eclipſe upon a Plane. For ſince ſo ſmall a part of the Heaven of the Moon, as the Shadow (or even the Penumbra) of the Earth, together with the Moon touching it at the beginning of the Eclipſe, may ſafely be taken for a Plane: Having drawn Right lines, which croſs one another in an Angle equal to that which the Way of the Moon contains with the Ecliptic, repreſenting thoſe Circles, and the next Node by their concourſe; in the Right line which repreſents the Ecliptic, take a Point diſtant from the common concourſe of the Right lines, ſo many equal parts of any kind, as the parts (as for Example, Minutes) are of the Circle, which meaſure the Diſtance of the Center of the Shadow, or Point oppoſite to the Sun from the Node; on that Center deſcribe a Circle repreſenting the Earth's Shadow, taking for the Semidiameter of it as many of the equal parts taken before, as the Semidiameter of the Shadow takes up Minutes in the Heavens; drawing Right lines from that Center perpendicular as well to the Ecliptic, as to the Moon's Way, you will in the Moon's Way have the Points of the true Full-Moon, and of the middle Eclipſe. If with the ſame Center, both within and without that Circle of Shadow, you draw two other Circles diſtant from the firſt, by the ſpace of ſo many equal parts taken before, as the Moon's apparent Semidiameter has Minutes; the interſections of thoſe Circles, with the Right line repreſenting the Moon's Way, will, in the Scheme, ſhew the Points in which the Moon's Center is, at the beginning and end of the Eclipſe, and of the total Obſcurity. Moreover, as the horary Motion of the Moon from the Sun, and the number of Hours in a given place, at the time of the true Full-Moon, and alſo of the Ecliptic, in its beginning, middle, and end, by the foregoing Propoſition;

poſition; the Right line, which repreſents the Moon's Way, will be divided into equal parts, which will be mark'd by numbers which denote the Hours, reckon'd in a given Habitation, in which the Moon is in homologous Points of the Heaven. And the Phaſis of the Eclipſed Moon happening at any time, will be made very viſible, if from a point in the Moon's Way, (mark'd with the number of the given Time) for your Center, you draw, with an Interval repreſenting the Moon's apparent Semidiameter, a Circle: This Circle, by that part which it has within the Circle repreſenting the Shadow, will repreſent the Eclips'd part of the Moon; and the part which ſtands out of the Circle of Shadow, (if any part does reach out of it) will ſhew the enlighten'd part: Therefore the Ecliptical Digits ſhew themſelves, as alſo the Ratio between the whole Diſc of the Moon and its bright part (if it has any;) namely, the difference between the two Segments of the given Circles, which is evident from the dimenſion of the Circle.

SECTION VIII.

Of the Sun's Eclipſe.

HAving thus ſpoken of the Lunar Eclipſe, we come now to the Solar one. And firſt we will ſpeak of it univerſally, and conſider the Moon's Shadow falling upon the Earth, and how it might truly and properly be called an Eclipſe of the Earth: For in this Eclipſe the Earth does the ſame as the Moon in the Lunar Eclipſe deſcribed above, and the Moon does in this caſe, what the Earth did in that. Afterwards we will conſider this Phænomenon of the Eclipſe of the Earth, as the Moon covers ſome part of the Sun from an Inhabitant of the Earth, immers'd in the Moon's Shadow and looking up to the Heavens, in which part the Sun appears to be Eclips'd for that reaſon; whence this Phænomenon has been call'd an Eclipſe of the Sun.

PROPOSITION XL. LEMMA.

IF the apparent Diſtance between the Sun and the Moon, ſeen from the Center of the Earth, be equal to the Moon's horizontal Parallax; and you encreaſe it by the Sun's horizontal Parallax, it will be equal to the diſtance between the Center of the Earth, and the Center of the Moon's Shadow, receiv'd upon the Diſc of the Earth, ſeen from a Point where a Right line joining the Centers of the Sun and Earth, meets with the Moon's Heaven: And proportionally, if the diſtance of the Sun and Moon be greater or leſs. [Fig. 38.]

Let the Earth's Center be *T*, the Sun's *S*, *TS* a Right line joyning the Center of the Sun and

Earth, *DT* a perpendicular to the Diſc of the Earth, meeting the Moon's Heaven at *M*. Let ☊*L* be the Moon's Orbit, in which *L* is the Moon; ☊*M* the Ecliptic; *LM* the apparent diſtance of the Sun and Moon, ſeen from *T*, equal to the Moon's horizontal Parallax. Let *SL* be drawn and produc'd, till it meets with the Earth's Diſc *DT* at *V*, where the Center of the Moon's Shadow, receiv'd upon the Diſc of the Earth, is. I ſay that the Angle *TMV* is equal to *STL*, which is the Sum of the apparent diſtance of the Sun and Moon, and the Sun's horizontal Parallax. Draw *LR* parallel to *MT*, and join *MR*. Becauſe the Angle *TMV* is equal to the Angles *TMR* and *RMV* taken together, and the Angle *TMR* (by *Prop.* 29. *El.* 1.) is equal to *LRM*, that is, (upon account of the nearneſs of the Points *R* and *T*) *LTM* and *RMV* (for the ſame reaſon) is equal to *RLV*, or (by *Prop.* 29. *El.* 1.) to *LSM*; *TMV* will be equal to the Angles *LTM* and *LSM* taken together. Moreover, becauſe the Angle *LTM* is ſuppos'd equal to the Moon's horizontal Parallaxis, and *L* the Moon; the diſtance of the Points *L* and *M* will be equal to the Earth's Semidiameter; (for the Moon's horizontal Parallax is the Angle under which the Earth's Semidiameter is ſeen, at a diſtance equal to the diſtance of the Earth and the Moon) therefore the Angle *LSM* is nearly the Sun's horizontal Parallax (namely, the Angle under which the Earth's Semidiameter appears at the diſtance of the Sun from the Earth, that is, almoſt at the diſtance of the Sun and Moon) and the Angle *LTM* is the Diſtance of the Sun and Moon ſeen from the Center of the Earth. And therefore the Angle *TMV* is equal to the diſtance of the

Sun and Moon, ſeen from the Center of the Earth, together with the Sun's horizontal Parallaxis.

After the ſame manner, if the Arc *LM* be equal to half, or one and a half, or twice the Moon's horizontal Parallaxis; the Angle *TMV* will be equal to *LTM*, and a half, or one and a half, or twice the Sun's horizontal Parallaxis. *Q. E. D.*

PROPOSITION XLI.

TO *determine whether the New-Moon of a Month ſhall be Ecliptical, that is, whether in any part of the Earth the Sun will appear to be partially or totally Eclips'd; and if there is to be an Eclipſe, to determine its kind.* [Fig. 39.]

Let *DCG* repreſent the Earth, in which the Poles of the Diurnal Revolution are *P*, and the latent Point oppoſite to it. Let the Hemiſphere in the Figure, be that which is enlighten'd by the Sun; and therefore the Circle *DCG*, whoſe Center is *T*, will divide the enlighten'd from the darken'd Hemiſphere, whoſe Plane is the Diſc of the Earth. Let the Shadow of the Moon, caus'd by the Sun, be ſuppos'd to fall on the Plane of the Diſc of the Earth: The Figure of the Shadow ſo receiv'd will be circular, as *HM* having *V* for its Center; becauſe the penumbrous Cone is right, and its Axis nearly perpendicular to the Plane of the Diſc; for we ſpeak of ſuch a caſe as *V* is but very little diſtant from *T*, becauſe it is only in that caſe that the Moon's Shadow can reach the Earth; that is, that ſome ſort of Eclipſe of the Sun may happen. In the Center of it there is a little Circle altogether dark if the Moon's total Shadow reaches ſo far: The remaining Crown round about is penumbrous or almoſt dark, thoſe parts of it being darkeſt which are neareſt to the aforeſaid ſmall Circle, by *Prop.* 34. It is plain, that if the Penumbra does not touch the Earth, there will not be an Eclipſe in any part of the Earth: If ſome part of the Earth be immers'd in the Penumbra, it will (properly ſpeaking, as we ſaid before, of the Moon) ſuffer ſome Eclipſe; or the Sun will, by reaſon of the Interpoſition of the Moon, appear to be Eclips'd, to the Inhabitants of that Tract of Land. But it is plain, that the Shadow reaches the Earth, if the Circle HM cuts the Circle DCG; otherwiſe it does not, becauſe DCG is a great Circle of the Earth.

Now, to know if, on the New Moon of a given Month, the Circle HM will cut the Circle DCG, you muſt ſuppoſe that Hemiſphere of the Earth which is enlighten'd by the Sun, and the penumbrous Circle to be ſeen from an Eye plac'd above, in that Point, where the Right line joining the Sun's and the Earth's Centers, meets with the Heaven of the Moon; in which caſe that Hemiſphere, tho' it be convex, will appear as the Plane of a Diſc, as it happens when we look at the Moon, the diſtance of the Eye naturally taking off the Convexity. The Ecliptic, upon the ſame account, will appear like a Right line, *viz.* ☊TK, when the Eye happens to be in its Plane: And becauſe the Eye is but a little way off from the Plane of the Moon's Orbit, the Moon's Orbit, receiv'd upon the Diſc, or rather the Way of the Center of the Circle of Shadow; that is, the Line which it deſcribes in that Plane of the Diſc of the Earth, will alſo be a Right line; namely, ☊l. From T, the Center of the Earth, draw TV perpendicular to ☊l, and V will be the neareſt of all the Points of ☊ *l* to the Center of the Earth. Therefore if the penumbrous Circle *HM* does at all reach *CDG*, the Earth's Diſc, it will reach it when its Center is in *V*: For in any other Point theſe Circles are farther from each other. And for the ſame reaſon, if *HM* reaches *CDG*, it will moſt of all run into it when the Center of the penumbrous Circle is come to *V*. But it is plain that the Circle *HM* does not touch *DCG*, if when the Center of *HM* is come to *V*, *VT* exceeds *TR* and *VM* taken together: But (by the foregoing *Prop.*) when the Place of the Eye is ſuppos'd as above *TV*, it appears under an Angle equal to the Diſtance of the Sun and the Moon, ſeen from the Center of the Earth, augmented by an Angle which is to the Sun's horizontal Parallax, as the ſaid Diſtance of the Luminaries to the Moon's horizontal Parallax; and *TR* appears to the ſame Eye under an Angle equal to the Moon's horizontal Parallax; and *VM* (by *Schol. Prop.* 34.) is ſeen from it under an Angle equal to the aggregate of the Semidiameters of the Sun and Moon, ſeen from the Center of the Earth: If therefore, when the Center of the penumbrous Circle is come to *V*, the diſtance of the Luminaries, ſeen from the Center of the Earth, augmented by an Angle which is to the Sun's horizontal Parallax as the ſaid Diſtance of the Luminaries is to the Moon's horizontal Parallax, be greater than the aggregate of the Moon's horizontal Parallax, and the Semidiameters of the Luminaries; it is plain, that in ſuch a caſe there will not be an Eclipſe of the Sun in any part of the Earth. And as in the New-Moons, in which there is a Suſpicion of an Eclipſe, or in thoſe in which *HM* touches *DCG*, the apparent Diſtance of the Luminaries is ſeſquialter (or as $\frac{1}{2}$ to 1) to the Moon's horizontal Parallax, (namely *TV* is ſeſquialter to *TR*) when *VM* the aggregate of the apparent mean Semidiameters of the Luminaries, is as it were ſubduple of *TR*, the Moon's Mean horizontal Parallax; there will be no Eclipſe on the whole Earth, if the Center of the Circle *HM*, being come to *V*; the diſtance of the Luminaries, ſeen from the Center of the Earth, encreas'd by one and a half of the Sun's horizontal Parallax, exceeds the Sum of the Sun's and Moon's Semidiameters, and the Moon's horizontal Parallax; that is, if the diſtance of the Luminaries, ſeen from the Center of the Earth, exceeds the Sum of the Semidiameters of the Luminaries, and the Moon's horizontal Parallax, once and a half the Sun's rallax being ſubducted from it.

And if that Diſtance be equal to this Sum, the Penumbra bruſhes againſt the Earth, but will not fall upon it, neither will the Sun appear Eclips'd to an Inhabitant of the Earth.

But if the diſtance be leſs, that is, if when the Center of the Penumbra [*Fig.* 40.] is in *V*, the diſtance of the Luminaries, ſeen from the Center of the Earth, wants of the Sum of their Semidiameters and the Moon's horizontal Parallax, by the Quantity of once and a half the Sun's Parallax; that is, if *TV* be leſs than *TR* and *VM* together, or the Circle *HM* cuts the Circle *DCG* (namely, in *V* and *Z*) and at the ſame time the little Circle which is quite dark in the Center of the Circle *HM*, either not touches at all, or does but juſt touch the Circle *DCG*; part of the Sun will appear Eclips'd to thoſe that inhabit that part of the enlightned Hemiſphere, which is taken in by the Segment *VRZM* common to both Circles.

But becauſe the Semidiameter of this little dark Circle to an Eye poſited as before, appears under an Angle equal to the Semidiameter of the Moon, ſeen from the Earth's Center, the Sun's apparent Semidiameter from it being taken from it, by *Schol. Prop.* 33. it is plain that this Circle (tho' at *V*) will not reach the Diſc *CDG*, if the diſtance of the Luminaries, ſeen from the Center of the Earth, together with the Sun's horizontal Parallax (for in that caſe *TV* does not much exceed *RT*) does exceed the Moon's horizontal Parallax and the Exceſs of the Moon's apparent Semidiameter above the Sun's apparent Semidiameter, or is equal to them; that is, if the ſaid diſtance of the Luminaries exceeds the Semidiameter and horizontal Parallax of the Moon, taking from it the Semidiameter and horizontal Parallax of the Sun; or if it be equal to them: But if it wants of that Sum, then the whole Shadow will fall upon the Earth, and make a whole Eclipſe of the Sun to the Inhabitants of that Tract of the Earth which it falls upon; but without any ſenſible *mora* or duration, becauſe this little Circle is very ſmall, when the Moon's apparent Diameter does but a little exceed the Sun's apparent Diameter, and therefore is commonly look'd upon as the Center of the Penumbra. Nay, ſometimes this Circle quite vaniſhes, when the Sun's apparent Diameter is greater than the Moon's apparent Diameter. Now in either caſe, ſome place of the Earth has a total (or a central, tho' not total) Eclipſe, when the Center of the Penumbra being at the leaſt diſtance from the Center, the apparent diſtance of the Luminaries is equal to, or leſs than the Sum of their horizontal Parallaxes.

But the whole Penumbra of the Moon darkens the Earth, firſt when *TV* (*Fig*. 41.) the leaſt diſtance of the Centers of the Earth and penumbrous Circle, that is, (by the foregoing *Prop.*) the leaſt apparent diſtance of the Luminaries half the Sun's horizontal Parallax being ſubtracted from it, (for *VM* is nearly equal to half *TR*,) is equal to the difference of *TR*, *HV*, or to the exceſs of the Moon's horizontal Parallax above the ſum of the Semidiameters of the Luminaries. And if that Diſtance is leſs than this Exceſs, the whole Penumbra falls upon the Earth, all the Tract of Land round about has no Eclipſe of the Sun.

Tho' the Moon's Shadow or Penumbra receiv'd upon the Plane of the Diſc be a Circle, yet on the Spheric ſurface of the Earth (where it really falls) it is circumſcrib'd by a line not at all Circular, (unleſs it be when the Axis of the Cone of Shadow is extended thro' the Center of the Earth,) and even not lying in the ſame Plane; (of which kind of Lines P. *Courſier* wrote a ſmall Treatiſe in the Year 1663.) which is alſo true of the Shadow of the Earth receiv'd on the Moon's ſpherical Surface. But in treating of the Moon's Eclipſe, there was no need of taking notice of this, becauſe its ſpherical Surface does not differ from a flat Diſc to an Obſerver on the Earth. But in the Earth, our Habitation, the thing is otherwiſe: For all the Spectators in the Earth which have an Eclipſe of the Sun of the ſame Quantity in the ſame moment of Time, lie under ſuch a line. But the determination of it does not belong to this Place, nor indeed properly to Aſtronomy.

But the neareſt diſtance between the Centers of the Luminaries near the New-Moon ſeen from the Earth is to be found the ſame way as the neareſt diſtance between the Centers of the Moon, and of the Shadow of the Earth near the Full-Moon, was found in *Prop.* 35; namely, finding (by *Prop.* 31.) the Time of a true Conjunction, happening on a given Month, *viz.* when the Moon is found in *N*, the Sun being at *S*, [*fig.* 42.] *SN* being drawn for a Circle of Latitude, and finding the Places of the Moon's Nodes for the ſame Time, (of which ☊ muſt be neareſt to *S*) as alſo the Inclination of the Moon's Orbit to the Ecliptic; upon which account, in the Spherical Triangle ☊ *SN* rectangular at *S*, you have given ☊ *S* the diſtance of the Node and the Sun, and the Angle ☊, which is the Inclination of the Lunar Orbit to the Ecliptic; whence you will find the Side *SN* and the Angle *N*. Again, in the Triangle *SLN*, (which, by reaſon of its ſmallneſs, may be look'd upon as rectilineal) beſides the Right Angle at *L*, (for *SL*, by conſtruction, is perpendicular to the Moon's Orbit, therefore the next diſtance of the Luminaries, namely, when the Moon is in *L*) you have given the lately found Side *SN*, and the Angle *N*; whence *SL*, the leaſt diſtance of the Luminaries, will be found, and alſo the Side *NL*, which will be of uſe afterwards: But you will readily find the Arc *SL* from the Tables of the Moon's Latitude, by taking it equal to the Moon's Latitude, when the Argument of the Latitude is equal to the Arc ☊ *S*, the diſtance of the Sun from the Node, the reaſon of which we have ſhewn in the ſame caſe, *Prop.* 35.

COROLLARY.

Hence will be defin'd the Terms for all manner of Eclipſes of the Sun, that happen in any part of the Earth. For if *TV* be leſs than the Sum of the Semidiameters of the Earth's Diſc, and the Moon's Penumbra, there will only be a partial Eclipſe in ſome part of the Earth. If *TV* be leſs than *TR* the Semidiameter of the Diſc, there will ſomewhere be a total Eclipſe of the Sun; at leaſt, a central one. Therefore in the rectilineal Triangle ☊*VT*, which is deſcrib'd in the Diſc of the Earth, having given, beſides the Right Angle at *V*, the Angle *V*☊*T*, and the Side *VT*, equal to the ſaid Sum of the Semidiameter, you will find the Side ☊*V*; namely, the diſtance of the Center of the Moon's Penumbra from the next Node, and of the Moon from the oppoſite Node; which being ſuch, at the Time of the true New-Moon, the Penumbra will go by the Earth without touching it: But if, at the ſaid moment of the New-Moon, the diſtance from the neareſt Node be leſs than the above found Arc ☊*V*, the Moon's Penumbra will involve ſome Tract of the Earth, and the Sun will appear under ſome ſort of Eclipſe to the Inhabitants of that Tract. After the ſame manner will the Terms be defin'd for a central Eclipſe of the Sun, happening ſomewhere in the Earth, by taking the Semidiameter of the Diſc itſelf, inſtead of the Sum of the Semidiameters of the Diſc and Penumbra, as was before ſhewn. But by reaſon of the ſmallneſs of the Angle at ☊, and of the Sides of the Triangles *VNT*, *TN* differs but very little from *TV*: Whence they will nearly be the Terms for the Sun's Eclipſe, whether total or of any kind happening in any place; namely, the diſtances of the Moon from the next Node, when at the Time of the true New-Moon the Latitude is equal either to the Sum of the Semidiameters of the Luminaries and the Moon's horizontal Parallax taking from it once and a half the Sun's horizontal Parallax, or to the difference of the Sun's and Moon's horizontal Parallaxes.

PROPOSITION XLII.

TO *define the Time of the Beginning, Middle, and End, as well of every Eclipſe, or leſſening of Light, upon account of the Moon's Penumbra, as of a total Eclipſe of the Sun, happening in any part of the Globe of the Earth, upon a given Month; and thence to determine their Duration.* [Fig. 43.]

Let *DCG* be the Diſc of the Earth deſcrib'd about the Center *T*, ſeen from a Point of a Right line joining the Centers of the Earth and Sun, where it meets with the Heaven of the Moon. Let the common Interſection of its Diſc and the Plane of the Ecliptic be the Right line ☊*K*; and *TN* perpendicular to it will be the common Section of the Planes of the Circle of Latitude, and of the Diſc of the Earth, in which the Point *N* is, where the common Interſection of the Moon's Orbit, and the Plane of the Diſc, or the Projection of the Moon's Orbit on the Plane of the Diſc, cuts the ſaid Circle of Latitude. When we ſuppoſe the Eye ſeeing this Eclipſe of the Earth plac'd in a Right line, which joins the Centers of the Sun and the Earth, it is plain that it will move together with the Sun; and therefore that it will ſee the Center of the Moon's Penumbra, deſcribing upon the Diſc of the Earth that Way from the Sun, of which we have amply treated in *Prop.* 36. Therefore let the Right line *lN*☊ be conceived to be drawn thro' *N*, making with *TN* Angles on each ſide equal to thoſe which the Moon's Way from the Sun makes with the Circle of La-

Latitude at the time of the true New-Moon; and let this way of the Moon from the Sun, defin'd for the New-Moon or Mean Eclipſe, be ſuppos'd to continue during the whole Eclipſe; which indeed nearly happens: But if we wou'd go on more exactly, we muſt draw it a new in other Diſtances of the Moon from the Node. From *T* draw *TV* perpendicular to *l*☊; and *V* will be the Point, to which, when the Center of the Moon's Penumbra is come, it will be the leaſt diſtant of any from the Center of the Diſc, and run upon the Earth moſt of any; as has been ſhewn in the foregoing *Prop.* Moreover (from *Prop.* 3. *El.* 3.) *V* is alſo the middle of that part of the Right line ☊*l*, which lies in the Circle *DCG*, or any other concentric to it; and therefore it is ſuch, that the Center of the Penumbra is found in it in the middle of any Eclipſe of the Earth, which laſts as long as the Center of the Penumbra is in running thro' the Right line *AB*, ſuppoſing *TA* to exceed *TR* by the Exceſs *AC*, equal to the Semidiameter of the penumbrous Circle; and alſo ſuch as the Center of the Penumbra may be found in it in the middle time, that the Center of the Penumbra going thro' the Right line *EF*, produces the total and central Eclipſe of the Sun in ſome part of the Earth. Therefore the Problem comes to this, that the Point of Time be determin'd when the Center of the Penumbra is found in *V*. For doing of which, find (by *Prop.* 31.) the Time of the true Conjunction, which is in *N*; and when in the Triangle *TVN*, beſides the Right Angle at *V*, you have the Side *TN*, and (by *Prop.* 36.) the Angle *TNV*, you will find *NV* in the ſame parts in which *TN* or *TR* is given; that is, it will be known what Angle at the Eye is ſubtended by *NV*.

But from the given horary Motion of the Sun and Moon, the Moon's horary Motion fromthe Sun is given, or the Arc which the Moon, going from the Sun, runs thro' in the ſpace of an Hour, that is, the Time neceſſary for the Moon to move from the Sun, thro' a given Arc; therefore the Time will be known which is requir'd for the Moon to move from the Sun thro' ſubtending an Angle ſubtended by the Right line *NV*, that is, that the Center of the Lunar Penumbra may run thro' the Right line *NV* upon the Diſc of the Earth; that is, laſtly, the Time between the moment of the true Conjunction, and the moment of the middle of the Eclipſe, in reſpect of the whole Earth; therefore the firſt being given, the laſt will alſo be given: For the moment of the Sun's Mean Eclipſe, (or of the middle of the Eclipſe) in reſpect of the whole Earth, and alſo that of the Moon's Mean Eclipſe, goes before the true Conjuction, when the Sun and the Moon have juſt paſs'd either of the Nodes, but follows it when the Luminaries are not yet come to the Nodes. To find this Time more readily we muſt make uſe of the Tables of the Time between the true Syzygy and the greateſt approximation of the Centers of the Diſc and of the Shadow, of which we ſpoke in *Prop.* 37.

Now having found the Time of a Mean Eclipſe, it will be eaſy to define the Beginning and the End of every ſort of Eclipſe, or the moments when the Moon's Penumbra firſt comes at, and laſt of all leaves the Diſc of the Earth. For in the rectangular Triangle *TVA*, you have given the Sides *TV*, *TA*, the firſt of them before found, and the laſt made up of the Semidiameters of the Diſc and of the Penumbra

bra before determin'd; and therefore *AV* will be given, and thence as before, from the Moon's horary Motion from the Sun you will find the Time requir'd for the Penumbra's Center to run over it; that is, half the Duration of every kind of Eclipſe; and as the moment of the Mean Eclipſe is given, the Beginning and End of it are known. The *Semi-mora* or half the Duration of the whole Penumbra in the Diſc of the Earth, will likewiſe be found, if inſtead of *TA* the Sum of the Semidiameters of the Diſc and of the Penumbra, you take their Difference, juſt after the ſame manner as it is done in the total Immerſion of the Moon in the Earth's Shadow in total Eclipſes of the Moon.

But the *Semi-mora* of the Center of the Penumbra between the Earth's Diſc (or of the whole Shadow, which, as it is always ſlender, ſometimes none at all, is look'd upon as a Center) is found by inveſtigating *EV* from the given Lines *TE*, *TV*, and the time requir'd for the Center of the Moon's Shadow to go over it, as before. But becauſe the Moment of the mean or middle of the Eclipſe, and half the Duration of the Center within the Diſc are known, the Time will alſo be known when the whole Shadow comes at or leaves the Earth; that is, when the Sun's total Eclipſe begins or ends to an Inhabitant of the Earth.

COROLLARY.

Hence you have all the Angles of the rectilineal Triangles *ATN*, *ETN*, *VTN*, *FTN*, and *BTN*, and from hence alſo ſuch a Scheme of the Eclipſe of the Diſc of the Earth, as this which we talk of, will be exactly deſcribed upon a Plane; as was done of the Moon's Eclipſe, *Corol.* 2. *Prop.* 39: For the 43d Figure is the very Scheme requir'd; therefore that Scheme will be rightly drawn. And as it is known what parts of the Right line ☊*l* the Center of the Moon's Shadow will go thro' in the Space of an Hour, and what a Clock it is in a given place, when the ſaid Center is at *N* or *V* or any other Point given in ☊*l*; it is plain that if we aſcribe to the Points of this Line rightly drawn, the Numbers by which the Hours are expreſs'd in a given Place, the Scheme requir'd will be perfect for a given Place.

PROPOSITION XLIII.

TO *determine the Quantity of a Solar Eclipſe in reſpect of a Spectator upon the Earth within the Penumbrous Circle, which is a Section of the Moon's Penumbra by the Diſc of the Earth, when the ſaid Penumbra is in a given Place.* [Fig. 44.]

Let *EFG* be a Circle dividing the Hemiſphere of the Earth, which is enlighten'd by the Sun from the darken'd one, whoſe Plane therefore is the ſame with the Plane of the Diſc mentioned before. Let *KT*☋ repreſent the Ecliptic on this Plane; *lN*☋ the Way of the Center of the Moon's Penumbra; *N* the Point to which when the Center of the Penumbra is come, there is a true New-Moon. Let *A* be put for the Center of this Penumbra, at a certain Point of time when the Shadow goes thro' the Right-line *l*☋, and the Penumbrous Circle *MQX*; *AN* will be that part of the Journey of the Penumbra which will be perform'd between that point of Time and the moment of the true Conjuction, which will happen when that Center will be come to *N*. Let us ſuppoſe the Obſerver plac'd within this Circle as at *O*, or rather in a Point of the enlighten'd Hemiſpherical Surface, which is ſhewn by a Right line let fall from *O* perpendicular to the Plane of the Diſc; and we muſt find what part of the Sun's Diameter is cover'd from the foreſaid Obſerver. Let the Right line *AO* be drawn and produc'd till it meets with the Circle *M Q X* at *M*; I ſay that that part of the Sun's Diameter which is cover'd by the Moon is to the whole Diameter as *OM* to *AM*. For if the Obſerver was at *M*, the Moon wou'd cover no part at all of the Sun's Diameter from it; but as the Obſerver goes gradually towards *A* the Center of the Penumbra, a greater and greater part of the Sun's Diameter is cover'd from it by the interpoſition of the Moon, till at laſt when it is come to the Center *A*, the whole Sun (that is, its whole Diameter) is cover'd by it, if the Sun's apparent Diameter does not exceed the Moon's apparent Diameter, (which I here ſuppoſe, and which commonly happens in their mean Magnitudes) and the Vertex of the total Shadow of the Moon reaches as far as the Earth, or the penumbrous Circle *M Q X* has its Center perfectly and totally dark: And therefore if the Sun's Diameter be divided into Twelfth parts, from the given Ratio between *AM* and *OM*, which is known from the given Situation of the Place between the Circle *M Q X*, you will have the Twelfths, or Digits, and parts of Digits of the Sun's darken'd Diameter in reſpect to the Obſerver.

But if to the Circle *MXQ* (in the 2d 44th *fig.*) be concentric, the little Circle *BC* which is either wholly dark, or not altogether dark, but more obſcure than the reſt and that equally; that is, if the Center or Point of the Moon's Shadow be either

ther extended beyond the Spectator, or falls ſhort of him, inſtead of *A M* you muſt in the firſt caſe take *B M*, the Difference of the Semidiameters *A M* and *AB*; in the latter, *C M* their Sum, *viz.* the Sun's whole Diameter is to its parts cover'd by the Moon at *O* in the firſt caſe, as *B M* to *O M*, and in the latter caſe, as *C M* to *O M*. For in the firſt caſe the Spectator at *B* will ſee the whole Sun cover'd by the Moon, in the latter, the Spectator, plac'd at *A* will ſee a bright Circle which is the remainder of the Sun's Diſc, encompaſſing the dark Moon, like a Crown, whoſe apparent Latitude is equal to the Semidiameter of the Sun, when the Semidiameter of the Moon is taken from it, which is the ſame as the Semidiameter *AB*, ſeen from the Moon; and the Spectator muſt be at *C*, to ſee the Margin of the Moon touch the inner Diſc of the Sun. Therefore the Quantity of the Sun's Eclipſe is determin'd in every caſe. *Q. E. F.*

PROPOSITION XLIV.

TO give an account of the chief Phænomena of a Solar Eclipſe, and to deſcribe the Places of the Earth where they will happen, whilſt the Shadow of the Moon, in an Ecliptic New-Moon, goes along the Earth. [Fig. 45.]

Let *CGD* repreſent the Globe of the Earth, revolving from Weſt to Eaſt by its Diurnal Motion upon the Pole *P* and the other Pole which is hid, or turning from *C* upwards towards *G*. Let the Hemiſphere of this Globe, ſeen in the Scheme, be that which is enlighten'd by the Sun, therefore the Sun will directly and perpendicularly ſhine on the Circle *C D G* whoſe Pole is *S*. Thro' the Pole *P* and the Point *S* let *PSR*, an unmovable

Circle be ſuppos'd drawn without the Earth's Surface, and alſo *KSY* another, *viz.* the common Section of the Plane of the Ecliptic with the Surface of the Earth, *ZSX* another, at Right Angles to this, both theſe being alike fix'd, between which the Earth performs its Diurnal Revolution. It is certain that it is Noon at every place of the Earth, when it is come to that half of the Circle *PSR* which is here viſible, becauſe the Sun is in the Plane of it. Let *AB* repreſent the Journey of the Center of the Moon's Penumbra, deſcribing the Track *ENF* on the Surface of the Earth: Tho' the Hemiſphere of the Earth which is enlighten'd by the Sun, and conſequently has Day, be ſuppos'd to be rais'd up above the Surface of the Scheme, yet the Way *AB*☊ is conceiv'd to be gone thro' by the Center of the Penumbra, on the Plane of the Diſc, upon which the ſaid Hemiſphere ſtands. Moreover, tho' *PSR*, *KSY*, *ZSX*, be Circles in the ſaid enlighten'd Hemiſphere, croſſing one another in *S*, the Point directly under the Sun, they alſo repreſent the Right lines upon the Diſc of the Earth, directly under theſe Circles, namely, thoſe which are the orthographical Projection of theſe Circles, when the Eye is ſuppos'd in a diſtant Point of the Right line which joins the Centers of the Earth and Sun.

It is plain, that when the Center of the Moon's Penumbra is come to *A* (namely, when the penumbrous Circle touches the Diſc of the Earth in *C*) the Eclipſe of the Sun will begin to an Inhabitant at *C*: Now he that is at *C* by the Diurnal Revolution of the Earth, begins to enter the enlighten'd Hemiſphere; that is, the Sun riſes to him; therefore to the Inhabitant of the Earth *C*, whom, firſt of any Inhabitant of the Earth, the Penumbra reaches, the riſing Sun will firſt of all appear to be Eclips'd in its Upper or Weſtern Limb or Edge. When the Center of the Penumbra it ſelf comes to the Earth at *E*, an Inhabitant at *E* ſees the riſing Sun totally Eclips'd, becauſe he is under the Center of the Penumbra. He that lives upon that Point of the Earth, where the whole Penumbra, receiv'd within the Diſc of the Earth, (if that happens) touches it within, will perceive an Eclipſe of the Sun to end at the lower Limb of the riſing Sun: So he which is in that Point, where the Penumbra does laſt of all touch the Diſc within, as it is ready to go out of it, will perceive the Eclipſe of the Sun to begin, as the Sun Sets. We have not expreſs'd thoſe Two Caſes in the Figure, leſt we ſhou'd make a confuſion.

When the Center of the Penumbrous Circle is come to *Z S X*, which is a Right line perpendicular to the Diſc of the Ecliptic, or when the Axis of the Moon's Penumbra is at *N* come to the Circle *Z S X*, which anſwers to the aforeſaid Right line, it being the Diameter of it; a Point of the Earth's Surface, which is under the Point *N* of the Circle *Z S X*, has a Central Eclipſe of the Sun, and to a Spectator at *N* or any other Point of the Circle *Z S X* the Sun appears in the higheſt Point above the Horizon. For ſince, by ſuppoſition, the Circle *Z X* goes thro' the Zenith of the Spectator, it will be perpendicular to this Horizon: But it is alſo perpendicular to the Ecliptic at *S*; and therefore (by *Elem. Spheric.*) is a whole Quadrant diſtant from the ſaid concourſe of the Ecliptic and the Horizon. But half the Ecliptic (becauſe it is a great Circle) is above any Horizon, whence *ZX* meets with the Ecliptic in a Point diſtant from the Eaſt Point a whole Quadrant, or the higheſt Point. Therefore an Inhabitant of the Earth which is then at the Point of the Earth's Surface at *N*, ſees the Sun centrally Eclips'd at the 90th Degree: But he that lives from *N* towards *Z* or *X*, but in the Circle *Z S X*, does indeed ſee the Sun Eclips'd at its 90th Degree, if he dwells within the Limits of the penumbrous Circle; but more or leſs according to his greater or leſs diſtance from *N*, as has been ſhewn in the foregoing Propoſition. Likewiſe when the Center of the Penumbra comes to the unmovable Circle *P S R* at *M*, an Inhabitant of the Earth, which is under it, ſees the Sun Eclips'd juſt at Noon: But thoſe that live from *M* towards either Pole under the ſaid Meridian *PR*, ſee the Sun at Noon in a Partial Eclipſe, if they are within the Limits of the Penumbra. And he that is remov'd from *M* towards the North Pole *P*, ſees the Sun's Southern part Eclips'd, and ſo on the contrary; which alſo happens in the firſt caſe, and univerſally in all caſes. At laſt the Center of the Penumbra being come to *F* the end of the Diſc, the Spectator which at the ſame time is at that Point, ſees the ſetting Sun (becauſe being ready to change Day for Night, he goes out of the enlighten'd into the darkned Hemiſphere) centrally Eclips'd. But he that, being at *D*, receives the laſt Stroke of the Moon's Penumbra, ſees the ſetting Sun as it were contiguous to the Moon in its upper Limb, and the end of the Eclipſe, be it of what kind it will.

PROPOSITION XLV.

TO determine the Place of the Earth (that is, its Longitude and Latitude) where the riſing Sun appears to be Eclipſ'd in its upper Limb. [Fig. 46.]

Things being as before, the Longitude and Latitude of the Point C is to be determin'd. Let the great Circle SC, which, in this as well as in the foregoing Figure, appears as a Right line, be drawn thro' the Point of the Earth S, to which the Sun is perpendicular, and thro' the Point C. Draw alſo PC, PZ Arcs of great Circles. In the Spherical Triangle PSZ, you have three Sides given, *viz.* PS, the diſtance of the Sun from the enlighten'd Pole, SZ, a Quadrant, and PS, the diſtance of Z, the Pole of the Ecliptic (becauſe the Arc SZ is perpendicular to the Ecliptic $KS\Upsilon$ and extended to a Quadrant) from P the Pole of the Equator; therefore the Spherical Angle between the Arcs SP, SZ, is known, being the ſame with the rectilineal Angle between the Right lines SP, SZ drawn in the Plane of the Diſc; as generally the Angle in the Surface of a Sphere, is equal to a rectilineal Angle, which the common Sections of the Planes of Circles, comprehending a Spherical Angle, make on a Plane perpendicular to both of them. Moreover (by *Corol. Prop.* 42.) the rectilineal Angle NSA and ZSC on the other Side of it are given, and therefore alſo PSC the Sum or Difference of it and of ZSP, or the Spherical Angle on the Surface of the Earth, comprehended by the Circles SP, SC: Now in the Spherical Triangle PCS beſides the Angle PSC lately found, you have the Sides SP, SC, of which the laſt is a Quadrant, and the firſt the diſtance of the Place C

from the Pole *P*, that is, the Complement of its Latitude; therefore the Angle *SPC* will alſo be given, in this Triangle. But (by *Prop.* 42.) the Time is given when the Moon's Penumbra firſt touches the Diſc of the Earth at *C*, reckon'd in ſome determinate Place: Let this Place be *H*, whoſe Meridian is *PH*; therefore the Angle *SPH* will be given, namely, that by which the Sun has already paſs'd the Meridian, or is ſhort of it. Therefore *HPC*, the Sum or Difference of the Angles *CPS*, *HPS*, will be given, which *HPC* is the Difference of the Meridians of the Place *C*, and the given Place *H*. Therefore the Longitude and Latitude of the Place is found. *Q. E. F.*

You may after the ſame manner find a Place, where the Setting Sun is Eclips'd in its upper Limb, which the Moon's Penumbra leaves laſt of any.

PROPOSITION XLVI.

TO determine what Place in the Earth is under the Center of the Moon's Penumbra at a given Hour, or in which the Sun is centrally Eclips'd at a given Time. [Fig. 47.]

Things remaining as before, ſince the Place where the Sun appears centrally Eclips'd is requir'd, and the Time of ſuch an Eclipſe is given, as well as the Time of the true Conjunction, by *Prop.* 31. this Interval is given, and therefore from the Sun and Moon's Horary Motion, the Moon's Motion from the Sun, agreeable to that Interval, is alſo given. Whence it happens, that in the rectilineal Triangle *SNA*, deſcribed in the Earth's Diſc, you have (by *Prop.* 36.) the Angle *SNA* equal to the Angle comprehended between the Circle of Latitude and the Way of the Shadow, the Side SN Sine of the Moon's Latitude in the true Conjunction, and AN the Motion of the Center of the Penumbra agreeable to the Interval between the propos'd Time and the Time of the true Conjunction; therefore the Side SA and the Angle ASN will be given, or its Complement to two Right Angles: But ZSP is given (as was ſhewn in the foregoing *Prop.*) therefore you will alſo have the rectilineal Angle PSA, and the Spherical Angle upon the Earth's Surface PSA which is equal to it. You have alſo the two Sides SA, SP of the Spherical Triangle PSA, this laſt equal to the Sun's Diſtance from the Pole P, and the firſt will be found after the following manner: To the Right lines SC, SA, drawn on the Plane of the Diſc, anſwer upon the Spherical Surface Two Arcs beginning from the Point S, which is directly under the Sun, of which SC is a Quadrant, SA is an Arc, whoſe Right Sine is the Right line SA; therefore the Arc SA ſide of the Triangle PSA, is that whoſe Sine is to the whole Sine as the given Right line SA is to the given SC; and therefore the Arc SA is known. Therefore in the Triangle PSA you will know PA the Complement of the requir'd Latitude of the Place A, and alſo the Angle SPA. But ſince the Time is given, as reckon'd in the determinate Place H, the Angle SPH is given; therefore you will alſo have the Angle HPA, which is the Difference of the Meridians of the determin'd Place H and of the requir'd Place A.

You may after the ſame manner find the Place of the Earth, where the Sun's central Eclipſe happens in the 90th Degree of the Ecliptic from the Eaſt. We have demonſtrated in the 44th *Prop.* that that happens to him who by the Earth's Diurnal revolution is brought to *N*, when the Center of the Moon's Penumbra falls on that Point: And we have ſhewn in *Prop.*31. the Time number'd in the Place *H* when this happens, (for it is the ſame with the time of the true Conjunction of the Luminaries;) therefore this is only a particular caſe of the foregoing Problem.

SCHOLIUM.

Hence therefore is defin'd that Tract of Earth, which the Center of the Moon's Penumbra goes thro' in the Solar Eclipſe of a given Month; that is, all the Places which have ſucceſſively a central Eclipſe of the Sun. If (by *Prop.* 46.) you would know in what Place of the Earth the riſing Sun is centrally Eclips'd, and then another Place where it is centrally Eclips'd half an Hour or an Hour after, and ſo on till you come to the Place where the Sun is Eclipſed after the ſame manner as it ſets; thoſe Places join'd by a Line or Lines on the Earth's Surface will ſhew that requir'd Tract. If on either ſide about this Line you draw as it were a Zone whoſe breadth is equal to the Diameter of the Moon's Penumbra; the whole Tract of the Earth involv'd in the Penumbra which can have any Eclipſe of the Sun will be taken in. In that Tract may be mark'd the Places where the chief Phaſes are, as where the Sun is Eclips'd in the 90th Degree, and where in the Meridian, *&c.*

PROPOSITION XLVII.

TO determine the Place in the Earth's Surface, where the Eclipſe of the Sun is either Total or Central, or where a given part of its Diameter towards a given Pole, will be hid by the Moon juſt at Noon. [Fig. 48.]

Things remaining as before, it appears, that that Point of the Earth which comes to *M*, together with the Center of the Moon's Penumbra, will have an Eclipſe of the Sun juſt at Noon. In order to find it out, let it be obſerv'd, that in the rectilineal Triangle *NMS*, in the Diſc deſcrib'd as before, you have given (by *Prop.* 36.) the Angle *SNM*, (namely, that which the Way of the Moon from the Sun contains with the Circle of Latitude) *NS* the Diſtance of the Centers of the Diſc and Penumbra, at the true Conjunction, and the Angle *NSM* equal to the Spherical Angle *ZSP* in the Spherical Surface contain'd between the concourſe at the Sun, of *SZ* and *PSR*, which anſwer to the Ecliptic and the Æquator; whence you will have the Right line *NM*, and therefore the time requir'd for the Center of the Penumbra to go thro' it in its Motion from the Sun: But the time reckon'd in a given Place, is given, in which that Center is in that Point *N*, namely, the ſame with the Time of the true Conjunction of the Luminaries; therefore alſo the Time number'd in the ſaid Place will be given, when the Center of the Penumbra is in *M*. Find therefore by the foregoing *Prop.* that Place of the Earth which at that time will ſee the Sun in a central Eclipſe; and it will be the ſame in which the Sun is centrally Eclips'd at Noon.

Suppoſe that about the Center *M*, in the Plane of the Diſc, there is deſcrib'd a penumbrous Circle, cutting the Right line *PSR* at C and *D*; then, as the Sun's whole apparent Diameter is to a given part of it, which is not Eclips'd in the Place requir'd, ſo let the Semidiameter of the Penumbra be to *MO*, and let *MO* be taken towards the Pole oppoſite to it on that ſide where the Eclips'd part of the Sun is, (as for example, if the Sun's Eclips'd part lies towards the hidden Pole in *Fig.* 49. let *O* be taken towards *P*, the Pole in ſight;) the Point on the Earth's Surface which a Right line drawn from O, perpendicular to the Plane of the Diſc, ſhews, will be the Place requir'd, in which the Sun juſt at Noon, in a given part, is Eclips'd towards the Pole oppoſite to *P*; as it is plain enough from the 43^d^ and 44^th^ Propoſitions. If inſtead of the dark Center *M*, either the whole Circle be totally dark, or there is altogether no total Shadow, but a Circle equally enlighten'd, (as in *Schol.* of *Prop.* 33.) inſtead of the Semidiameter *MC* you muſt take the Difference or the Sum of the Semidiameters of the Circle *C D*, and another interior and concentric to it; as it has been ſhewn in the ſaid 43^d^ *Prop.* To find the Latitude of that Place O, (for the Longitude is known from knowing the Hour in a given Place, when it is Noon to an Inhabitant at O) obſerve that the Right line *S M* is given, namely, a ſide in the Triangle *S N M*, alſo *MO*, each of them (and therefore their Sum or Difference) in parts of *S Z*, the Semidiameter of the Diſc of the Earth; therefore you will alſo have the Arc which is over it in the Surface of the Earth, of which *S* O is the Sine: But the Arc *SP*, *viz.* the Diſtance of the Sun from the Pole, *P* is given; and therefore the Arc *PO* will be known, and its Complement to the Quadrant of a Circle, is the Latitude of the Place *O*, which was requir'd. *Q. E. F.*

If the Point *O* falls out of the Diſc of the Earth *KZYX*, the Problem is impoſſible. After the ſame manner may be determin'd the Place where the Eclipſe of the Sun in its 90th Degree, or of it riſing or ſetting, is of a given Quantity, or where the Sun upon a given Time, reckon'd in a given Place, is Eclips'd a determinate number of Digits. In the laſt caſe there are innumerable Places, but in each of the former caſes, there are, at moſt but two, which anſwer to what is requir'd.

PROPOSITION XLVIII.

TO determine whether there will be a viſible Eclipſe in a given Place of the Earth, on the New-Moon of a given Month; and if there is one, to determine the Time of the greateſt Obſcurity, and the Quantity of the Eclipſe. [Fig. 49.]

We have hitherto conſider'd the Sun's Eclipſe in reſpect of the whole Earth, (or rather, properly ſpeaking, the Eclipſe of the Earth,) as we did the Moon's before: Now we muſt conſider it as it is ſeen from a given Place of the Earth's Surface. Since the time that *Kepler* gave Aſtronomers the Moon's Shadow on the Earth's Diſc, to be look'd upon from the Lunar Heaven, they have with emulation endeavoured to deduce an Eclipſe of the Sun, viſible in a given Place, and all the Phænomena attending it from the ſame Scheme of the Diſc of the Earth, over which the Moon's Penumbra paſſes. But as this is done in a leſs Geometrical manner, (to which a Trigonometrical Calculation can ſcarce be applied) and certainly a leſs natural one, and (as Mr. *De la Hire*, who makes uſe of it, confeſſes) ſomething more tedious; we, according to the example of *Kepler* himſelf, in the *Rudolph.* Tables (who certainly knew how far his Invention wou'd reach) ſhall follow another way, ſhewn by *Ptolemy* (in the 6th *Book* of his *Almageſt.*) and his excellent Interpreter *Regiomontanus*, that it may alſo be known to the Reader; at the ſame time not forgetting to commend the Method given by the ingenious Mr. *Flamſtead* in his *Doctrine of Spherics*, by which the Solar Eclipſe, or the coming of the Moon to the fix'd Stars in a given Place, is graphically and handſomely protracted, the Places in the other Hemiſphere of the Earth, turn'd from the Sun or Star, being orthographically projected on its great Circle.

Let *C N* repreſent the Ecliptic, *S* the Sun's, and *L* the Moon's true Place at the time of the true Conjunction; find (by *Prop.* 63. *B.* 2.) the Sun's and Moon's Parallaxes in reſpect of their Longitude and Latitude, by which their Places in the Ecliptic or Diſtances from the Ecliptic may be corrected; and this for the ſaid Time, tho' not very accurately: Then, from the true foregoing Places, or Places ſeen from the Center of the Earth, *S* and *A* ſeen from a given Place in the Surface of the Earth (or ſimply ſeen, as they call it) will be had. We don't make the Sun's Place differ from that mention'd before, to avoid Confuſion in the Scheme, as well becauſe the Sun's Parallax is ſo ſmall, that it may well be overlook'd in this Scheme, and alſo chiefly becauſe in taking the Moon's Parallax from the Sun (as it has been ſhewn in the ſaid 63d *Prop.* of *Book* 2.) the Line *N C* may be taken for the apparent Way of the Sun, or for the Ecliptic a little irregular. The Moon'sapparent Place *A* will be reduc'd to the Ecliptic *C*; and its Latitude *AC*. Now to determine the Point in which the Center of the Moon will appear to the propoſed Spectator to be but little diſtant from the Center of the Sun, that is, when the greateſt Obſcurity (if any will be) ſhall happen; find the Situation of the Moon's apparent Way from the Sun, by ſeeking again, from what has been laid down, (for a Time little enough diſtant from that in which the Moon appears in *A*) the Places of the Sun and Moon as they appear to the given Obſerver, and imagining the Sun to be drawn back to its old Place *S*, and the Moon at the ſame time drawn back as much. The Place of the Moon ſo found will be *K*, and being reduc'd to the Ecliptic will ſhew its Longitude at *D* and its Latitude *KD*. Now the Arcs *CA*, *DK* and *CD* being given, you may (by *Lemma* of *Prop.* 36.) determine the Situation *AN* of the Moon's apparent Way in reſpect to the Ecliptic; namely, the Point *N* of concourſe with the Ecliptic and the Angle *ANC*. From the Sun's Place *S* let *SM* fall perpendicularly upon *AN*, and the Point *M* is neareſt to *S* of all the Points in *AN*: To find which let it be obſerv'd that in the Triangle *NSM* beſides the Right Angle at *M* the Angle *N* and the Hypothenuſe *NS*, each lately found, are given, whence *MS* and *MN* will be known; therefore alſo *MA*, becauſe the Arc *AN* is known. Now becauſe *M* is a Point of the Lunar Way which is the fartheſt diſtant of any from the Sun *S*; it is plain that if a propos'd Spectator is to have any Eclipſe of the Sun, that it will happen when the Moon's Center appears to be in *M*, and that then alſo the greateſt part of the Sun's Diſc will be hid by the Moon; and it is alſo plain, that if the before found *SM* be greater or equal to the Sun and Moon's apparent Semidiameters *SR* and *MH* together, then a propos'd Obſerver will have no Eclipſe; if *SM* be leſs than *SR* and *MH* taken together, then there will at leaſt be ſome Eclipſe. But *SM* is found for the New-Moon of a given Month, and (in the ſame parts, *viz*, parts of a Circle) the apparent Diameters of the Sun and Moon are above determin'd; whence it may be defin'd, whether the New-Moon of a given Month will be Ecliptic or no.

If there is to be any Eclipſe, in order to determine its Quantity, we muſt take notice that [*fig.* 50.] *SR* and *MH* taken together are equal to *MS* and *RH* together: If therefore, from the known Sum of the Semidiameters of the Sun and Moon be taken the known Arc *SM*, there will be left *RH*, part of the Sun's Diameter cover'd from the Moon at the time of the greateſt Obſcurity, which is eaſily reduc'd to Digits and parts of Digits, becauſe we already have the Sun's whole Diameter and the cover'd part of it in ſuch Digits.

But it is plain from Obſervation, that the Moon's apparent Diameter is leſs (or perhaps the Sun's apparent Diameter is greater) when there is an Eclipſe of the Sun, than at the ſame diſtance from it when there is no Eclipſe, (for reaſons ſhewn in *B.* 1. *Prop.* 19.) and that that Difference is about the 60th part of the whole Diameter: We muſt therefore have regard to this Correction when we define the Quantity and Phaſes of the Eclipſe.

Laſtly, to determine the Time that the Moon's Center will be found in *M*, let it be obſerv'd, that the Time is given (becauſe it is taken) that the Moon by its apparent Motion from the Sun (compounded partly of the Moon's true Motion, partly of its apparent Motion ariſing from the Parallax) goes thro' the known part *AK* of the apparent Way *AN*, or rather the known Arc *CD* of the Ecliptic, and that beſides there is given the Arc *AM*, namely, the Sum or Difference of the known Arcs *AN* and *MN*; whence by Analogy you will have the Time neceſſary for the Moon, by its apparent Motion from the Sun to go thro' *AM*: But the Time is given in which the Moon is ſeen in *A*; therefore the Time will alſo be known when it will appear in *M*, where it hides the Sun moſt of all from a given Obſerver. *Q. E. F.*

For the Time found in which the Moon is in *M*, find again the Place of the Luminaries, to know whether all is rightly perform'd, or whether there is need of a farther Correction.

SCHOLIUM. [*Fig.* 49.]

In the foregoing Operations we have ſuppos'd the Moon's Motion from the Sun in the apparent Way *AN* to be equable, which is not true; for tho' the Moon's Motion from the Sun in its true Way *LB*, during the whole time of the Eclipſe, differs but little from an equable one, yet ſince there is a Parallax in its apparent Motion in its apparent Way (as has been ſhewn at large in the Definitions of the Parallaxes *Prop.* 45. *B.* 2.) for that reaſon it differs very little from an equable one. Therefore for the determination of the apparent Way *AK*, you muſt take the Time after that the Moon has been in *A*, if *C* is after *S*; but the Time before, if *C* be before *S*, according to the order of the Signs, and, as far as we can conjecture, almoſt equal to that which the Moon muſt take up to move from *A* to *M*; which will be nearly equal to that which is requir'd for the Moon by its Motion from the Sun along the Ecliptic, to run thro' the known Arc *CS*; for they only differ by the apparent Motion ariſing from the Parallax.

If from *S* be drawn *SV* perpendicular to the Ecliptic, meeting the Moon's apparent Way at *V*, *V* will be the Point where the Moon's Center is, at the Time of an apparent Conjunction. But *AV* may be thus found: In the Triangle *NSV*, beſides the Right Angle *NSV*, there are given the Side *NS* and the Angle *N*; wherefore the Side *NV* will be known; and ſince the Arc *NA* is alſo known, *VA* will likewiſe be found; therefore the Time neceſſary for going thro' it will be known, and therefore the very Moment of the apparent Conjunction. But becauſe the Points *M* and *V* are very little diſtant from each other, therefore the Time of the apparent Conjunction is commonly taken for the Moment of the greateſt darkneſs.

PROPOSITION XLXIX.

TO *determine the Time of the Beginning and End of a Solar Eclipſe, happening upon a given Month and ſeen in a given Place, and ſhew what its whole Duration will be.* [Fig. 51.]

Things being as before, let us ſuppoſe *XR* to be the Moon, going on its Way *AN*, and *HR* to be the Sun at *R*: It is requir'd to find the Time when they will be in this Poſition. Let *S* and *L*, the Centers of the Luminaries, be join'd by *SL*, the Arc of a great Circle. In the Triangle SML, beſides the Right Angle at M, you have given the Sides SM, SL, the firſt of them determin'd in the foregoing Propoſition, and the laſt, the aggregate of SR, LR, the Semidiameters of the Luminaries; therefore the Arc ML will be known, which is in appearance to be run thro' by the Moon's Motion from the Sun between the beginning of the Eclipſe and the Moment of the greateſt Obſcurity (known by the foregoing *Prop.*) happening when the Center of the Moon comes to M: Therefore alſo you will know the Quantity of its correſpondent in the Ecliptic, *viz.* SE, reckon'd back from S. In order to determine the Time requir'd for this, find the Time aſſum'd (which is an Hour or Two, as the Duration of the Eclipſe is great or ſmall, which will be known from the greatneſs or ſmalneſs of the Arc SM; and here again all will be ſo much more exact, as the Time aſſum'd comes nearer to the half Duration of the Eclipſe) being the before known Moment of the greateſt Obſcurity, the apparent Places of the Luminaries, which being reduc'd to the Ecliptic (the Sun's Place being carried to S) will make the diſtance CS reckon'd on the Ecliptic. Knowing then the Time (becauſe it is taken) that CS, the known Arc of the Ecliptic, is run thro' by the Motion of the Moon from the Sun; you will alſo know the Time in which another known one ES is gone thro' by the ſame Motion of the Moon from the Sun; that is, the Time between the Moment of the greateſt Obſcurity, and the Beginning of the Eclipſe: But the former is determin'd by the foregoing Propoſition; therefore this latter will be alſo known.

If the Moon's apparent Motion was equable, there would be as much Time requir'd from the Moment of the greateſt Obſcurity to the end of the Eclipſe, as from the Beginning to the ſaid Moment, becauſe the Right line *MP* is equal to *LM*, ſuppoſing the Moon *HY* to touch the Sun *RH*, at the End of the Eclipſe; and the greateſt Obſcurity wou'd coincide with the middle of the Eclipſe, as it happens in the Moon's Eclipſe, where there is no Parallax of the Moon from the Shadow of the Earth; for they touch one another. But becauſe here it happens otherwiſe, as has been ſhewn in the foregoing *Prop.* in order to find the Time of the End of the Eclipſe, you muſt repeat the ſame Operations, finding what a Portion of its Way the Moon goes thro' in a Time taken after the greateſt Obſcurity, and finding by Analogy how much Time is requir'd for the Arc *PM* (equal to the before found *LM*) to be gone thro' by the Moon's apparent Motion from the Sun. For this being added to the before found Moment of the greateſt Obſcurity, will give the Moment of the End of the Solar Eclipſe ſeen from a propos'd Place: And knowing the Time of the Beginning and End you will know the whole Duration. *Q. E. F.*

SCHOLIUM.

Whilſt the greateſt part of the foremention'd Triangles is reſolv'd by Trigonometry, they may be look'd upon as Plane: For by reaſon of the ſmalneſs of their Sides, the part of the Spherical Surface which they take up differs little from a Plane, as was ſaid before in the like caſe, *Prop.* 38. And this gives a compendious Way of Calculating Eclipſes of the Sun, which is very valuable, becauſe that Calculation is of it ſelf as tedious as any whatever, upon account of the Parallaxes which muſt be ſo often found; but we muſt be cautious how we uſe this ſhort Method.

COROLLARY.

Hence alſo may we exactly draw a Scheme of a Solar Eclipſe for a given Place. For ſince ſo ſmall a part of the Heavens as comprehends the Sun, (reckoning the ſpace it takes up from the beginning to the end of the Eclipſe) may be look'd upon as a Plane, the Ecliptic *CN* together with the Moon's apparent Way *AN* (which ſometimes, by reaſon of the Parallax, will ſenſibly vary from a great Circle, or a Right line) and the Points *S*, *M*, *L*, and *P*, where the Sun's and the Moon's Centers are at the ſaid Times, will be rightly protracted after the common Method: And deſcribing Circles about the Points *S* and *M* whoſe Diameters are proportional to the apparent Diameters of the Luminaries at that Time, and are taken in ſuch parts as the Portions of the Ecliptic and the apparent Way were before taken, you will have the Sun's Phaſis, at the Time of the greateſt Obſcurity. And as (by *Prop.* 36. *B.* 2.) you have the Angle which the Ecliptic comprehends with the Sun's Vertical, and the Situation of the Vertical in reſpect to the Meridian; if the Scheme above drawn be inclined to the Plane of the Meridian and to the Horizon in the due Angles already found, the Scheme of the greateſt Obſcurity will be perfect. Moreover, if the Line *AN*, repreſenting the Moon's apparent Way, be divided into ſuch parts as the Moon appears to go thro' in Hours and parts of Hours (all the while [illegible] and you give [illegible] Hours as they are rec-[illegible] given Place, which may be easily done, because the [illegible] greatest Obscurity, of the Beginning [illegible], which are to be [illegible] and P, are known [illegible] Propositions; you will have [illegible] a Phasis agreeable to a given [illegible] may immediately be drawn, [illegible] the given Hour set down [illegible] draw a Circle with a Diameter equal [illegible] Moon's apparent Diameter; [illegible] the Circle about the Center S, [illegible] cut off a Segment, [illegible] the Ecliptic, with [illegible] at that time covers of the Sun.

The [illegible] necessary for Calculating the Eclipses [illegible] we have described in the [illegible] Propositions, their Foundation, [illegible] Order, as far as our Design [illegible] in their proper Places; [illegible] of the Luminaries apparent [illegible] of the Inclination [illegible] Way from the Sun to the Ecliptic, [illegible] and that of the Time between the true Syzygy and the Mean [illegible].

PRO-

PROPOSITION L.

TO give the reaſon why the Lunar Tables hitherto Calculated ſhew the Moon's Place in the Eclipſes of the Luminaries, generally more accurately in any Syzygies than in any other Aſpects, eſpecially than in the Quadratures.

Beſides that, in order to determine the Lunations, the Syzygies of the Sun and Moon have been more accurately obſerv'd by Aſtronomers, than any other Phaſes, and the Moon's Place in reſpect of the Sun has been moſt nicely obſerv'd in Eclipſes of the Luminaries, and thence a Theory has been particularly made and fitted for finding the Moon's Place in thoſe caſes by Calculation; all the Monthly Inequalities, whether thoſe that are to be made up in the Syzygies with the Sun, or thoſe that affect the Motion of the Moon and its Orbit, ceaſe in the Syzygies; and have ſenſible Effects in the other Aſpects. But all of them (except the Variation alone) are neglected by the Conſtructors of Lunar Tables, as has been ſhewn in the 29th *Prop.* whence we ſee the Reaſon why the Moon's Place taken from thoſe Tables is more exact in the Syzygies than in any other Aſpect with the Sun. For all theſe Inequalities become greater by how much the Moon is farther remov'd from the Syzygies, ſo that they are greateſt when the Moon is in the Quadratures: For the Apſes of the Lunar Orbit, which, abſolutely ſpeaking, go forward, do in the Moon's Quadratures either go back or at leaſt go forward more ſlowly, by *Prop.* 8. Likewiſe the Nodes, whoſe abſolute Motion is regreſſive, go back ſlower when the Moon is in Quadrature with the Sun, by *Prop.* 14. And the Excentricity of the Lunar Orbit is leſs, and its Inclination to the Ecliptic greater (by the 12th and 15th Propoſitions) when the Moon is in Quadrature with the Sun, than in any other Aſpect, *cæteris paribus*; whence the Lunar Tables hitherto conſtructed are very far from determining the Moon's true Place in the Sun's Quadrature. Again, ſince there is no account in the Tables of the Moon's greater Diſtance from the Earth in the Quadratures than in the Syzygies, (of which we have treated in the 2d *Prop.* and its Corollaries,) and of the Moon's Parallax and apparent Place which depend upon it, theſe laſt are not properly to be taken from the Tables.

Moreover, as many Inequalities of the Moon depend upon the Aſpect of the Nodes of the Moon's Orbit with the Sun, and tho' there is notice taken of them in ſome Tables that have been hitherto conſtructed, yet it is not done with the greateſt exactneſs: Therefore the Moon's Place will, even from theſe Tables, be had more accurately when thoſe Inequalities ceaſe; but they ceaſe when the Nodes are in the Syzygies; for (by *Prop.* 14.) the Nodes are then at reſt, and the Inclination of the Limits is then the greateſt, by *Prop.* 15. Beſides, in the Eclipſes of the Luminaries the Nodes are at a ſtand in or near the Syzygies; hence the reaſon is plain why in theſe the Moon's Place is more accurately found in the Tables: For the Errors of the Tables in the various regreſs of the Nodes, and the various Inclination of the Lunar Orbit to the *Magnus Orbis* according to the various Aſpect of the Nodes to the Sun (which are certainly ſo, in ſome meaſure) have no Place in this caſe. For at the Time of the Eclipſes there is a Ceſſation of the Cauſes that produce thoſe Inequalities which are not ſo exactly ſet down in the Tables, and therefore the Effects of the Errors of the Tables in the Moon's Place, for either Eclipſe (and for a whole Month before and after either Eclipſe) by being wrong defin'd, alſo ceaſe. Therefore ſince in the Eclipſes of the Luminaries, both the foremention'd caſes obtain, namely, the Moon itſelf and the Nodes of the Moon's Orbit alſo are in the Syzygies, the Moon's Place at that time taken out of the Tables (as wanting fewer Æquations becauſe it is liable but to few Inequalities) will be more accurate and exact than in any other Situation or Aſpect to the Sun of the Moon or its Nodes, where there are Inequalities, of which there is no account, or no exact one in the Tables.

SECTION IX.

Of the Motions of Satellites revolving about other Primary Planets beſides the Earth.

PROPOSITION LI.

THE *Diſtance of a Primary Planet from the Sun remaining the ſame, if ſeveral Secondary Planets revolve about the Primary one in ſimilar excentric Orbits, and ſimilarly inclin'd to the Plane in which the Primary revolves about the Sun, all the Angular Errors in the Motion of the Satellites, ſeen from the Center of the Primary, and generated in any Revolution of the Satellites, are directly as the Squares of the Periodical Times reſpectively.*

Becauſe all Bodies have the ſame accelerating Force towards the Center of the Sun or that of a Primary Planet at the ſame Diſtance from it, it is plain that ſeveral Secondary Planets revolving about a Primary are affected the ſame way as one Secondary Planet, whoſe Orbit is dilated and contracted as much as the Orbit of thoſe different Secondary Planets are diſtant from each other. But in this caſe it is plain (from *Prop.* 18.) that the ſaid Angular Errors ſeen from the Center of the Primary, and generated in every Revolution, are directly as the Squares of the Periodical Times. Whence the truth of the Propoſition is evident. *Q. E. D.*

COROLLARY.

Since all the Errors abovemention'd, generated in the different Times of the different Re-

Revolutions, are as the Squares of the ſaid Times; thoſe that are generated in the ſame and a determinate Time, will be as the Times of the Revolutions themſelves, that is, as the Periodical Times; namely, taking away on either ſide one of the compounding Ratio's, *viz.* the ſimple Ratio of the Periodical Times.

But becauſe ſome of the aforeſaid Errors, namely, the Motion of the Apſes and the Nodes, is ſometimes *in antecedentia*, ſometimes *in conſequentia*; therefore their mean Motions made in ſome remarkable Time towards the ſame parts will in the preſent caſe be as the Periodical Times of the Satellites.

PROPOSITION LII.

IF two Primary Planets revolve about the Sun at different diſtances from it, and one Secondary Planet revolves about each Primary, and their Orbits be equal, ſimilar and ſimilarly inclined to the Orbits of their Primaries; the Sun's Forces that diſturb the Motion of the Satellites and their Effects, that is, all the angular Errors, reſpectively ſeen from the Centers of their Primaries, are reciprocally in a Duplicate Ratio of the Periodical Times of the Primary Planets about the Sun.

For, becauſe the accelerating Force of any Primary Planets whatever about the Sun, is the ſame, at the ſame Diſtance, it is plain that thoſe Secondaries which revolve in equal Orbits, about different Primary Planets are affected the ſame way as one Secondary Planet revolving about its Primary, which revolves about the Sun in an Orb ſo excentric, that its Diſtances from the Sun, in divers Points of ſuch an Orbit, differ as much from one another as the Diſtances of the two Primaries in the preſent caſe.

But it is plain (by *Prop.* 19.) that in ſuch a caſe, the ſaid Errors, both in Longitude and Latitude, ſeen from the Center of the Sun, are inverſly as the Cubes of the Diſtances of the Primary from the Sun. And therefore the ſaid angular Errors of Secondary Planets revolving in equal ſimilar and ſimilarly inclin'd Orbits about two Primary Planets mov'd about the Sun in different Diſtances, ſeen from the Centers of the reſpective Primaries, are inverſly as the Cubes of the Diſtances of the Primaries from the Sun. But, by the 40th and 42d Propoſitions, the Squares of the Periodical Times of the Primary Planets about the Sun are as the Cubes of the mean Diſtances of them from the Sun: And therefore all the angular Errors of the Secondary Planets reſpectively, ſeen from the Centers of their Primary ones, are inverſly as the Squares of the Periodical Times of the Primaries about the Sun. *Q. E. D.*

PROPOSITION LIII.

THE *Angular Errors of the Satellites generated in every one of their Revolutions about their reſpective Primary Planets, and ſeen from their Centers, are in a Ratio compounded of the direct duplicate Ratio of the Periodical Times of the Satellites about their reſpective Primary Planets, and of the duplicate reciprocal Ratio of the Periodical Times of the Primary Planets about the Sun.*

For, by *Prop.* 51. if the Diſtances of the Primary Planets from the Sun were equal, the foreſaid Errors wou'd be in a direct duplicate Ratio of the Periodical Times of the Satellites about their Primary Planets reſpectively: And by *Prop.* 52. if the Diſtances of the Satellites from their Primaries were equal, the ſaid Errors wou'd be in a reciprocal Duplicate Ratio of the Periodical Times of the Primaries about the Sun: And therefore, tho' neither the Orbits of the Primaries about the Sun are equal, nor thoſe of the Satellites about the Primary Planets; the Ratio of the foreſaid angular Errors, generated in every reſpective Revolution of the Satellites, and ſeen from the Centers of their Primary Planets, is compounded of the foreſaid Ratios; namely, a direct Duplicate Ratio of the Periodical Times of the Satellites about their reſpective Primary Planets, and a Duplicate reciprocal Ratio of the Periodical Times of the Primary Planets about the Sun. *Q. E. D.*

SCHOLIUM.

From the ſame Reaſoning you may (by *Prop.* 52. and the Corollaries of *Prop.* 51.) gather, that the mean Motions of the Apſes and Nodes of Two Satellites are in a Ratio compounded of a direct Ratio of the Periodical Times of the Satellites about their reſpective Primary Planets, and a reciprocal Duplicate Ratio of the Periodical Times of the Primary ones about the Sun.

PROPOSITION LIV.

TO *determine the unequal Motions of Satellites revolving about any other Planets.*

From the before determin'd unequal Motions of the Moon, ſeen from the Center of the Earth, the analogous Motions of any Satellite may be the help of the premis'd Theorems be determin'd, if the Form of the Orbits of the Satellites and the Inclination of the Orbits to the Plane, in which the reſpective Primaries are carried about the Sun, do not much differ from the Form of the Lunar Orbit, and the Inclination of its Plane to the Plane of the Ecliptic, as it happens in the Satellites of Jupiter and Saturn: For their ſmall difference does not ſenſibly change the Motions, as it has been taken notice of in the 11th and 14th Propoſitions.

Let (for Example) the Motions of the outtermoſt of the Circumjovial Planets be to be determin'd: It is plain, from the Scholium of the foregoing Propoſition, that the regreſſive Mean Motion of the Nodes of the ſaid Satellite is to the Mean Motion of the Moon's Nodes, in a Ratio compounded of the Ratio of the Periodical Time of the ſaid Satellite about Jupiter, to the Periodical Time of the Moon about the Earth, and a Duplicate Ratio of the Periodical Time of the Earth about the Sun, to the Periodical Time of Jupiter about the Sun; and therefore that it is given, when the ſaid Periodical Times are known by Obſervation.

Likewiſe from the Mean progreſſive Motion of the Apſes of the Moon, may be found the Mean progreſſive Motion of the Apſes of a Satellite.

After the ſame manner, from the foregoing Propoſition, the Variation of the ſaid Satellite is to the Moon's Variation above found, in a Ratio compounded of the Duplicate Ratio of the Periodical Time of that Satellite about Jupiter, to the Periodical Time of the Moon about the Earth, and a Duplicate Ratio of the Periodical Time of the Earth about the Sun, to the Periodical Time of Jupiter about the Sun; that is, (by the ſaid foregoing *Prop.*) as the whole Motions of the Nodes in the Periodical Times of the Satellite and the Moon to one another.

And after the ſame manner the greateſt Æquations of the Nodes and the Apſes of a Satellite are to the greateſt Æquations of the Nodes and of the Apſes of the Moon reſpectively, as the Motions of the Nodes and Apſes of a Satellite during the Time of one Revolution of the former Æquations, to the Motions of the Nodes and Apſes of the Moon, during the Time of one Revolution of the latter Æquations.

All theſe Things likewiſe hold true of any other Satellite. But having once found the ſaid Inequalities in one of the Satellites, the analogous Inequalities in any other revolving about the ſame Primary Planet are more eaſily determin'd, by *Prop.* 51. and its Corollaries; for there will be no need of making uſe of an inverſe Duplicate Ratio of the Periodical Time of the Primary about the Sun, the Primary remaining the ſame.

PROPOSITION LV.

TO correct the Motions of the Apſes of the Satellites of Jupiter and Saturn found by the foregoing Propoſition.

Becauſe the Apſis of the Moon, in the Syzygies with the Sun goes forward, and alſo the Apſis of a Satellite, by *Prop.* 11. but ſlower by reaſon of the ſhort Periodical Time of the Earth about the Sun, in compariſon of the Periodical Time of Saturn or Jupiter about it; it is plain that the Moon's Apſes ſtick longer in the Syzygy of the Sun and near it, than the Apſis of a Satellite, becauſe it does more cloſely follow the Sun, which is carried *in conſequentia* faſter than either Apſis. And on the other hand each Apſis of a Satellite goes back in the Sun's Qua-

drature,

drature, but that of the Moon faſter; therefore it remains a ſhorter Time in the Sun's Quadrature, which it meets. And as for the firſt Cauſe, the Apſis of a Satellite goes forward for a leſs time than the Apſis of the Moon, and, by the ſecond Cauſe, it is longer a going back than the Apſis of the Moon, and the abſolute Progreſs of each ariſes from a greater Progreſs than Regreſs; it is plain that thoſe two Cauſes conſpiring diminiſh the progreſſive Motion of the Apſes of a Satellite, more than in proportion to the other Cauſes. Therefore upon this Foundation we muſt build the Correction of the before found Motion of the Apſes, and diminiſh the ſaid Motion according to the permanency of this Cauſe and its Effects on each of the Satellites of Jupiter or Saturn.

SCHOLIUM.

Theſe Inequalities which affect the Motion of a Satellite revolving about any Primary Planet being determin'd, it will be eaſy (according to the foregoing Methods about the Moon) to find, by Calculation, the Places of thoſe Satellites for a given Time, their Immerſions into the Shadow of their Primary Planet, their Emerſions out of it, the journey of the Shadow of a Satellite over the Diſc of his Primary, and other things concerning their Motions, as they are ſeen from their proper Primary Planets. And the Form of the Tables for determining their Motions, ſeen from thence, will be the ſame as that of the Moon's Tables above deſcribed; from which, together with the Tables of the Primary, (deſcrib'd in the foregoing Book) one may readily find the Phænomena of thoſe Satellites. But becauſe the Eclipſes of the Satellites,

lites, as they run into the Shadow of their Primary Planet, are very remarkable Phænomena to be obſerv'd by the Inhabitants of the Earth; Aſtronomers have conſtructed Tables for the immediate Calculation of them, whoſe Foundation and Form appear eaſy and ſimple to any one that underſtands the foregoing Propoſitions; and therefore we ſhall only give a few Examples to illuſtrate them over and above; which yet do not properly belong to theſe *Elements*, becauſe they are only Phyſical and Geometrical, not Arithmetical.

But yet we ought to take notice that regard is to be had to the different Diſtance of the Earth (the Place of the Obſerver) from theſe Phænomena of the Immerſion and Emerſion of the Satellites of Saturn and Jupiter: For ſuch a Phænomenon is ſooner ſeen when the Earth is nearer to it, than when it is remov'd from it. Let *S* [*fig.* 52.] repreſent the Sun; *CDEF* the *Magnus Orbis*, or the Way of the Earth; *G*♃*H*, the Way of Jupiter, ♃ Jupiter caſting his Shadow ♃*V*; *BAK* the Orbit of a Satellite of Jupiter. Let an Inhabitant of the Earth at *C*, ſee the Emerſion of the Satellite out of the Shadow of Jupiter at *A*; for if the Satellite be one of the neareſt to Jupiter, or the Earth be after the Oppoſition or before the Conjunction of Jupiter with the Sun, then the Immerſion at *B* will not be ſeen by reaſon of the Interpoſition of Jupiter's Body: If the Spectator ſhould remain in the ſame Place, after ſome Time the like Emerſion of the ſame Satellite wou'd again appear, and another ſuch Emerſion of it after the ſame ſpace of Time, and ſo on. But as the Earth goes from *C* to *D*, it is plain that there is need of a longer Time for the ſame

Appearance to reach to it at *D*; namely, as much more Time as is needful for the Light (which, as it were, brings the Phænomenon) to go thro' the Interval of the Diſtances *AD*, *AC*, or the Diſtance *CD*. For the ſame reaſon the Immerſion of the Satellite at *B*, ſeen by an Inhabitant of the Earth at *E*, will, after a determined Time, be again ſeen, and ſo on; but the Earth, as it were, meeting the Phænomenon, as it goes from *E* to *F*, a Spectator will ſooner ſee the Phænomenon when he is at *F* than when he was at *E*. And this proceeds from the different Diſtance of the Earth from the Bodies whoſe Phænomena are obſerv'd, to which we ſaid that regard was to be had: For by reaſon of this different Diſtance, they are ſeen ſooner or later, as alſo all the Inequalities which really happen in the Motion of the Bodies themſelves. And from the comparing ſuch Phænomena ſeen ſooner or later with the Earth's annual Orbit, we, on the other hand, diſcover the ſucceſſive Motion of Light, and from it find out its Velocity by Obſervations.

SECTION X.

Of the Motions of Satellites about their own Centers reſpectively.

PROPOSITION LVI.

TO *deſcribe the Motion of the Moon about its own Center, as it is carried at the ſame Time round the Earth.*

For the Example of the Motion of the Satellites about their own Centers, we ſhall give an inſtance in our Moon, to whoſe Motion 'tis reaſonable to believe the Motions of the Satellites of other Planets to be analogous; for we cannot make minute Obſervations of thoſe ſmall Bodies which are at ſo immenſe a Diſtance. The Motion of the Moon is in all things like the Motion of the Earth about its own Center, whilſt it is carried round the Sun, excepting the Inclination of its Axis and the Period of its Revolution. For as the Moon moves round the Earth, it moves alſo about its own Axis: This Axis is every where parallel to itſelf, therefore not perpendicular to the Plane of the Moon's Orbit, (for this Orbit is continually chang'd by reaſon of the action of the Sun which diſturbs it) but inclined to it, and almoſt at right Angles with the unmov'd Plane of the Ecliptic. The Moon's Motion about this its own Axis (as alſo that of the Earth about its Axis, and of the other Primary Planets about their Axes, ſhewn in *Prop.* 30. *B.* 1.) is uniform and equable, always of the ſame tenour; and its Period is the ſame with the Periodical Month, different from what happens in the Earth and the other Primary Planets carried about the Sun, every one of which performs ſeveral Revolutions about its own Axis, whilſt it goes once thro' its Orbit.

PROPOSITION LVII.

THE Plane of the ſame Lunar Meridian (or of a Circle made by the Section of a Plane along the Axis of its Rotation) being produc'd, does always nearly paſs thro' that Focus of the Elliptic Orbit (deſcrib'd about the Earth) in which the Earth is not. [Fig. 53.]

Let *ABDC* be the Ellipſe, whoſe Periphery is deſcrib'd by the Moon moving about the Earth: Let the Earth *T* be plac'd in one of its Foci, to which a Ray or Radius (which we have call'd a *Vector*) being drawn, deſcribes Areas proportionable to the Times in which they are deſcrib'd. And by *Prop. 5. B. 3.* the Right lines *AF*, *BF*, being drawn from any two Points of an Ellipſis to one of its Foci *F*, (namely, that in which the Earth is not) the Angle *AFB* is to four Right ones nearly as the Time in which the Moon runs thro' the Arc *AB* to the whole Time of the Moon's Period. If *AM* repreſents the Plane of the Lunar Meridian, which (the Moon's Center being any where, as for Example, at *A*) being produc'd goes thro' *F*; I ſay that the ſame Plane being likewiſe produc'd will go thro' *F*, in whatſoever Point of the Ecliptic Orbit the Moon is. For, becauſe the Rotation of the Moon is uniform about its own Axis, and it alſo has the ſame Period as the Moon has about the Earth; it is evident that the Plane of the Meridian which in the Moon's Situation *A* was *AM*, when the Moon's Center is got to any other Point (as *B*) will have ſuch a Situation *BE*, that the Line *Bm* being taken parallel to *AM*, the Angle *mBE* will be to four Right Angles as the Time in which the Moon is carried thro' the Arc *AB* to the Moon's whole Periodical Time: Therefore (by *Prop.* 11. *El.* 5.) *mBE* is to four Right Angles as *AFB* is to four Right Angles; whence (by *Prop.* 9. *El.* 5.) the Angle *mBE* is equal to *AFB*. And as *AF* and *Bm* are parallel, *BE* will be *in directum* to *BF*; that is, in a Situation of the Moon *B* taken at pleaſure, the Plane of the ſame Meridian, which produc'd did in the former Situation *A* go thro' *F*, produc'd in this does alſo go thro' *F*. Therefore in what Point ſoever of the Orbit the Moon's Center is found, the Plane of the ſame Meridian will go thro' *F*, the Focus in which the Earth is not. *Q. E. D.*

PROPOSITION LVIII.

TO deſcribe the Motions which appear in the Moon's Face, ſeen from the Earth, ariſing from the Moon's Motion about its own Center.

Firſt, it is plain that nearly the ſame Face of the Moon is always turn'd towards the Earth, and that nearly the ſame Lunar Spots are ſeen by an Obſerver on the Earth. For ſince the produc'd Plane of the ſame Meridian goes thro' the empty Focus *F* of the Lunar Orbit, and the Lunar Orbit is not very excentric, that is, *F* is not very far from *T*, or the Angle *FBT* (when greateſt) is not very great, it is plain that nearly the ſame Face of the Moon is always turn'd to the Earth.

But if that Face of the Moon which is turned towards the Earth be nicely conſider'd, by looking at the Spots, ſo as to draw the Figure of it by the help of a Teleſcope, it will appear that we don't preciſely ſee the ſame Face. And ſince the Plane *A M* of the ſame Meridian is not directed to the Earth *T*, but to the other Focus *F*, it is plain that the Hemiſphere of the Moon when it is at *A*, ſeen from the Earth *T*, is ſomething different from that which is ſeen from it when the Moon is come to *B*, ſome part of the Moon's Hemiſphere which is drawn towards *D*, and was hid from the Eye in *T* when the Moon was at *A*, being now diſcover'd, and part of the Hemiſphere towards *C*, which was then ſeen, being now out of Sight. And this apparent Motion of the Moon from the Earth, by which ſome of its Spots in the Hemiſphere turn'd from the Earth, come in ſight, and ſome of thoſe in the Hemiſphere towards the Earth get out of ſight, is properly enough call'd, the *Libration of the Moon*, and that which happens for the reaſon juſt given is call'd, the *Libration in Longitude*, ariſing from the Moon's Motion in Longitude, or along the Zodiac. This Libration is twice reſtor'd in every Periodical Month; namely, when the Moon is in the Apogæum *C* or Perigæum *D*: For in either caſe the Plane of the ſame Meridian, which being produc'd falls on *F*, falls alſo on *T*. And this Libration is from all the Inequalities of the Motion affected in Longitude, and, amongſt others, by that Inequality, by which the Moon, in Quadrature with the Sun, is remov'd from the Earth, of which mention has been made in the Corollary of *Prop.* 2.

Moreover, becauſe the Axis of the Moon's Rotation is not perpendicular to the Orbit which the Moon deſcribes about the Earth, but inclined to it, it is plain that ſometimes one Pole of the Moon, ſometimes another, points towards the Earth, (for the ſame reaſon that ſometimes one Pole of the Earth, and ſometimes another is turn'd towards the Sun) and therefore ſometimes we ſee from the Earth the Lunar Spots which are near one Pole, and ſometimes thoſe which are near the other, and conſequently we ſee another Libration of the Moon from the Earth; which, as it depends upon the Situation of the Moon, in reſpect of the Nodes of its Orbit with the Ecliptic, (for its Axis, as has been obſerv'd, is nearly perpendicular to the Plane of the Ecliptic) is rightly call'd, the *Libration in Latitude* This is perform'd in the ſpace of a Periodical Month, or rather as the Moon returns to the ſame Situation in reſpect of the Nodes.

Again, there is a third kind of Libration of the Moon, by which, tho' no other part of it be turn'd to the Earth than what is occaſion'd by the two foregoing Cauſes (namely, the Libration in Longitude, and the Libration in Latitude) yet another part is enlighten'd by the Sun. For ſince the Axis of the Moon is nearly perpendicular to the Plane of the Ecliptic, when the Moon goes fartheſt from the Ecliptic towards the South (or is in the Southern Limit) the Moon's North Pole, and alſo ſome part of the Lunar Globe beyond that Pole, are enlighten'd by the Sun, when at the ſame time the South Pole of it, and ſome parts on this ſide of it (that is, on the ſide towards the Sun) are in darkneſs; if therefore in this caſe it happens that the Sun is on the ſame ſide as the Moon's Southern Limit, the Moon, going from the Conjunction with the Sun to the aſcending Node will appear by little and little to draw away its

Northern parts and the Spots near the North Pole, into the darken'd Hemiſphere, when at the ſame time on the oppoſite ſide other Spots about the South Pole emerge out of darkneſs into Light: And the contrary will happen half a Month after, when the New Moon deſcends from the Northern Limit; that is, the more Northern parts of the Moon will by little and little break out into Light from Darkneſs, at the ſame time that the moſt Southern parts hide themſelves in Darkneſs. It is plain that this Libration, or rather theſe Effects of the Libration in Latitude, depend upon the Sun's Light, and therefore on the Synodical Month.

SCHOLIUM.

From this Propoſition it appears how manifold this Libration of the Lunar Globe ſeen from the Earth, is; which is produc'd by its moſt ſimple Motion about its own Axis, deſcrib'd in *Prop.* 56. for the foregoing kinds of Librations, according to the various Situation of the Lunar Apſes, of the Nodes and of the Sun (upon all which they depend) are variouſly mix'd and combin'd, and make that moſt intricate Libration of the Moon obſerv'd by Aſtronomers.

And that all theſe things obtain likewiſe in the Satellites of Jupiter and Saturn is very probable, becauſe they go round thoſe Primary Planets, and, together with them, are carried about the Sun, juſt as the Moon and Earth; and alſo at Jupiter's and Saturn's Satellites have that very remarkable Phænomenon, in reſpect of their Primaries, which has ever been obſerved of the Moon, namely, that it always turns the ſame Face to the Earth, as was ſaid at the be-

beginning of the laſt *Prop.* For it is obſerv'd of the outward Satellite of Saturn, that it is at the Weſt of this Planet; but that it is always hid when at the Eaſt: It is likely that it happens for this reaſon, that the greateſt part of its Surface is darker (that is, does not copiouſly enough reflect the Rays of Light every way) than the other. This Satellite, like our Moon, has very ample Plains or Vallies, which being the darkeſt part, when turn'd towards us, can't be ſeen upon account of the ſmall Quantity of the reflected Light. And as this Saturnian Moon is found to be darken'd when it is in the Eaſtern part of its Orbit, and never in the other; it is a plain ſign that the ſame Face of that Moon always looks towards Saturn, that Phænomenon being a conſequence of this Phænomenon, as is evident. And now ſince this ſame thing is true both of that remoteſt Moon, and of our Moon, *viz.* that the ſame Face is always turn'd to the Primary Planet, who can doubt of its obtaining in the other Moons about Jupiter and Saturn, tho' we have not Obſervations to evince the truth of it?

SECTI-

SECTION XI.

Of the Magnitude and Denſity of the Satellites.

IN endeavouring to find out the Magnitude of the Satellites, we ſhall alſo ſuppoſe that they, like the Primaries, (about which they revolve) and all other great Bodies in the World, did at firſt (by reaſon of the Gravitation of the parts towards the whole, and the Center of the whole) when their matter was fluid, put on a Spherical Figure. This abundantly appears to be true in the Moon's Satellite of our Earth, from its Monthly Phaſes, Eclipſes, and all the other Arguments which can be deviſ'd, as may be ſeen in all Writers of Natural Philoſophy and Aſtronomy: This alſo appears, namely, in the Satellites of Jupiter and Saturn; for whatever Face they turn to a Terreſtrial Obſerver (for they turn all their Faces ſucceſſively to the Earth) they appear to have a circular Diſc, which only happens in Spherical Figures.

Tho' the Figure of the Planets be Spherical, conſidering any whole Planet, yet ſome Inequalities (and great ones if compar'd with thoſe on our Earth) are found on their Surfaces, ariſing from particular Cauſes near their Surfaces, exerting their Forces on ſuch parts as are already hard and ſolid; ſuch as are found in our Earth and in the Moon, but are not look'd upon as conſiderable, in reſpect of their Globes, and therefore do not take off from their Roundneſs.

In inveſtigating the Magnitude and Denſity of the Satellites, we ſhall take our Moon for an Example, and compare them with it.

PROPOSITION LIX.

TO *define the Moon's Magnitude.* [Fig. 54.]

Becauſe from foregoing Propoſitions we know both the Moon's Diſtance and the Angle under which it appears, that is, (*ABC* repreſenting the Moon, *L* its Center, and O the Eye) the Right line *OL* and the Angle *AOB*, and therefore the half of it *AOL*; in the rectangular Triangle *LAO*, rectangular at *A*, *LA* the Moon's Semidiameter will be found: Or more readily thus: The Diameter of the Moon is to the Diameter of the Earth, as the known apparent Semidiameter of the Moon to the Moon's known Horizontal Parallax, as it is plain. Now the Moon is to the Earth (by *Prop.* 18. *El.* 12.) in a Triplicate Ratio of its Diameter to the Diameter of the Earth, and therefore it is defin'd. *Q. E. F.*

The Magnitude of the Satellites of Jupiter and Saturn will be likewiſe defin'd, from knowing their Diſtance, and the Angle which each of them ſubtends.

PROPOSITION LX.

TO *define the Quantity of Matter in the Moon, in reſpect of the Quantity of Matter in the Earth.* [Fig. 55.]

Let *S* denote the Sun; *ABC* the *Magnus Orbis*, in whoſe Circumference the common Center of Gravity of the Earth *T*, and of the Moon *L* is carried, as has been explained in the 1ſt Book. Let *A* repreſent that Center at the known Time of ſome New-Moon; and *B* the ſame Center near the known Time of a Quadrature (the next, for Example.) And from theſe two times given you will (by *Prop.* 18. *B.* 3.) have the apparent Places of the Sun, ſeen from the Points *A* and *B*: For (by *Prop.* 63. *B.* 1.) that common Center of Gravity of the Earth and Moon, goes thro' the *Magnus Orbis* in ſuch manner that the Areas cut off by Radii drawn to the Sun are proportionable to the Times. Obſerve the Places of the Sun and Moon from the Earth at *t*, and then you will know the Angles *BtS*, *ASt*; but *ASB* was known before, therefore *BSt* will be known. Therefore in the Triangle *BSt* the Angles *BtS* *BSt*, being given, and the Side *SB* (which is a Line of the *Magnus Orbis*, drawn from the Sun, and given in Poſition, whoſe Quantity is from the Sun's Theory and Parallax known in Diameters of the Earth) the Side *tB* will alſo be had in ſuch known Meaſures; but *tl*, the Diſtance of the Moon from the Earth, is known from the Theory of the Moon; therefore (by the 1st *Datum*) the Ratio between *tl* and *tB* is known, and therefore that between *Bt* and *Bl*. Moreover, it is known from *Mechanics*, that there is the ſame Ratio of the Gravity of the Moon to the diſtant Sun, to the Gravity of the Earth to it, that is, (by *Corol. Prop* 47. *B.* 1.) of the Quantity of Matter in the Moon, to the Quantity of Matter in the Earth, as of the Right line *Bt* to the Right-line *Bl*; therefore the Ratio of the Quantity of Matter in the Moon to the Quantity of Matter in the Earth is defin'd. *Q. E. F.*

To make this Obſervation we take one of the Times at the Syzygy of the Sun and Moon, and the other near the Quadrature, that the Angle BSt may be the greater.

The ſame thing may alſo, according to the ſame Principles, be gather'd from the different apparent Diameter of the Sun, in the New-Moon and the Full-Moon; for it is (*cæteris paribus*) leſs at the Time of the New-Moon, and greater at Full-Moon.

SCHOLIUM.

The foreſaid Inequalities in the Motion of the Earth, ariſing from this, that not the Earth it ſelf, but the common Center of Gravity of the Earth and Moon is carried in the *Magnus Orbis*, and deſcribes Areas (by Rays drawn to the Sun) proportionable to the Times, is alſo to be obſerv'd in the other Planets. For this Monthly Parallax of Mars, when *Achronical*, is twice, and that of Venus, in Conjunction with the Sun and Retrograde, three times greater than the Sun's, (becauſe in ſuch a caſe Mars is twice and Venus three times nearer the Earth than the Sun;) but in the other Planets it is almoſt inſenſible. There will likewiſe be the ſame Inequalities in the Motions of Jupiter and Saturn, (which have Satellites) eſpecially when all or moſt of the Satellites are on one ſide of the Primary: But theſe Primaries are ſo very great in reſpect of their Satellites, that this Inequality becomes leſs ſenſible.

PROPOSITION LXI.

TO *define the Ratio of the Moon's Denſity to the Earth's Denſity.*

It has been univerſally demonſtrated in the 49th *Prop. B.* 3. that the Ratio of the Denſity of two Bodies is compounded of the direct Ratio of the Quantity of Matter in them and the inverſe Ratio of their Magnitudes; that is (in the preſent caſe) the Denſity of the Moon is to the Denſity of the Earth, in a Ratio compounded of the Ratio of the Quantity of Matter in the Moon, to the Quantity of Matter in the Earth, and the Ratio of the Magnitude of the Earth to the Magnitude of the Moon: But the two compounding Ratios are given, by the 59th and 60th Propoſitions; therefore the Ratio compounded of them is given, namely, the Denſity of the Moon to the Denſity of the Earth. *Q. E. F.*

PROPOSITION LXII.

TO define the Ratio of the accelerating Gravity towards the Moon on its Surface, to the accelerating Gravity towards the Earth on its Surface.

It is plain from what has been before ſhewn, that in like diſtances from Bodies, however unequal, ſo that the Place be taken without them, the accelerating Gravities are as the Quantities of Matter in the ſaid Bodies; and in unequal Diſtances from the Centers of equal Bodies, the accelerating Gravities are inverſly as the Squares of the ſaid Diſtances; and therefore, tho' neither the Diſtances nor Bodies are equal, the Ratios of the accelerating Gravities towards them is compounded of the direct Ratios of the Bodies themſelves, towards which heavy Bodies tend, and the inverſe Ratio of the Squares of the Diſtances from their Centers; that is, (in the preſent caſe) the accelerating Gravity towards the Moon on its Surface, is to the accelerating Gravity on the Earth's Surface, in a Ratio compounded of the Ratio of the Maſs (or Quantity of Matter) of the Moon to the

Maſs of the Earth, and a Duplicate Ratio of the Semidiameter of the Earth to the Semidiameter of the Moon, (for the diſtances of the Surfaces from their Centers are the Semidiameters of the Spheres) now the former of the compounding Ratios is given, by *Prop.* 60. and the latter of them, by *Prop.* 69. and therefore the compounded Ratio will be known, namely, the Ratio of the accelerating Gravity on the Surface of the Moon, to the accelerating Gravity on the Surface of the Earth.

By the ſame Obſervations and Reaſoning may be defin'd the Maſſes and Denſities of the Satellites of Jupiter and Saturn, in reſpect of their Primary Planets: And ſince their Maſſes and Denſities, in proportion to thoſe Qualities in the Earth, are defin'd, in the 48th and 49th Propoſitions of *B.* 3. we have defin'd the Maſs, Denſity and accelerating Gravity near the Surface of the Planets, contain'd in the Solar Syſtem.

SECTION XII.

Of the Figure of the Satellites and of the Primary Planets ariſing from their mutual Gravity.

ALtho' the Mountains and Vallies upon the Earth, and the Eminencies and Cavities of the Moon (which appear to be ſuch upon account of their Shadows projected according to the different Situation of the Sun) do not hinder thoſe Bodies from being round, as has been ſaid before; yet there are ſome univerſal Deviations or Differences from a Spherical Figure, which are not to be omitted; as for Example, the prolate Spheriodical Figure, which has in *Prop.* 31. *B.* 1 been ſhewn to belong to all fluid Bodies turning round their own Axes. But by reaſon of the ſlowneſs of the Rotation in the Lunar Globe, which is not perform'd in leſs than a Periodical Month, the deviation from a Spherical Figure ariſing from it will be almoſt inſenſible, if it be compar'd with the like deviation in the Earth, as will be ſhewn in the following Propoſition.

For another reaſon the Moon is of an oblong Spheroidical Figure, whoſe Axis produc'd goes thro' the Center of the Earth. For ſince the cauſes of this Figure of it are remarkable, and its effects no leſs ſo, it is fit to take the matter higher; eſpecially ſince it will be a means to explain the Phænomenon of the Tide, which is the moſt remarkable thing of thoſe, which happening on the Earth, are produc'd by Celeſtial Bodies, as we ſhall ſhew it to be.

PROPOSITION LXIII.

TO *define the Moon's prolate Spheroidical Figure.*

This Problem may be ſolv'd after the ſame manner as in *Prop.* 53. *B.* 3. where the Figure of the other Planets is determined: For we have the Ratio between the Diameters of the Earth and Moon about their own Axes, by *Prop.* 56. therefore we have (by *Prop.* 26. *B.* 1.) the Ratio of the Centrifugal Force at the Æquator of the Moon to the like Force in the Æquator of the Earth: But the Ratio between the Centrifugal Force at the Æquator of the Earth, and the Gravity on the Earth's Surface is given, (by *Prop.* 52. *B.* 3.) and (by *Prop.* 42.) you have the Ratio of the Gravity on the Surface of the Earth, to the Gravity on the Surface of the Moon; therefore the Ratio compounded of thoſe Three will be given; namely, the Ratio of the Centrifugal Force in the Moon's Æquator, to the Gravity towards the Center of the Moon, which obtains on its Surface, which (in the ſaid *Prop.* 52. *B.* 3.) has been ſhewn to be the ſame as the Ratio of the Exceſs of the Moon's Æquatorial Diameter above its Axis. Therefore (by the 6th *Datum*) we ſhall have the Ratio between the Moon's Axis and any Diameter of it, which is perpendicular to the Axis; that is, between the Axis of an Ellipſe by whoſe Rotation about the ſhorter Axis a prolate Spheroid like the Moon is generated.

The aforeſaid Ratio comes ſo near the Ratio of equality, that the prolate Spheroidical Figure of the Moon, ariſing from it, differs very little from a Spherical one.

PROPOSITION LXIV.

IF the Globe of the Earth was every where covered with a deep Sea, it would nearly put on an oblong Spheroidical Figure, whoſe Axis produc'd wou'd paſs thro' the Moon. [Fig. 56.]

Let a Circle, whoſe Center is *T*, repreſent the Earth, and a Circle whoſe Center is *L* the Moon; It is plain, that if neither the Moon nor any other Body acted on the Earth, if the Earth was every where cover'd with a Fluid, it wou'd be Spherical, except ſo much as its Figure wou'd differ from a Sphere upon account of the reaſons ſhewn in *Prop.* 31. *B.* 1. which deviation is not to be taken notice of here. But becauſe all the parts of the Earth gravitate towards the Moon, and the Gravity towards the Moon (as well as towards any other Body) is reciprocally as the Square of the Diſtance from its Center; if (as it has been explain'd at large in the 60th *Prop.* of *B.* 1. and in *Prop.* 1. *B.* 4.) the Right line *LT* is put for the accelerating Gravity of a Body plac'd in the Earth's Center towards the Moon, and *E* (in the firſt *fig.* 56.) be any Particle of the marine Fluid or of the Sea, and if in the Right line *LE* (or *LE* produced) *LK* be taken equal to *LT*, and *LF* be taken ſuch as to be to *LK* in a duplicate Ratio of *LK* to *LE*; *LF* will expreſs the Gravity of a Body plac'd at *E* towards *L*, which (as is commonly known) is divided into Forces, which are as *FG*, and *GL*, and in the Directions of thoſe Lines: But if from that Force, by which the Body, poſited at *E*, is urg'd, you take away a Force which is as *TL*, by which the Center of the Earth, together with the whole Earth, is urg'd towards the Moon; there will be left Forces as *FG* and *GT*, with the Directions *FG* and *GT*, by which a Body at *E* is urg'd, beſides the Force of the proper Gravity, by which *E* tends towards the Center of the Earth. In the ſecond *fig.* 56. things remaining as before, let *C* be a Point of the Earth, to which the Moon is in the Zenith, and *A* one to which the Moon is in the Nadir, *B* and *D* the Points round about (or rather a Circle) to which the Moon is in the Horizon. It is plain that the Point *G* is very much diſtant from *T*, when the Point *E* is either in *C* or in *A*: In the firſt caſe, *G* will change its Place and get to *M*, and in the latter caſe to *N*. But when the Point *E* is found in the Circle *BD*, the Point *G* nearly coincides with *T*, and in the parts of the Fluid in the ſaid Circle *BD*, there is no Force left beſides that of their own Gravity, and the Force *FG* in the direction *FG*; but *FG* in this caſe becomes *BT* or *DT*, the Point *F* falling in with *K*. Since therefore the particles of the Fluid at *C* and *A*, beſides their own Gravity towards the Center *T*, are urg'd by Forces, as *MT* and *NT*, that is, thoſe at *C*, with the Force *MT* in a direction from *M* towards *T*, that is, from *C* towards *L*; and thoſe at *A* with the Force *NT* in a direction from *N* towards *T*, that is, from *A* towards *M*; they will not gravitate ſo much towards the Center *T* as if the Force ariſing from the Moon was away: But the Particles at *C* are more attracted towards the Moon, than the whole Earth, which is look'd upon as plac'd in the Center *T*; and the Particles at *A* are leſs drawn towards the Moon than the whole Earth at *T*, and therefore being as it were left behind after the Earth, are affected after the ſame manner as if they were drawn to contrary parts towards *M*: But the Particles in the Circle *BD*, to whoſe Inhabitants the Moon is in the Horizon, have more Gravity towards *T*; for beſides their proper Gravity, another, as *BT* or *DT*, ariſing from the Force of the Moon, is ſuperadded in the direction from *B* or *D* towards *T*. But in the intermediate Places between *A* or *C*, and *B* or *D*, partake of both theſe conditions, of which the firſt or the laſt prevails as they are nearer to the firſt or to the laſt Places; namely, by how much nearer the parts of the Terreſtrial Fluid are to the Points *C* and *A*, by ſo much leſs heavy are they, becauſe the action of the Moon (that is, a Force as *GT*) diminiſhes the Force of their own Gravity towards *T*, and by how much nearer they are to the Points *B* and *D*, by ſo much heavier are they, the ſame Action of the Moon encreaſing their proper Gravity in that caſe by a Force as *FG*. But as the Globe *ABCD* is ſuppos'd every where cover'd with a pretty deep Fluid, that is, ſuch a one whoſe Particles eaſily yield to each other, and whoſe proper Motion is very free; it is plain that thoſe parts of the Fluid which are at *A* and *C*, will be thruſt out by the parts which are at *B* and *D*, namely, the Lighter by the Heavier, according to the Laws of Hydroſtatics. Therefore the Fluid at *A* and *C*, and near thoſe Points, will ſtand up or be rais'd, but in the Circle *BD* and near it, it will ſubſide, till the greater Quantity and Height of the one is compenſated by the greater Gravity of the other; ſo that Earth if every where cover'd with a deep Fluid, will form itſelf as it were into an oblong Spheroid, whoſe Axis is *AC* a Right line which produc'd will paſs thro' the Moon. *Q. E. D.*

SCHOLIUM.

For the ſame reaſon the Terreſtrial Fluid will put on nearly an oblong Spheroidical Figure, whoſe Axis produc'd goes thro' the Sun: For if, in the foregoing reaſoning, the Body *L* does not ſignify the Moon, but the Sun, the Propoſition is plain. But the Axis of this Spheroid does not ſo much exceed the Diameters which are perpendicular to it, as it does in the Spheroid that the Moon produces. For, becauſe the Elevation of the Fluid at *C*, under the Moon *L*, ariſes from this, that it gravitates more towards the Moon than the Earth itſelf (which is to be look'd upon as plac'd at *T*,) becauſe it is nearer to it; but its Elevation at the Point *A* ariſes from this, that the Earth at *T* gravitates more towards the Moon than the Fluid at *A* as being nearer to it; the Fluid at C drawn by the Moon (to uſe *Kepler*'s Expreſſion) leaves the Earth behind it, but the Fluid at *A* is as it were left by the Earth: If this is occaſion'd by the Sun's Action, the Effects won't be ſo ſenſible, becauſe the Semidiameter *TC* or *TA* of the Earth almoſt vaniſhes, in reſpect of the immenſe Diſtance between the Earth and the Sun, which, in reſpect of the Diſtance between the Earth and the Moon (which ſcarce exceeds thirty Diameters of the Earth) is ſenſible enough; and therefore the Gravity of the Fluid at *C* is but inſenſibly greater towards the Sun than at *T*, and the Gravity of that part of the Fluid which is at *A* inſenſibly leſs than the Gravity of the part of it which is at *T*; whence the Spheriod thus produc'd by the Sun will be but little rais'd above a

Sphere at its Vertices *C* and *A*, and little depreſs'd below the ſurface of a Sphere in the Circle *BD*. Yet this Elevation of the Terreſtrial Fluid in *C* and *A*, and the Depreſſion of it in the Circle *B D*, will be ſomething ſenſible, upon account of the immenſe Forces of the immenſe Body of the Sun, tho' at a vaſt diſtance.

PROPOSITION LXV.

THE *Flowing and Ebbing of the Sea are owing to the oblong Spheroidical Figures of the Fluid that covers the Globe of the Earth, whoſe Axes produc'd go thro' the Sun and Moon.*

J. Kepler was the firſt (that I know of) that diſcover'd the true cauſe of the Tide, and he explains it largely in his *Introduction to the Phyſics of the Heavens,* given in his *Commentaries to the Motion of the Planet Mars,* where after he has ſhewn the Gravity or Gravitation of all Bodies towards another, he thus writes: *The Orb of the attracting Power, which is in the Moon is extended as far as the Earth, and draws the Waters under the Torrid Zone, acting upon places where it is vertical, inſenſibly on included Seas, but ſenſibly on the Ocean, whoſe Beds are large, and the Waters have the liberty of reciprocation, that is, of riſing and falling*; and in the 70th *Page* of his *Lunar Aſtronomy,* —— *But the cauſe of the Tides of the Sea appear to be the Bodies of the Sun and Moon drawing the Waters of the Sea.* Afterwards that incomparable Philoſopher Sir *Iſaac Newton,* improv'd the hint, and wrote ſo amply upon this Subject as to make the Theory of the Tides his own, by ſhewing that the Waters of the Sea riſe under the Moon and the Place oppoſite to it: For *Kepler* believ'd *that the* Impetus *occaſion'd by the preſence of the Moon, by the abſence of the Moon occaſions another*

another Impetus; *till the Moon returning, ſtops and moderates the Force of that* Impetus, *and carries it round with its motion.* Therefore this Spheroidical Figure which ſtands out above the Sphere (like two Mountains, the one under the Moon and the other in the place oppoſite to it) together with the Moon (which it follows) is carried by the Diurnal Motion, (or rather, according to the truth of the matter, as the Earth turns towards the Eaſt it leaves thoſe Eminencies of Water, which being carried by their own motion ſlowly towards the Eaſt, are as it were unmov'd) in its journey makes the Water ſwell twice and ſink twice in the ſpace of 25 Hours, in which time the Moon being gone from the Meridian of any Place, returns to it again. But becauſe (by the *Schol.* of the foregoing *Prop.*) the Terreſtrial Fluid riſes alſo in the Places where the Sun is in the Zenith or Nadir, tho' leſs than when the Moon is in thoſe Places; in the Conjunction and Oppoſition of the Luminaries the ſaid Eminences of Water will alſo be joyn'd, the greateſt Elevation will happen when both the Luminaries are (at the Conjunction) in or near the Zenith, or (at the Oppoſition) the one in the Zenith and the other in the Nadir; alſo the greateſt Depreſſion will happen when the Luminaries are both at the Horizon, becauſe the Water is at one time raiſed by both the Forces, and at another time hinder'd by them. In the Quadratures of the Luminaries the Sun raiſes the Water where the Moon depreſſes it; and from the differences of the Forces the leaſt elevation of the Water ariſes, and alſo the leaſt depreſſion. Between the Syzygies and Quadratures of the Luminaries, the Effects appear to be intermediate, as the Cauſes require they ſhould.

If the Moon is in the Æquinoctial Circle, the two oppoſite Eminences of Water will be in the Terreſtrial Æquator, and each of them deſcribing with its Motion that Circle of the Earth willbe more agitated,and therefore will be rais'd up higher againſt the Shore when it reaches it; beſides that ſince the Earth's Diameter is greater than any other Right line drawn thro' the Center of the Earth and terminated on either ſide by its Surface, the Water will by the Forces of the Luminaries be raiſed (*cæteris paribus*) to a greater height in that Circle than any where elſe. And therefore when the two Luminaries are in Conjunction or Oppoſition in the Æquator, they joyn their Forces to raiſe the Water in the Terreſtrial Æquator,which happens in the Syzygies near the Æquinox; that is, ſuch as happen on the Months of *March* and *September*; the Tides then produc'd will be the greateſt and the moſt agitated, and the Tides which in the ſaid Months happen in the Quadratures will be the leaſt of all and the leaſt agitated, as being the Effects of the difference of the Forces of the Moon in its greateſt Declination from the Æquator; (namely,when the Moon is in Quadrature to the Sun in the Æquator, and therefore in the Solſtitial Points of the Zodiac) and therefore produces the leaſt and leaſt agitated Tide, and ſo being the occaſion that the Sun, which has no Declination, produces its greateſt and moſt agitated Tides; the contrary of which happens about the Solſtices, when the leaſt Tides happen whilſt the Luminaries are in the Syzygies, (I mean the leaſt of thoſe that do happen in the Syzygies) and the greateſt (taken in the ſame Senſe) in the Quadratures.

Again, the Tides (*cæteris paribus*) are greateſt when the Luminaries are in their Perigæum, and leaſt when in their Apogæum; and as that happens to the Moon at the time of every Lunation, but to the Sun only in Winter; this, joyn'd with the former cauſes, occaſions the greateſt Tides to happen in the Syzygies, and the leaſt in the next Quadratures before the Vernal and after the Autumnal Æquinox.

We have hitherto conſider'd the Properties of the Tides as they affect the whole Globe of the Earth: Now we muſt conſider them according as they happen in Habitations or Places of different Latitudes. In order to which let *BEKAQH* [*fig.* 56.] repreſent the Earth, whoſe Center is *T*, the Pole *A* and *B*, this the North Pole and the other the South Pole, *EQ* the Æquator, and the Circles *CD*, *FG*, two Parallels to it, the laſt being on the North-ſide, and the firſt on the South-ſide of it. Let the Fluid circumfus'd about the Earth, form it ſelf into an oblong Spheroid pointing to the Moon, whoſe Axis *KH*, being produc'd beyond *H*, will go thro' the Moon, as has been ſhewn in the foregoing *Prop.* the Right line *TH*, or *TK*, will be the greateſt Height of the Water (reckoning from the Center) and *TM* or *TN* the leaſt, namely, the Height of Water that is found in any Point of the Circle *NM*; as for *Example*, *X* or *Y* where it meets with the Parallels *CD* and *FG*. The Right lines *TC*, *TF*, *TD* and *TG*, will ſhew the Height of the Water in the reſpective points *C*, *F*, *D*, and *G*; and the Circle *PO* being drawn parallel to the Circle *NM*, the Right line *TO* will be equal to the Height of the Water in the Points *O*,*V*,*S*,*R* and *P*, where the ſaid Circle *PO* meets with the Æquator and the reſpective Parallels. Things being thus laid down, let us conſider the Place which by its Diurnal Motion deſcribes the Parallel *CD*: It is plain, that when this place is in *D*, the Height of the Sea *TD*, is the greateſt; that is, that it is high Tide or Flood when the Moon *L* is in the Meridian; but afterwards that in the ſame Place the Height of the Water is the leaſt, when the Place by its revolution is come to *X*, as has been ſhewn before; and again, that it is Flood when the Place is come to *C*. But becauſe *TD* is greater than *TC* (as being nearer to the longeſt line *TH*) in the preſent caſe when the Moon declines to the elevated Pole, the Height of the Sea will be greater when the Moon is in that part of the Meridian above the Horizon, than when it is in the part of the Meridian which is below it. Likewiſe *TG* is leſs than *TF*, being nearer to *TM*; that is, in the Place, which by its Diurnal Motion deſcribes the Parallel *FG*, the greateſt Height of Water, which happens when the Moon declining towards the depreſs'd Pole *B*, is in that part of the Meridian which is above the Horizon, is leſs than the greateſt Height which happens whilſt the Moon is come to the other half of the Meridian.

Moreover, the difference of the ſaid Tide, which is alternately higher or lower (in a Place ſituated out of the Æquator, as the Moon comes to the viſible or hidden part of the Meridian) will be greater, if it be joyn'd with ſome of the cauſes before explain'd which produce the ſame kind of effects, as for *Example*, at the time of the Solſtices; for then each Luminary very much declines from the Æquator, on which Declination it has been ſhewn that the alteration of the Tides depend. And the Effects will yet be encreas'd, if the aſcending Node of the

Moon's Orbit be in the Vernal Æquinox; for then the Moon being in Conjunction with the Sun, beſides a Declination equal to the Sun's greateſt Declination, does then (being in the Limit) decline ſtill more to the North or South by the Quantity of its greateſt Latitude.

Things would be exactly thus, if the Globe of the Earth was cover'd with Water of a ſufficient depth; but by reaſon of great and ſmall Iſlands which ſtop the Tide, and the Streights between them, alſo the Shelves and Shallows, along which the Tides are to be propagated, the variety of this Phænomenon is almoſt infinite. But when you know the ſituation of them in reſpect of each other, and juſt obſervations of the Tide are exactly had, you may find out the Reaſons of all the particular Phænomena of the Tide, becauſe they naturally flow from what has been laid down. But as theſe are not to our purpoſe, and even what we have hitherto ſhewn, are only as Lemma's for determining the Figure of the Moon, we ſhall not enter upon the detail of ſuch Particulars.

PROPOSITION LXVI. LEMMA.

IF the Moon's Globe was every where covered with a deep Sea like our Sea, it wou'd put on the Figure of an oblong Spheroid whoſe Axis produc'd wou'd paſs thro' the Earth: And the Height of the Lunar Fluid, in the parts of the Moon which are towards us, and thoſe that are turn'd from us, above a Sphere inſcrib'd in the Spheroid, wou'd be to the analogous Height of our Sea in the parts which are under the Moon, and thoſe which are diametrically oppoſite to them, above a Sphere inſcrib'd in the Spheroid, in a Ratio compounded of the Ratio of the accelerating Gravity of the Moon towards the Earth, to the accelerating Gravity of the Earth towards the Moon, *and the Ratio of the Moon's Diameter to the Earth's Diameter.*

That the Lunar Fluid wou'd riſe up into a Spheroidical Figure, whoſe Axis wou'd paſs thro' the Earth is plain from the ſame Arguments, which we made uſe of in the 64th *Prop.* to prove that the Terreſtrial Fluid wou'd put on ſuch a Figure. For ſince it has been ſufficiently ſhewn in the firſt Book, that all Bodies are heavy, we may very well ask with *Kepler*, in his *Lunar Aſtronomy*, *Why we may not ſay that the Earth draws the Lunar Waters, as well as the Moon draws the Terreſtrial Waters?* Moreover, if the accelerating Gravity of the Earth towards the Moon was equal to the accelerating Gravity of the Moon towards the Earth, (that is, if there was the ſame Quantity of Matter in the Moon as there is in the Earth;) it is plain that the Fluids of the ſame Denſity with which theſe two Globes are covered, will put on ſimilar Spheroidical Figures. For there can not be any reaſon thought of why the Fluid of one Globe ſhou'd riſe into a more ſharp Spheroid than that of the other: For, excepting the Maſs alone, in the preſent caſe, all things are equal on both ſides. And when all things neceſſary for producing every kind of Equality are ſuppos'd, except the Maſs, inſtead of all kind of Equality a Likeneſs or Similarneſs will be produc'd. The two Schemes [*fig.* 58.] will ſhew the ſimilar Spheroidical Figures in which the Moon (whoſe Center is *L*) and the Earth (whoſe Center is *T*) wou'd raiſe alternate Fluids of equal Maſs or Denſity; *LB*, *TA* the Axes of thoſe Figures wou'd lye *in directum*, by *Prop.* 64. If the Spheres *FD*, *EC*, cutting the Axes in *C* and *D*, be ſuppos'd to be inſcrib'd in theſe Spheroids, by reaſon of the ſimilarneſs of the Spheroidical Figures in the preſent caſe, *B L* will be to *L F* as *TA* to *TE*, and by dividing, *B D* to *L F*, as *A C* to *T E*, and again, *B D* to *A C*, as *L F* to *T E*; that is, if the accelerating Gravity of the Earth towards the Moon, and of the Moon towards the Earth was equal, the Elevation of the Lunar Fluid in the neareſt and the remoteſt parts above the inſcrib'd Sphere, wou'd be to the analogous Elevation of the Terreſtrial Fluid in the places under the Moon and thoſe that are oppoſite to it, as the Diameter of the Moon to the Diameter of the Earth. Again, if the Earth and the Moon had equal Diameters, the Heights above the inſcrib'd Spheres, to which Fluids of equal Denſity wou'd riſe, wou'd be as the accelerating Gravities in the attracting Bodies themſelves; that is, the Height of the Spheroid to which the Lunar Fluid is rais'd above the Sphere, in the parts neareſt to and fartheſt remov'd from the Earth, wou'd be to the like or ſimilar Height of the watry Spheroid above the inſcrib'd Sphere, to which the Terreſtrial Fluid is rais'd, as the Moon's accelerating Gravity towards the Earth, to the accelerating Gravity of the Earth toward the Moon. And therefore tho' neither the accelerating Gravity of the Earth towards the Moon be equal to the accelerating Gravity of the Moon towards the Earth, nor the Diameter of the Moon equal to the Diameter of the Earth; yet the Height of the Lunar Fluid above the Sphere (inſcrib'd in a Spheriod) on thoſe parts of the Moon which are turn'd towards us, and thoſe which are turn'd from us, will be to the analogous Height to which our Sea, in thoſe places that are under the Moon, and ſuch places as are diametrically oppoſite to them, does riſe above the Sphere inſcrib'd in the Spheroid, in a Ratio compounded of the Ratio of the Moon's Diameter to the Diameter of the Earth, and the Ratio of the accelerating Gravity of the Moon towards the Earth, to the accelerating Gravity of the Earth towards the Moon. *Q. E. D.*

PROPOSITION LXVII.

TO *define the oblong Spheroidical Figure of the Moon.*

It has in former Propoſitions been ſhewn, that the Moon was ſome time every where cover'd with a Fluid (which appears from its Spherical Figure) and that that Fluid has been of an oblong Spheroidical Figure, whoſe Axis goes thro' the Earth. But becauſe almoſt the ſame Face of the Moon is always turn'd towards the Earth, (which happens upon account of the Moon's Period about its own Axis being equal to the Moon's Period about the Earth, as was ſhewn in the 68th *Prop.*) the Situation of the Eminencies of that Spheroid beyond the Globe of the Moon will always be the ſame on the Moon's Surface. Then as the Fluid or ſoft Matter of the Moon, became hard by degrees, the Eminences of this Spheroid on the Moon's Surface being at reſt, alſo became hard, and the Moon from ſoft becoming hard, retains this oblong Spheroidical Figure; otherwiſe than it happens in the Earth, in which the Fluid Eminences of the Spheroid above the inſcrib'd Sphere, upon account of the ſwift Motion of the Earth about its own Axis are in a perpetual Rotation about the Earth and preſerve their Fluidity.

To determine of what kind this Spheroidical Figure is, we muſt obſerve that in the foregoing *Prop.* it has been demonſtrated that the Exceſs of the Axis of an oblong Spheroid above the Diameter perpendicular to it, is to the Exceſs of the greateſt Height of our Sea, occaſion'd by the Forces of the Moon alone above the leaſt Height of it in a Ratio compounded of the Ratio of the accelerating Gravity of the Moon towards the Earth, to the accelerating Gravity of the Earth towards the Moon, and the Ratio of the Moon's Diameter to the Diameter of the Earth: But each compounding Ratio is given, the firſt from the 62ᵈ *Prop.* the laſt from *Prop.* 59; therefore the compounded Ratio is given, namely, the Ratio of the Exceſs of the Axis of the Moon's Spheroid above the Diameter which is perpendicular to it, to the Exceſs of the greateſt Height of our Sea, occaſion'd by the Force of the Moon alone above the leaſt. But this Exceſs is given by Obſervation; therefore the other alſo, namely, the Exceſs of the Axis of the Lunar Spheroid above the Diameter perpendicular to it: But the Moon's Diameter is given, by *Prop.* 59; therefore alſo the requir'd Axis of the oblong Spheroid. And therefore alſo the Moon's oblong Spheroidical Figure is defin'd. *Q. E. F.*

The Figure of the Moon is compounded of the prolate Spheroid, defin'd in *Prop.* 63. and of the oblong one determin'd in this 67ᵗʰ Propoſition. But if its Surface was cover'd with a Fluid, there would beſides be a third oblong Spheroidical Figure, made by the diſturbing Force of the Sun, whoſe Axis produc'd wou'd go thro' the Sun, and therefore whoſe Eminences, in one Rotation of the Moon about its own Axis, wou'd go over the whole Compaſs of the Moon like our Tide. The higheſt Elevation of this Fluid above the defin'd Figure of the Moon, wou'd be to the Elevation of our Sea, produc'd by the Sun's Force only, as the Diameter of the Moon to the Diameter of the Earth, and therefore known.

What has been ſaid of the Earth and of the Moon its Satellite, is alſo true of Jupiter and Saturn and their Satellites, if thoſe great Bodies either now are, or have formerly been Fluid; For the mutual Gravity of Bodies, the Endeavour to recede from the Axis of Motion in rotated Bodies, and the nature of a Fluid, are the ſame at Jupiter and Saturn as at the Earth.

SECTION XIII.

Of Saturn's Ring and its Phaſes.

HAving ſhewn and demonſtrated the Geometrical Methods by which the reaſons of what has been obſerv'd of the Secondary Planets and their Phaſes are explain'd, and their Quantities are determin'd, the ſame muſt be done in reſpect of the Ring or Arch which encompaſſes Saturn, becauſe it may be look'd upon as a Satellite, and therefore ought here to be ſpoken of, now that we have explain'd other things concerning Satellites.

PROPOSITION LXVIII.

TO define the Magnitude, Figure, and Situation of the Ring which encompaſſes Saturn.

Saturn's Ring is thin, plane, and no where cohering to Saturn's Globe; [*fig.* 59.] the laſt of which is plain, becauſe we can ſee the Fix'd

Stars thro' the Space *GCB*, between the Ring and Saturn.

The Semidiameter of the Ring is to the Semidiameter of Saturn as Two and a half to One, and the Interſtice between the Globe of Saturn and *GCH*, the inner Limb of the Ring is equal to *CD*, the breadth of the Ring. Its two Surfaces, *viz. GDFHC* ſeen here, and the other hidden one parallel to it, are Planes, but yet not poliſh'd like a Speculum, but reflecting the Light of the Sun that falls upon it, every way. The thickneſs of the Ring, or the diſtance between the two ſaid Planes, is ſenſible, tho' not very great: It either does not at all, or does but very little reflect the Sun's Rays falling upon it; or perhaps like a Speculum, reflects the Light diverging from one Point, and not coming at the Earth.

This Ring, (which, upon account of its Gravity towards the Center of Saturn, is like an Arch every where ſuſtain'd at an equal Diſtance from Saturn) if it has any Revolution, has it about its own Axis, or a Right line drawn from the Center perpendicular to its parallel Surfaces; and therefore, by reaſon of its Uniformity, appears to a diſtant Obſerver to be at reſt. It is very probable that Saturn itſelf revolves about the ſaid Axis upon the Poles *P*, *p*; ſo that the Ring, in the produc'd Plane of Saturn's Æquator, encompaſſes this Planet at a Diſtance from its Surface ſomething greater than the fourth part of its Diameter.

All theſe things appear to an Eye aſſiſted with a good Teleſcope, and are meaſur'd by the help of a Micrometer; that is, are compar'd with the Globe of Saturn.

Saturn encompaſs'd with his Ring, is carried about the Sun in its Orbit, after the ſame manner as the Earth in its Orbit, which we have deſcrib'd in the 30th *Prop.* of *Book* 1. namely the Ring, and therefore perhaps the Æquator of Saturn, are carried in the Plane of the Orbit, the Axe always being parallel to itſelf; and the Plane of the Ring is always inclin'd in the ſame Angle, namely, of $23\frac{1}{2}$ Degrees, to the unmov'd Plane of the Ecliptic, and therefore always in the ſame Angle to the Orbit of Saturn.

About the Year of our Lord 1659, the common interſection of the Ecliptic and of the Plane of Saturn's Ring, was obſerv'd by *Chriſtian Hugens*, to be in the $20\frac{1}{2}$ Degree of Virgo, and the $20\frac{1}{2}$ Degree of *Piſces:* For at that time, when Saturn was in thoſe Points, it appear'd without Handles or its Ring; in the $20\frac{1}{2}$ Degree of *Gemini* and *Sagittarius*, namely, the moſt remov'd Points from the former, the Handles appear'd the moſt open and drawn aſunder, becauſe the Circle of the Ring being then ſeen with very little Obliquity, puts on the Figure of a broad Ellipſis. But becauſe the ſaid Points at $20\frac{1}{2}$ Degrees of ♍, and of ♓, are not very far diſtant from the Points where the common Interſection of the Terreſtrial Æquator, with the ſaid Plane of the Ecliptic, going thro' the Sun meets with the Ecliptic, and theſe two Planes are inclin'd in equal Angles to the Plane of the Ecliptic; theſe two Planes, namely, that of the Æquator, and that of Saturn's Ring, will be nearly parallel to each other. And therefore if this Ring be obliquely look'd upon from the Earth, and for this reaſon appears Elliptical, the greater Axis of this Ellipſe will be nearly parallel to the Celeſtial Æquator.

It is likely that the ſaid Right line (which is the common Interſection of the Plane of the Ring of Saturn, and the Plane of the Ecliptic) is carried *in antecedentia*, by an angular Motion. If this Ring be connected with Saturn's Æquator magnetically, or by any other imperceptible manner (for it is plain from Obſervation that it touches it no where,) ſo that the Planes of them may always coincide in one; Saturn's Equinoctial Point will go backward: For this Ring is ſo much redundant Matter about Saturn's Æquator; whence (by *Prop.* 64. *B.* 1.) we find the Motion of thoſe Points *in antecedentia*, after the ſame manner as that of the Terreſtrial Æquinoctial Points. If the Ring, like a Secondary Planet, performs its Motions, not depending upon Saturn's Globe, the Regreſs of the foreſaid Line will be plain, from *Prop.* 14. *Book* 4.

Moreover, the Angle which the Plane of the Ring contains with the Plane of Saturn's Orbit, will be greater, by *Prop.* 15. when Saturn is in their common Interſection, than when it is any where elſe; and leaſt of all when Saturn is moſt diſtant from it: But theſe Changes are ſo ſmall as to be ſcarce obſervable from the Earth.

PROPOSITION LXIX.

TO *deſcribe and define the Phaſes of Saturn's Ring, ſeen from the Sun.* [Fig. 60.]

We begin by what Phænomena happen when the Spectator's Eye falls in with, or is at the illuminating Body, as being the moſt ſimple and leading to the others. Firſt, it is plain, that when Saturn is ſeen from the Sun, as at *A* or *I*, (in the Figure where we have repreſented Saturn,

turn, together with its Ring carried parallel to it ſelf, with the Figure of the Diſc's Appearance drawn reſpectively behind each Repreſentation of Saturn) that we can ſee neither Face of the Ring, and that upon a double account: For firſt, ſince in that caſe the Plane of the Ring produc'd goes thro' the Sun, neither Face of it will be enlighten'd, the Sun's Rays only ſliding by on each ſide of it, therefore it can't be ſeen, as being obſcure; then, ſince the Eye is alſo plac'd in the Sun, that is, in the Planes to be ſeen produc'd, thoſe Surfaces, tho' enlighten'd, cou'd not be ſeen. If the Edge of the Ring ſhou'd, like the Planets, every way reflect the Sun's Rays falling upon it, two *bright Arms* wou'd ſtand out in a Right line inclin'd to the Ecliptic, in an Angle of 23½ Degrees; but ſince this Edge does not reflect the Rays (as has been ſhewn in the foregoing Propoſition) the Arms will not appear extended beyond Saturn's Diſc; but becauſe the Ring croſſes Saturn's enlightned Diſc, it will look like a dark Line inclin'd in the manner before mention'd, drawn thro' the Center of the Diſc: But this dark Line alſo will appear in other Poſitions, ſometimes above the Center of the Diſc, and ſometimes bent below it, as the Situation of the dark Edge requires.

As Saturn goes from *A*, thro' *B*, *C*, &c. towards *E*, or from *I*, thro' *K*, *L*, &c. towards *N*, the Sun is by degrees rais'd above the Face of the Ring, either the one or the other, till Saturn being come to *E* or *N*, a Plane drawn thro' the Sun and Saturn, perpendicular to the Plane of the Ecliptic, is alſo perpendicular to the Plane of the Ring, and the Sun is rais'd about 23½ Degrees above the Plane of the Ring, in which caſe that Face of the Ring, which the Sun enlightens, is moſt illumiated of any, by reaſon of the incidence of the Sun's Rays, which are then very direct: And the Circular Limb of the Ring, the Eye being alſo plac'd in the Sun, will, from the Laws of Optics, appear an Ellipſe, whoſe greater Axis is three times the length of the leſſer, as the Radius is almoſt three times the Sine of $23\frac{1}{2}$ Degrees. And then as Saturn goes from *E*, thro' *F*, *G*, &c. towards *I*, or from *N*, thro' *O*, *P*, &c. towards *A*, the Sun's Elevation above the enlighten'd Face, is by degrees diminiſh'd, till at laſt it enlightens neither Face, but throws its Rays along or parallel to both of them. In any intermediate Situation the enlighten'd Face will be more or leſs ſo, according to the greater or leſs Diſtance of Saturn from the Situation *A* or *I*, and the circular Ring will appear under the Shape of a more contracted or more dilated Ellipſe; namely, its greater Axis will be every where to its leſſer, as the Radius to the Sine of the Angle in which a Right line, drawn from the Eye to its Center, is inclin'd to the Plane of the Ring. The foreſaid Angle in which the Sun (or an Eye in the Sun) is rais'd above the Plane of the Ring, in any Situation of Saturn (namely *C*) is equal to the Angle in which the Sun is rais'd above the Plane of the Terreſtrial Æquator, (that is, to the Sun's Declination from the Æquator) when the Earth is diſtant from the Point where it is Æquinox to the Inhabitants of the Earth, by an Angle equal to *ASC*, and therefore it may eaſily be found by the Table of Declination of the Points of the Ecliptic: For (by the foregoing *Prop.*) the greateſt Elevation of the Sun above the Plane of the Terreſtrial Æquator is equal to its greateſt Elevation above the Plane of Saturn's Ring.

When Saturn ſticks near the Point *A*, the Ring will leaſt of all be perceiv'd, not only becauſe the leſſer Axis of the Ellipſe is very little in ſuch a caſe, but chiefly becauſe it will be but very little enlighten'd, by reaſon of the obliquity with which the Sun's Rays fall upon it, and therefore will be ſcarce perceivable to the Eye. But it is plain by experience, that to have the Face of the Ring enlighten'd, ſo that it may be viſible, the Sun muſt be at leaſt three Degrees above its Plane; in order to which the Heliocentric Place of Saturn muſt be ſeven or eight Degrees diſtant from *A* or *I*, and therefore all the time that Saturn, ſeen from the Sun, is on this ſide or beyond the Point *A* or *I* (already often deſcribed) it ſticks between ſeven or eight Degrees, that is, when in the oppoſite part of its Orbit, it runs thro' thoſe fifteen Degrees, which laſts about as many Months, its Ring and Arms or Handles which it makes will not appear; but Saturn will appear round and alone without any Ring. But theſe Limits will be different according to the different goodneſs of the Obſerver's Eye, and length and goodneſs of his Teleſcope. Theſe are the chief Phænomena of the Ring that encompaſſes Saturn, ſeen from the Sun; and this is the Manner of determining them.

SCHOLIUM.

For the ſame reaſon, the Ways of the Satellites revolving about any Primary Planet, ſeen from the Sun, appear ſometimes ſtreight, ſometimes Elliptical: Streight when the Line of the Nodes of the Satellites produc'd goes thro' the

Sun; when alſo the Satellite ſeen from the Sun been very much diſtant from its Primary Planet, declines alſo very much from the way of the Primary ſeen from the Sun, in which caſe the Eclipſe of that Satellite running into the Shadow of the Primary, will be central, and therefore (*cæteris paribus*) very laſting, and the Shadow of the Satellite, running thro' the lower part of its Orbit, falling upon the Primary, runs over the Center of the Diſc ſeen from the Sun. Whence the Phænomena of the Sun and of a Satellite ſeen from its Primary, will be defin'd, according to the foregiven Rule. Out of this Situation the Way of a Satellite, ſeen from the Sun, will appear to be an Ellipſe, (that is, a Circle ſeen obliquely,) whoſe greater Axis is not yet in the apparent Way of the Primary. This Ellipſe will have its ſhorter Axis, the leſs by how much nearer the Primary is to thoſe Points where the former Phaſis happens, and the greater by how much it is more remov'd from them; and therefore the longeſt of all (that is, this Ellipſe, which the Circular Orbit of the Satellite appears to be, will be broadeſt) when the Primary is in the Places intermediate between thoſe mention'd; namely, when a Right line joining the Sun and a Primary Planet is very little oblique to the Plane of the Orbit of a Satellite; in which caſe the Satellite, in its greateſt Elongation from its Primary, will appear to be in the Way of the Primary ſeen from the Sun; but in its Conjunction with the Primary, it declines very much from the ſaid Way, on the one or the other ſide, as it is in the inferior or ſuperior Semicircle of the Orbit. Whence it happens that a Satellite in the upper part of its Orbit, will be very far diſtant from the Axis of the Shadow of the Primary, and in the lower part of it, the Shadow of the Satellite will alſo be very diſtant from the Center of the Diſc of its Primary ſeen from the Sun; therefore according to the Inclination of the Plane of the Orbit of the Primary, and according to the Magnitude of the Satellite's Orbit, the Eclipſes ariſing hence will be (*cæteris paribus*) either very ſhort, or perhaps none at all. In the intermediate Place the Satellite's Way about the Primary ſeen from the Sun, will be an Ellipſe of leſs breadth, and all the foreſaid things will be mix'd. And all theſe Phaſes will happen twice in one Revolution of the Primary about the Sun, if the Line of the Nodes always remains parallel to itſelf; if it moves, all variety of Phaſes will happen, from the time that that Line of Nodes produc'd falls into the Sun, till it be again directed towards it. And converſly, from theſe Obſervations of theſe longer or ſhorter Eclipſes, the Poſition of the Line of Nodes of a Satellite, and the Inclination of the Plane of the Orbit of a Satellite to the Plane of the Orbit of a Primary, will be defin'd.

PROPOSITION LXX.

TO *deſcribe and define the Phaſes of Saturn's Ring ſeen from the Earth.*

Tho' the Phaſes of Saturn's Ring ſeen from the Sun are nearly the ſame with the Phaſes of it ſeen from the Earth, (becauſe Saturn's Geocentric Place, at moſt, ſcarce differs ſix Degrees from its Heliocentric Place) and therefore are to be defin'd by the foregoing *Prop.* Yet ſince they differ in ſome things, eſpecially becauſe

Saturn appears round, we ſhall in a few Words ſhew wherein they differ.

Let *S*, [*fig.* 61.] be the Sun, *ABIN* the Orbit of Saturn, *DCT* the annual Orbit of the Earth; (for we don't here conſider its Inclination to the Orbit of Saturn as being very ſmall) and let *A* and *I* be the Places of Saturn's Orbit, in which Saturn is when the Plane of its Ring produ'd goes thro' the Sun, and it is Æquinox in Saturn, if the Ring be in the Plane of the Æquator and Saturn revolves about its own Axis. It is plain that when Saturn is in *A* or *I* the Ring will not be ſeen by an Eye in the Earth, whatever part of its Orbit the Earth is in: For ſince in that caſe neither Face of the Ring is enlighten'd by the Sun, neither Face will be ſeen by the Eye, tho' rais'd high enough above its Plane. The Edge only of the Ring is enlighten'd, which yet is not ſeen, becauſe it does not reflect the Rays, at leaſt ſo as they may reach the Earth: Yet ſince it croſſes the bright Diſc of Saturn, it appears obſcure or like a dark Line, for the Reaſons explain'd in the foregoing Propoſition.

Moreover, if when Saturn is got to *B* the Earth be in ſuch a Point *C* of its Orbit, that *BC* be parallel to *AI*, Saturn will alſo appear round: For tho' one face of the Ring be in that caſe enlighten'd by the Sun, an Eye on the Earth is alſo found in the Plane of the Ring produc'd, and therefore it does not ſee it but has only the dark Edge turn'd towards it: This happens when Saturn ſeen from the Earth, has behind it the ſame Fix'd Stars, which it had when the Eye was ſuppos'd in the Sun, and the Sun in the Plane of the Ring produc'd; that is, when the Geocentric place of Saturn is the ſame with the Heliocentric Place, (which happens when it is in the Plane of the Ring produc'd) as may eaſily be known from Aſtronomical Tables.

Again, let Saturn be ſuppoſed in the ſituation *H*, the Earth in *E*, ſo that the Plane of the Ring, or rather its common Section with the Plane of Saturn's Orbit, which is the Right line *HM*, parallel to *IA*, being produc'd, falls between the Sun *S* and the Earth *E*, which will happen when Saturn's Geocentrical Place and its Heliocentrical Place are at different parts of the Place deſcrib'd above in *Prop.* 68, where the Plane of the Ring produc'd falls on the Sun or Earth, which is readily known by Aſtronomical Tables, eſpecially Ephemeris's; for the Right line *SH* ſhewing the Heliocentric Place of Saturn, falls on one ſide of the Right line *HM*, and *EH*, ſhewing its Geocentric Place falls on the other ſide of the ſame Right line: I ſay, that in ſuch a caſe an Obſerver on the Earth will not ſee the Ring, (only that a dark line will appear to croſs Saturn's Diſc) becauſe he can't ſee that face of the Ring which is expos'd to the Sun's Rays, as being turn'd from him. For in that caſe, that face which is as a ſhadow is turn'd towards the Earth.

On the contrary, when the Right lines *TN*, *SN*, drawn to Saturn from the Sun and the Earth, fall towards the ſame parts of the ſaid Line *NR*, (the common Section of the Planes of the Ring and of the Orb of Saturn) as it happens when Saturn is in *N*; that is, when Saturn's Geocentric Place and its Heliocentric Place are on the ſame ſide in reſpect of a Place above deſcrib'd, where the Plane of the Ring produc'd falls into the Sun; then the ſame face of the Ring is turn'd to the Sun and to the

Earth, and therefore (if the Reaſons given in the foregoing *Prop.* don't interfere) the enlightened Face ſeems to give Saturn Arms or Handles; namely, if the Sun does enough enlighten the Ring, that its Light may affect the Eye; that is, if it be rais'd three or more Degrees above the Plane of the Ring, or if Saturn be diſtant from the Point *A* or *I* eight Degrees or more, and the Earth alſo be rais'd one Degree or more above the Face of it, (for ſome ſenſible Elevation is neceſſary for the Eye to have above the Plane of the Ring, leſt the Ellipſe, which the Ring repreſents, ſhou'd appear like a very ſmall Line, and ſo not reflect Light enough to affect the Eye;) that is, if the Geocentric Place of Saturn be about three Degrees or more from the place where the Plane of its Ring produc'd falls upon the Earth, the Ring will be viſible, as has been ſhewn in the foregoing *Prop.*

To define what kind of Ellipſe the Ring at any time repreſents, we muſt obſerve, that when its Plane is inclin'd to the Plane of the Orbit of the Earth about 23 Degrees, where the Circle of the Ring appears the moſt direct of all, a Line joining the Earth and the ſaid Circle will be inclin'd to the Plane of the Circle in the ſaid Angle; and therefore in that caſe the Circle will appear to be an Ellipſe, whoſe greater Axis is three times longer than the leſſer Axis. Moreover, in any other Inclination of a Right line joining the Earth and Saturn to the Plane of the Circle, (we muſt find the Diſtance of the Geocentric Place of Saturn from that Geocentric Place of it where the Plane of the Ring produc'd goes thro' the Earth, by help of the Table of Declination of the Points of the Ecliptic, as above) its greater Axis will be to its leſſer, as the Radius to the Right Sine of the lately found Angle, in which the Eye is rais'd above the Plane of the Ring. And theſe are the chief Phaſes of the Ring ſeen from the Earth, and the manner of defining them. *Q. E. F.*

The Ellipſe alſo which repreſents the inner Circle of the Ring is of the ſame kind, as being in the ſame Plane with the outer one: But this Ellipſe is not the ſame with the perforated Figure, upon account of the thickneſs of the Ring. The hithermoſt half of this Figure is the ſame with part of the foreſaid Ellipſe; but the furthermoſt part of it is equal and ſimilar to a part of the Ellipſe, brought a little nearer to the Axis, as is plain from *Fig.* 59.

SCHOLIUM.

After the ſame manner from the Way or Track of a Satellite about its Primary Planet, ſeen from the Sun, and determin'd by the foregoing *Prop.* may be determin'd the Figure of it, ſeen from the Earth at any time propos'd. For it is plain that a Satellite will, to an Inhabitant of the Earth, appear to have deſcrib'd a Right line, when the Earth is found in the Plane of the Orbit of a Satellite, tho' the Sun at that time is not in it; and that the Ellipſe (which, in all other caſes, it repreſents) comes neareſt to a Circle, when a Right line joining the Earth and a Primary Planet, is the leaſt oblique to the Plane of the Orbit of a Satellite, tho' at that time a Right line drawn from the Sun to the Primary Planet, is not the leaſt oblique that it can be to the ſaid Plane. And on the other hand, from the Obſervation of the ſaid Phaſes, may be defin'd the Situation of the Line of Nodes, and the Inclination of the Plane of the Orbit of a Satellite, to the Plane of the Orbit which its Primary deſcribes about the Sun.

Moreover, the ſame things are true of the apparent Way of a Spot or any other viſible Mark in the Surface of a Primary Planet, or of the Sun itſelf, in its Rotation about its Axis, or of a Circular Order or Series of ſuch Marks: For ſuch a Circle will, by the Laws of Optics, appear ſometimes a Circle, ſometimes an Ellipſe, as the Earth is in its Plane produc'd, or out of it.

Plate 1 Book 4.

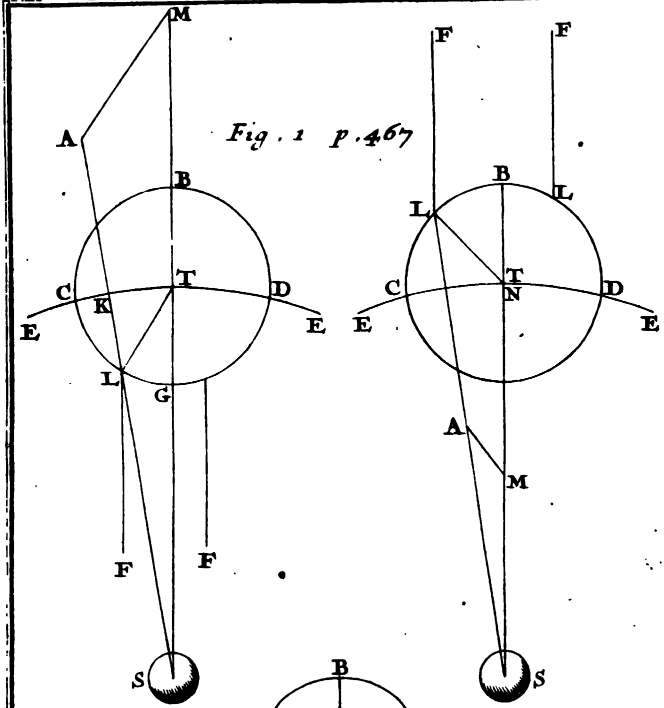


Fig. 1 p. 467

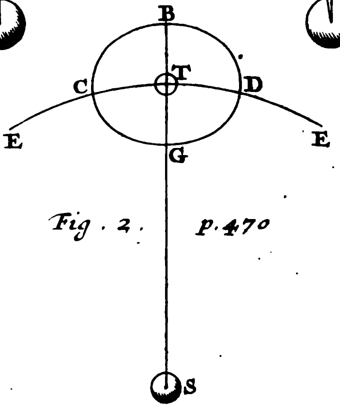


Fig. 2. p. 470

Plate 2 Book 4.

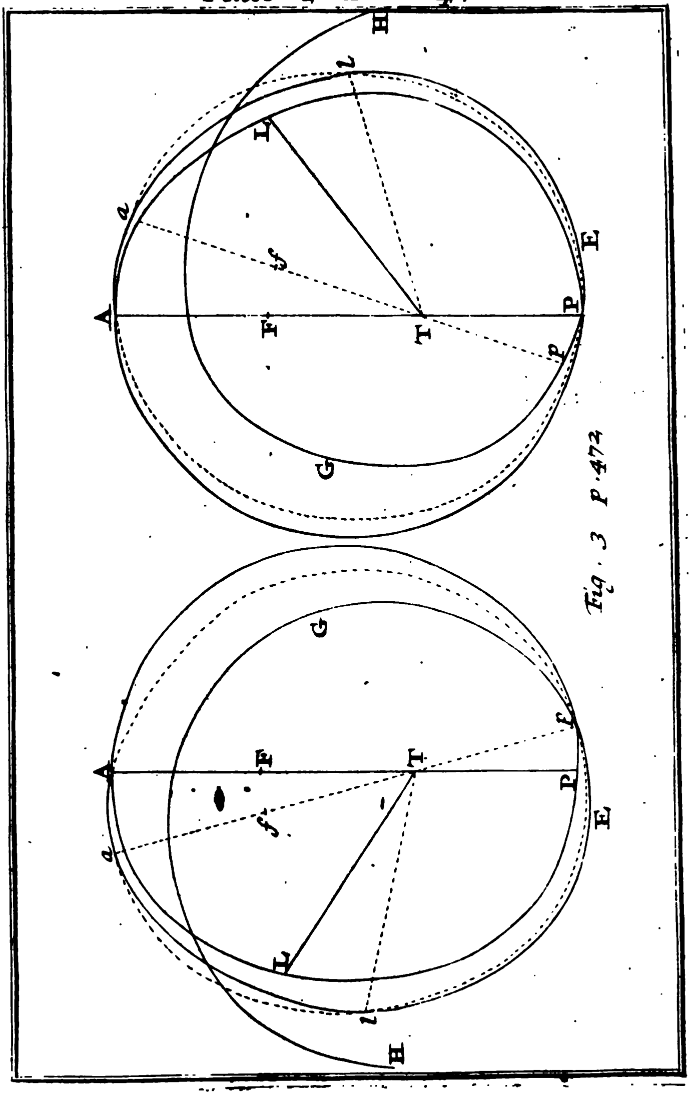


Fig. 3 P 472

Plate 3 Book 4.

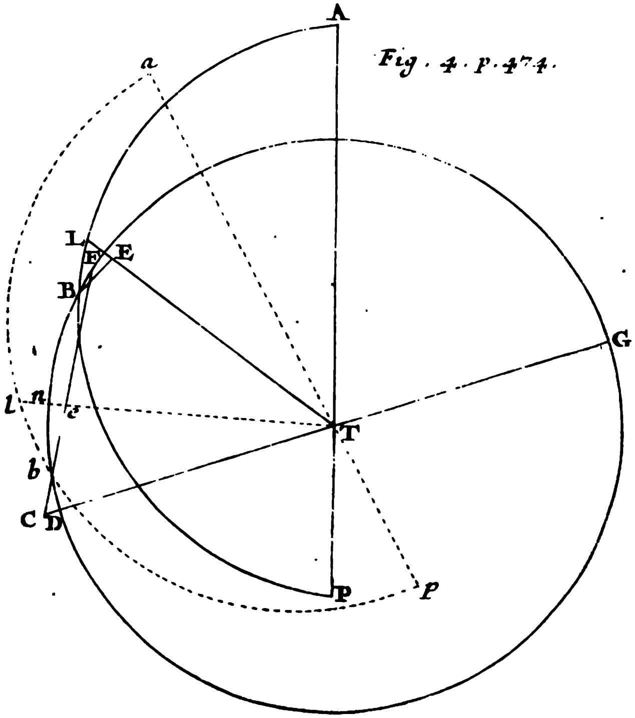


Fig. 4. p. 474.

Plate 4 Book 4.

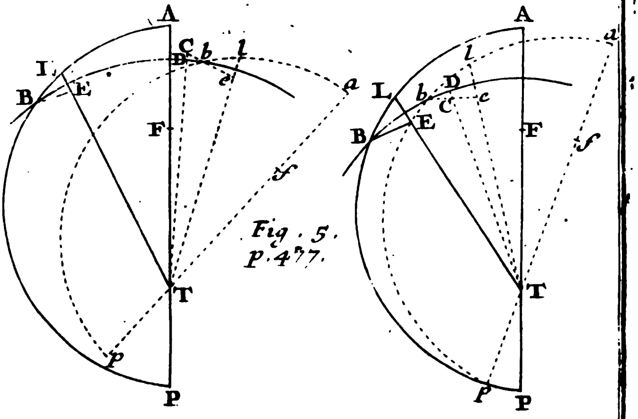


Fig. 5. p. 477.

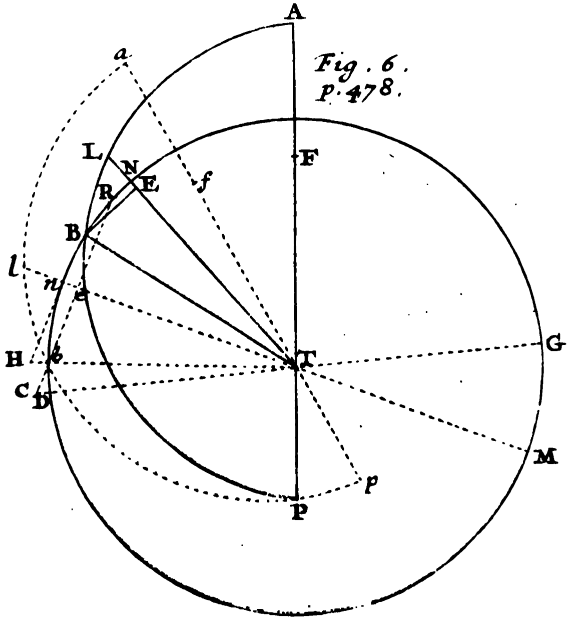


Fig. 6. p. 478.

Plate 5 Book 4.

Fig. 7.
p. 479.

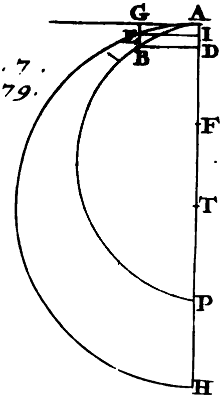


Fig. 8.
p. 480.

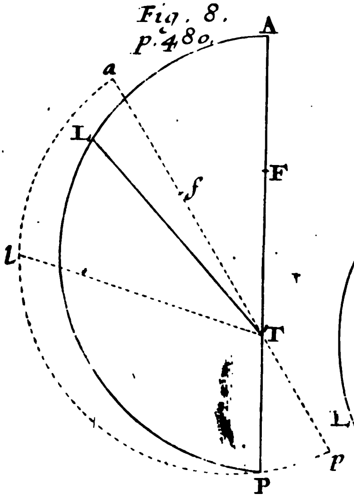
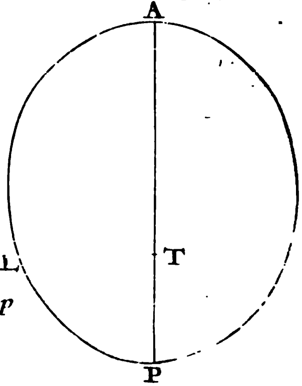


Fig. 9.
p. 482.



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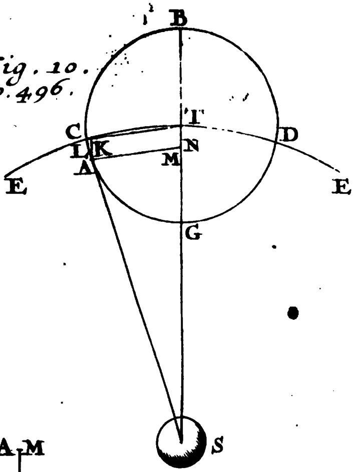


Fig. 10. p. 496.

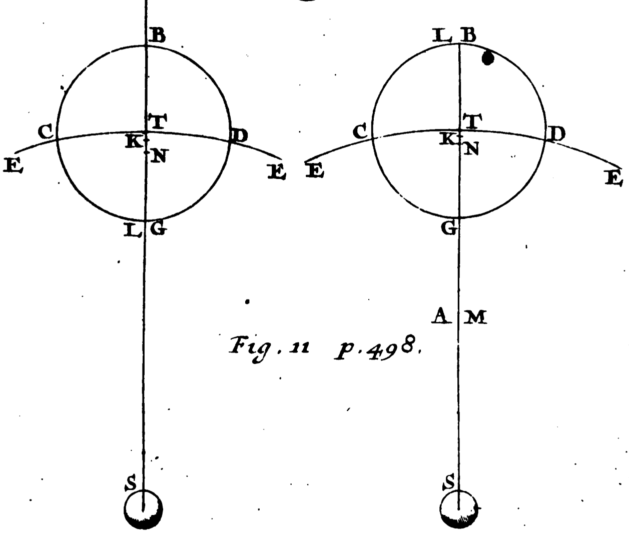


Fig. 11 p. 498.

Plate 7 Book 4.

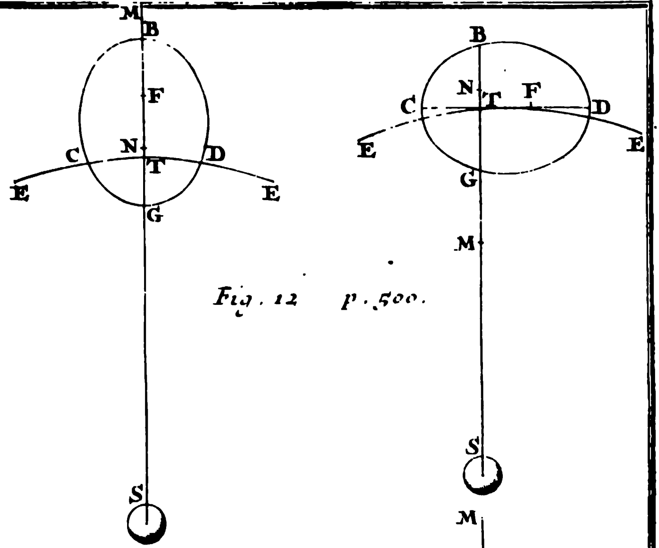


Fig. 12 p. 500.

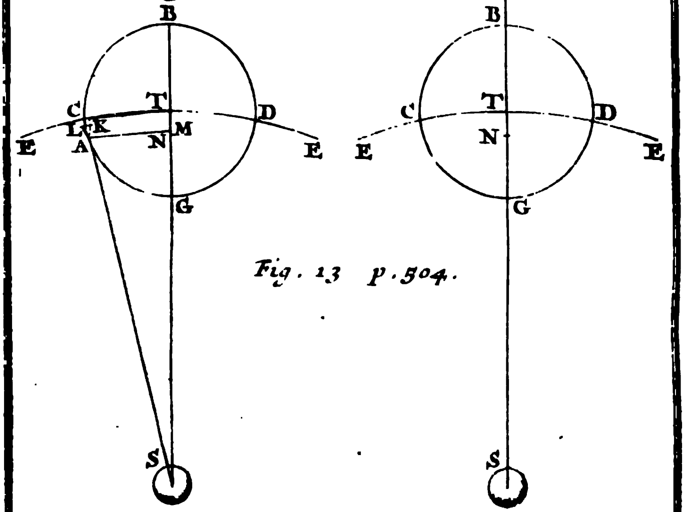


Fig. 13 p. 504.

Plate . 8 Book 4.

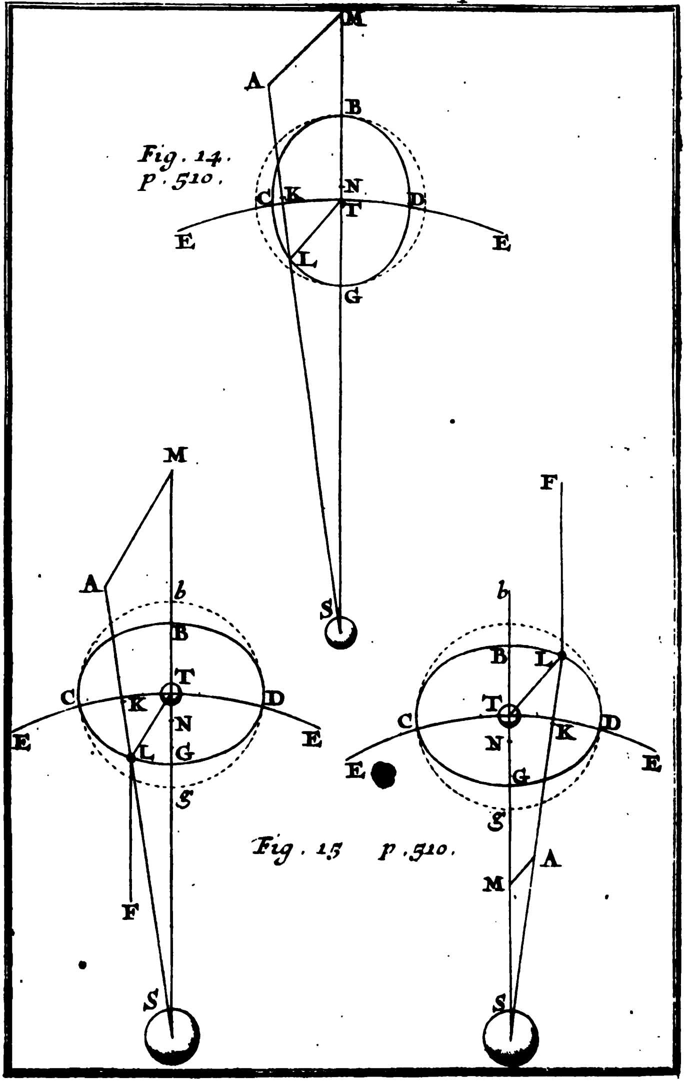
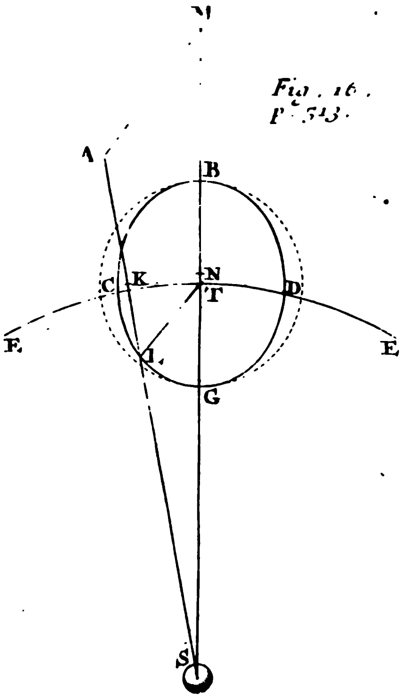


Fig. 14. p. 510.

Fig. 15 p. 510.

Plate 9 Book 4.

Fig. 16. P. 313.



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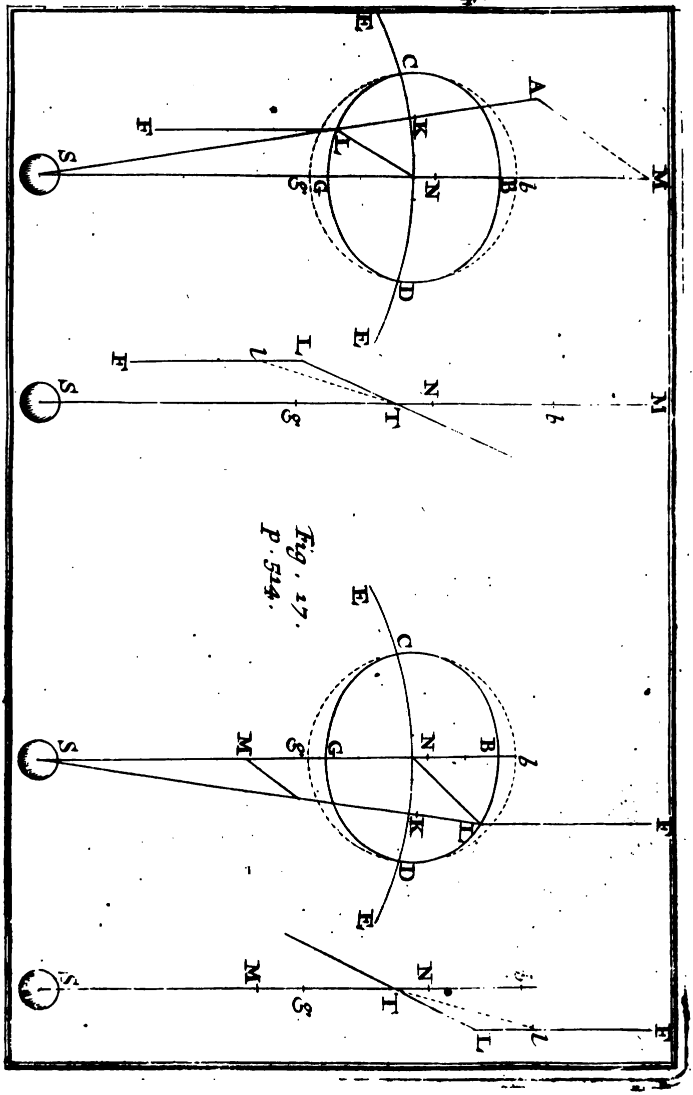
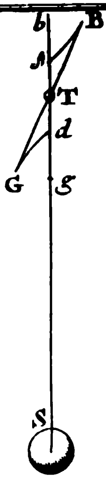
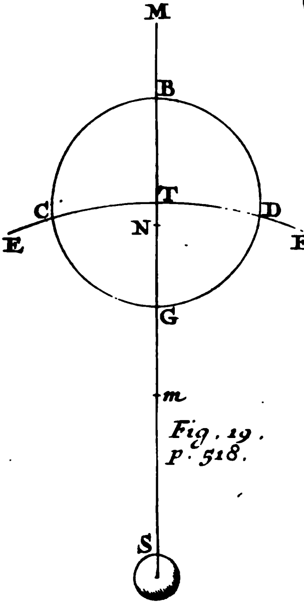


Fig. 17.
P. 514.

Fig. 18. p. 515.





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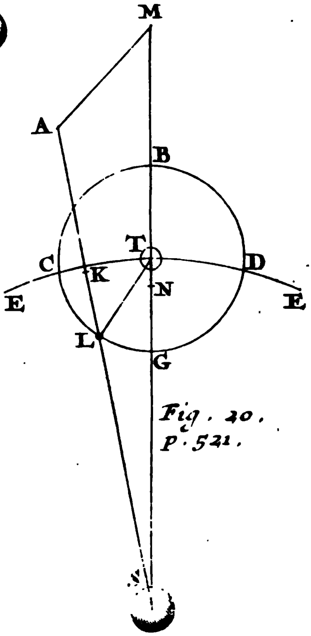

Fig. 20. p. 521.

Plate 12 Book 4.

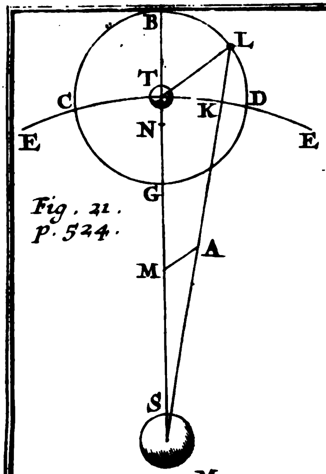

Fig. 21.
p. 524.

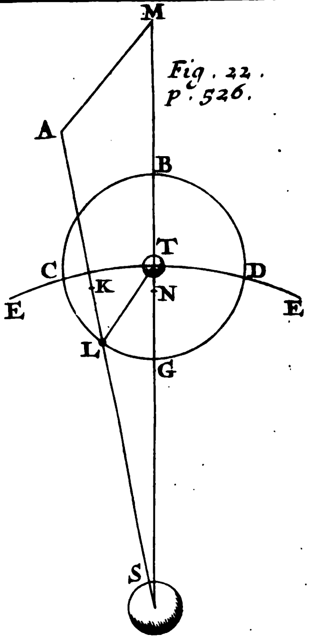

Fig. 22.
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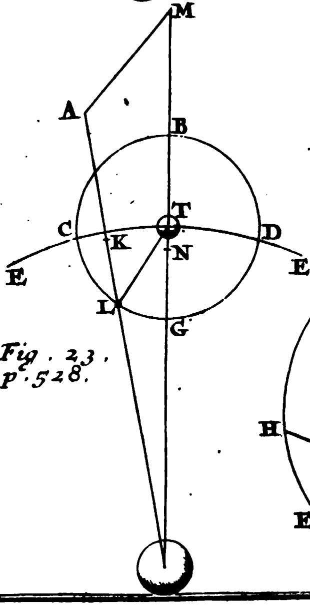

Fig. 23.
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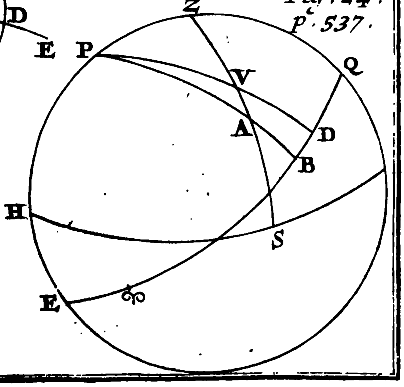
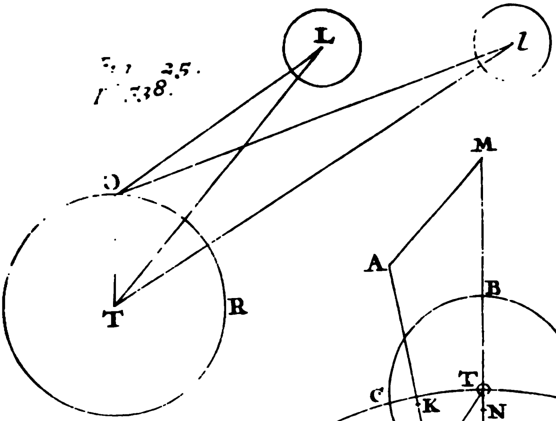


Fig. 24.
p. 537.

Plate. 13 . Book 4



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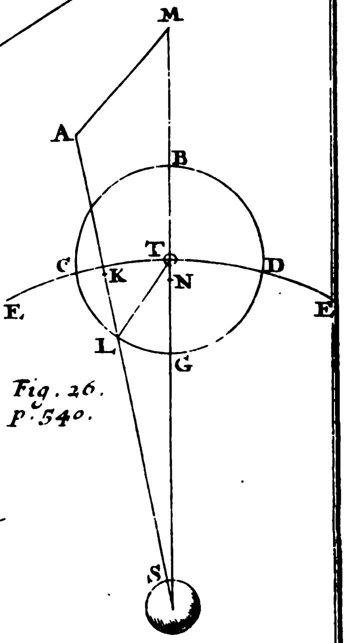

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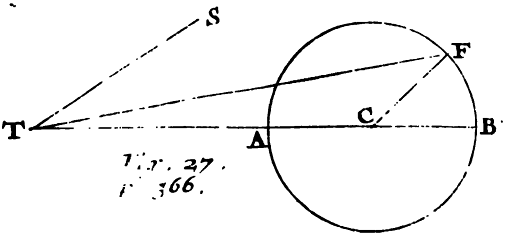


Fig. 27.
P. 566.

Plate 14 Book 4.

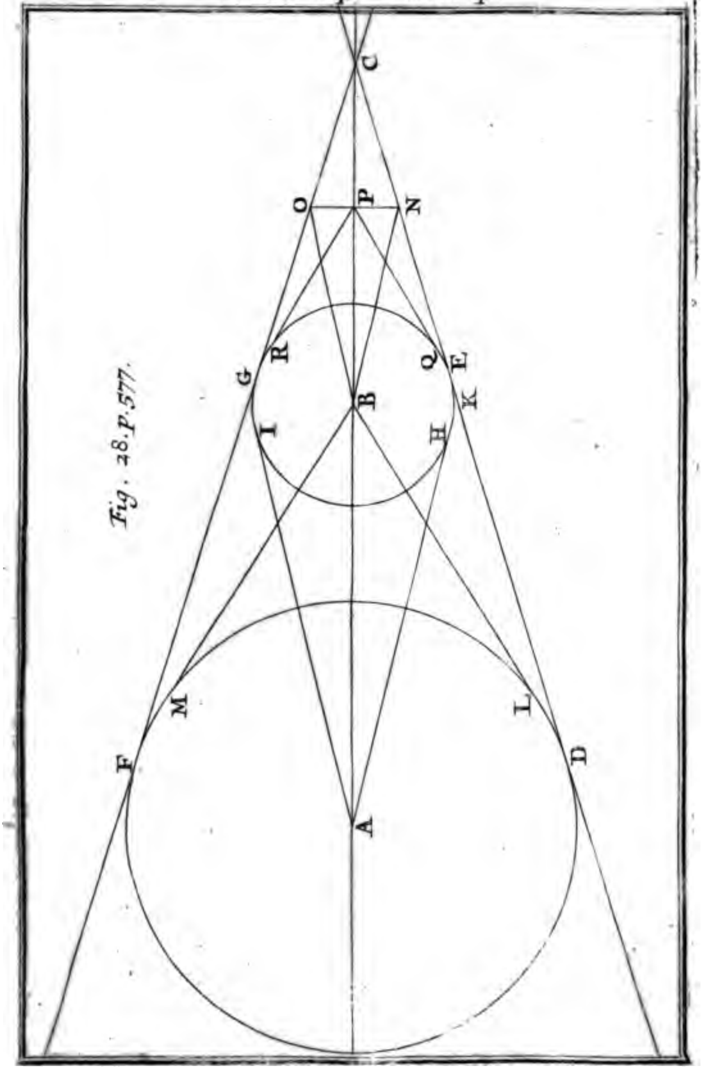


Fig. 28. p. 577.

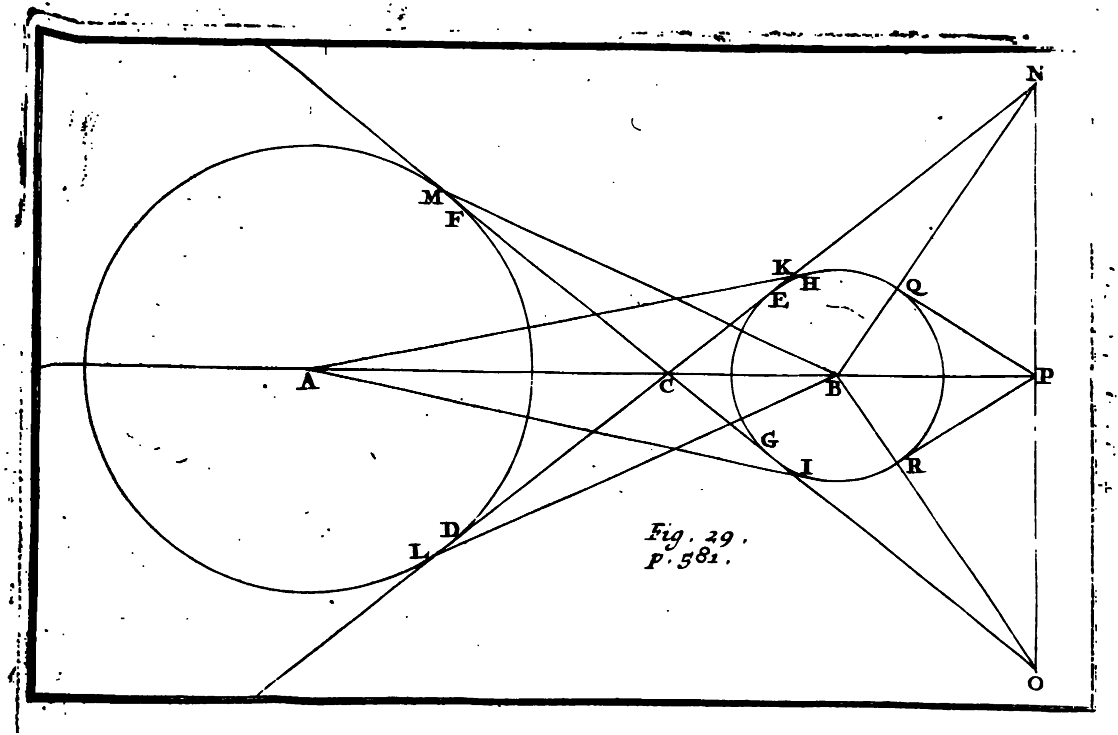
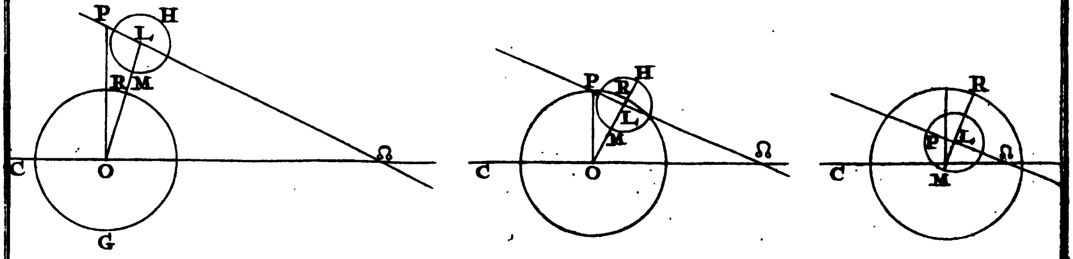


Fig. 29.
p. 581.

Plate 16 Book 4

Fig. 30. p. 584.



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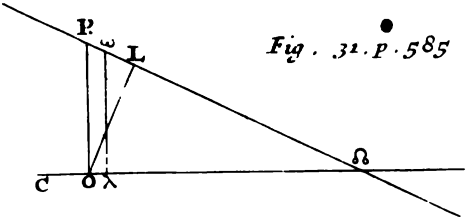


Fig. 31. p. 585

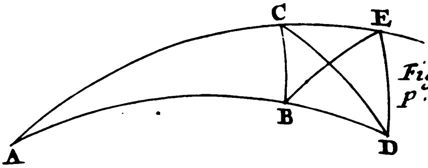


Fig. 32. p. 588.

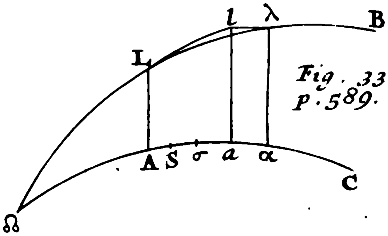


Fig. 33. p. 589.

Plate 18. Book 4.

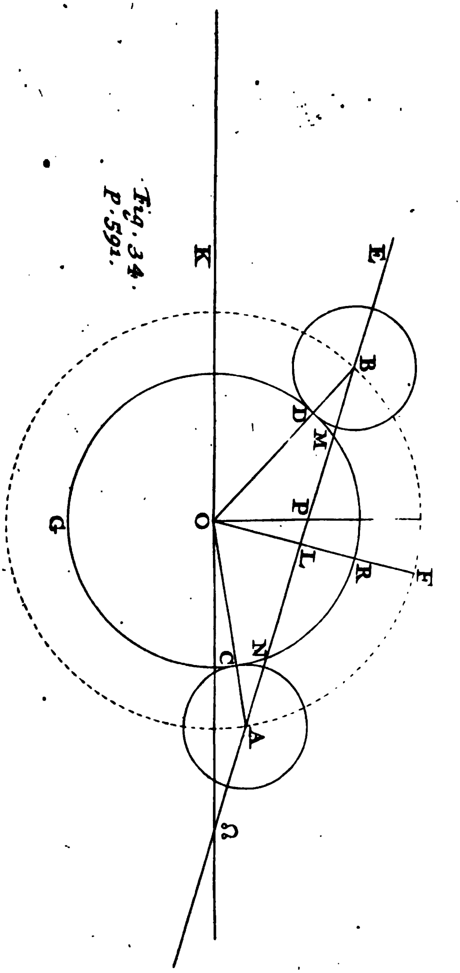
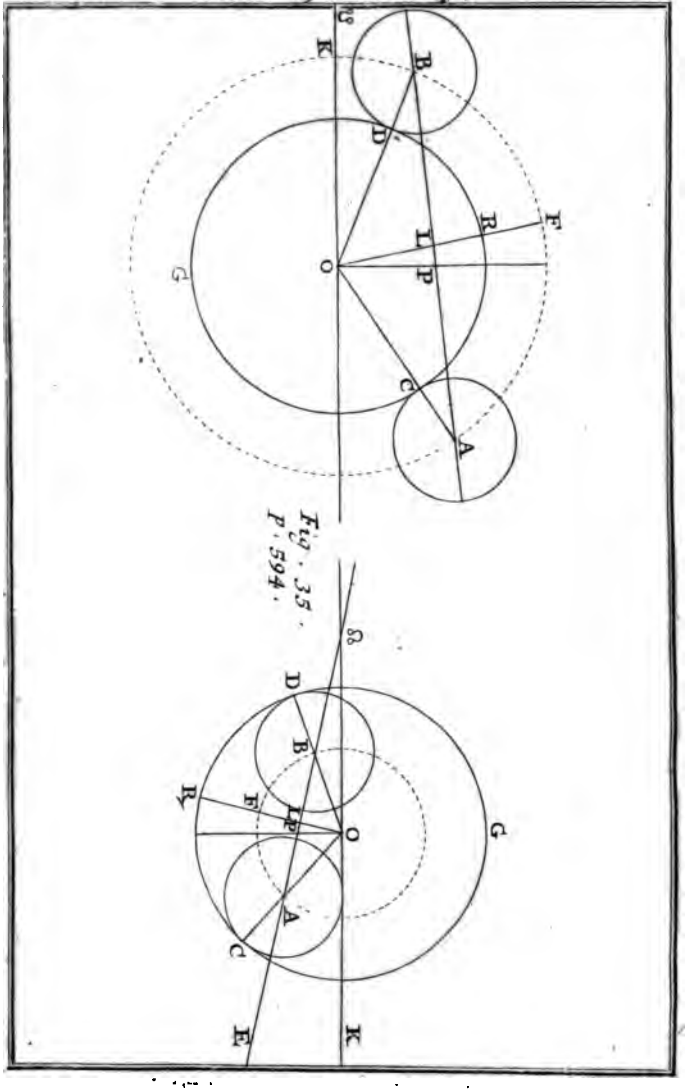


Fig. 34.
P. 591.



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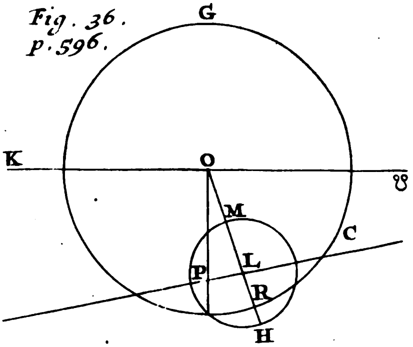

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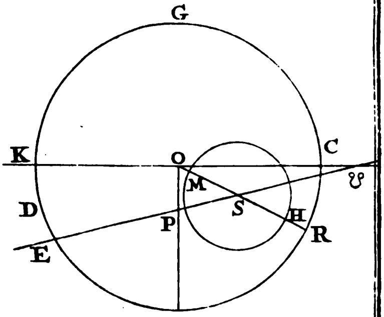

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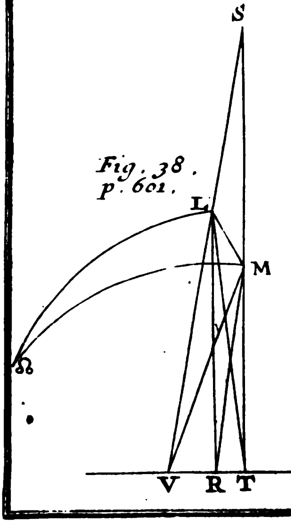
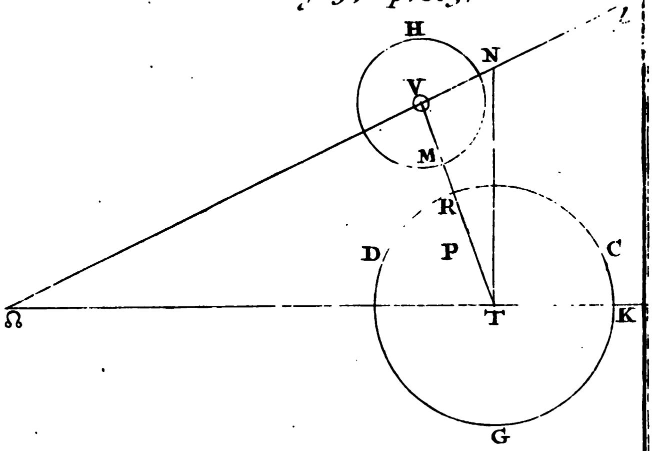


Fig. 38.
p. 601.

Plate 21. Book 4.

Fig. 39. p. 603.



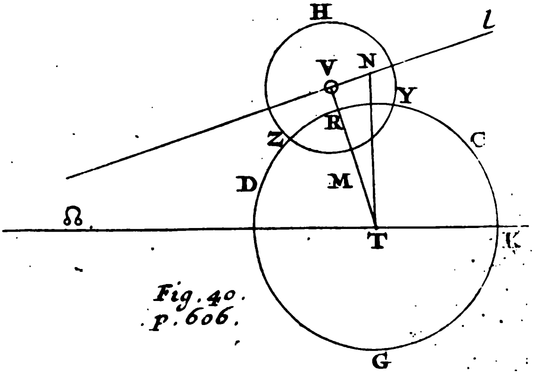
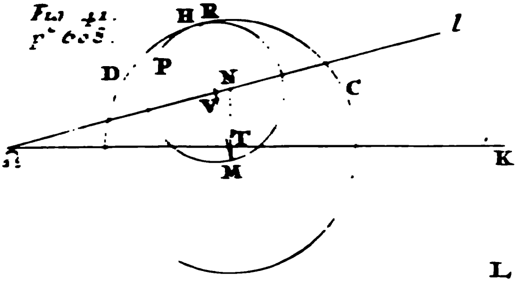
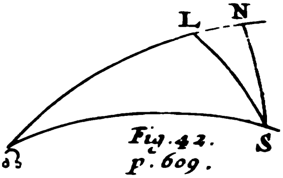


Fig. 40.
p. 606.

Fig. 41.
p. 605.





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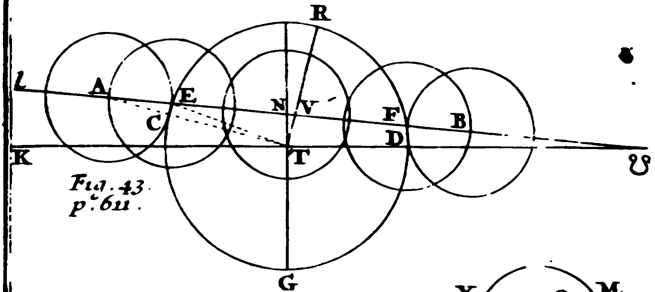

Fig. 43.
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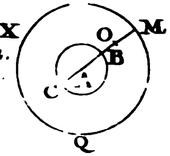
Fig. 44.
p. 615.



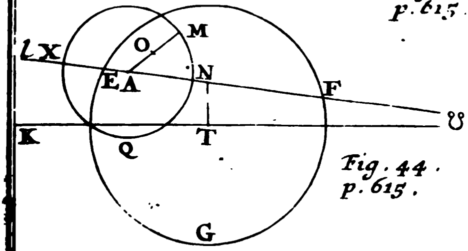


Fig. 44.
p. 615.

Book 4. Plate. 23.

Fig. 45.
p. 617.

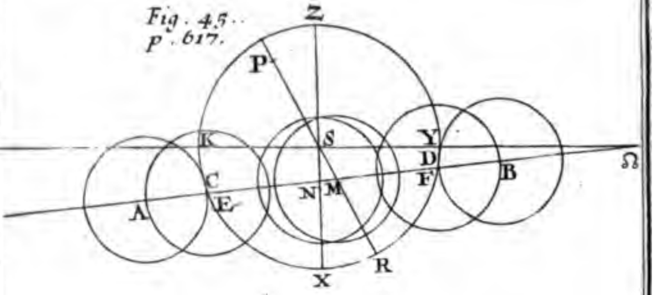


Fig. 46.
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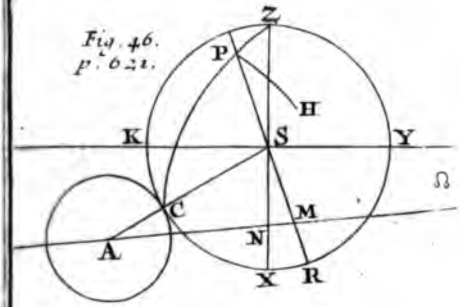

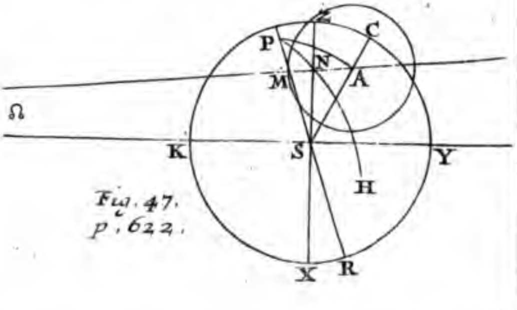

Fig. 47.
p. 622.

Book 4. Plate 24.

Fig. 48.
p. 625.

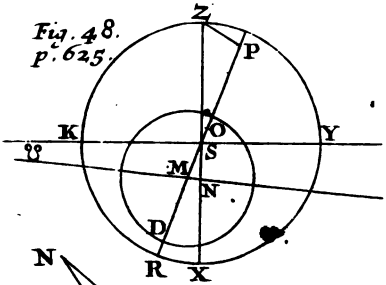

Fig. 49.
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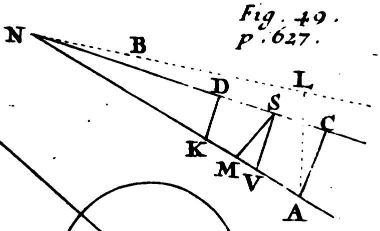
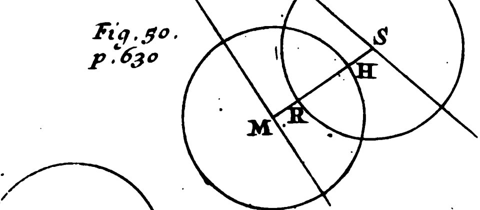


Fig. 50.
p. 630



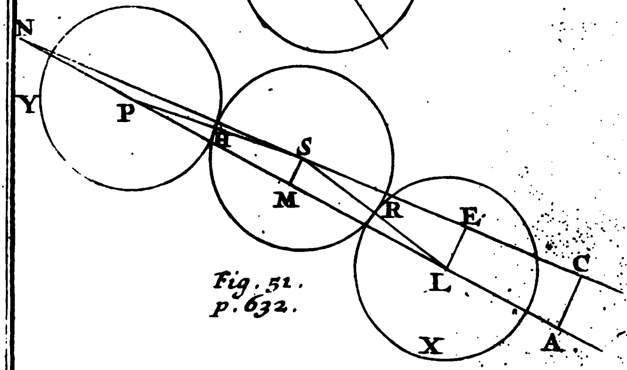


Fig. 51.
p. 632.

Plate 25. Book 4.

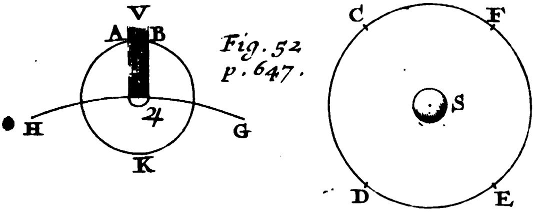


Fig. 52
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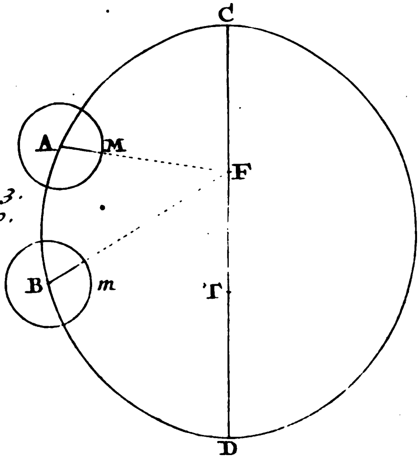

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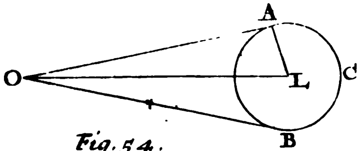


Fig. 54.
p. 657.

Plate 26. Book 4.

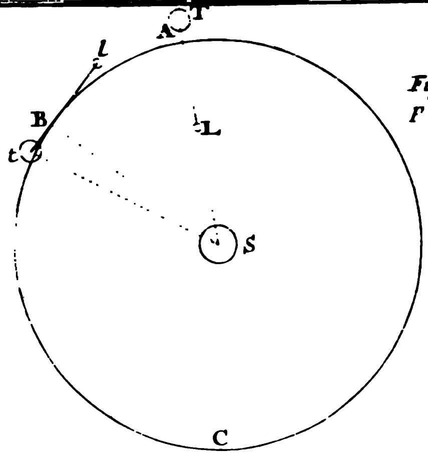


Fig. 55. p. 657.

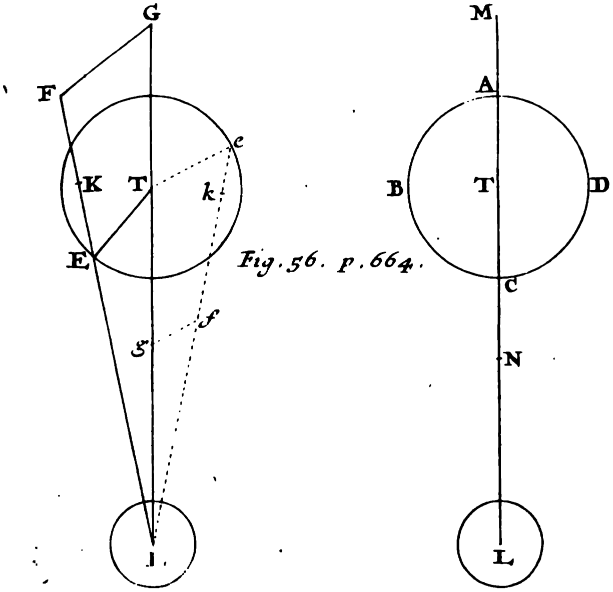


Fig. 56. p. 664.

Plate . 27 . Book 4 .

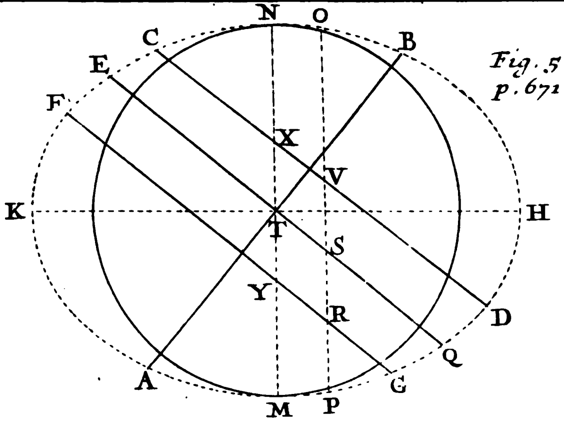


Fig . 57 .
p . 671 .

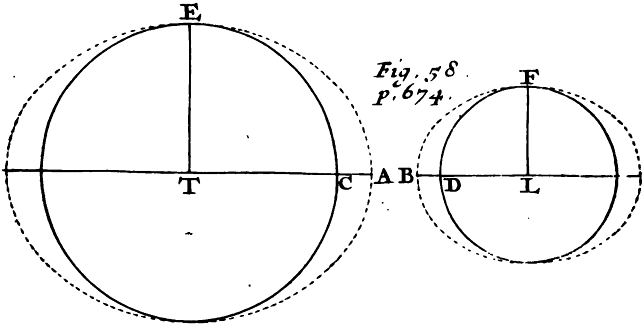


Fig . 58
p . 674 .

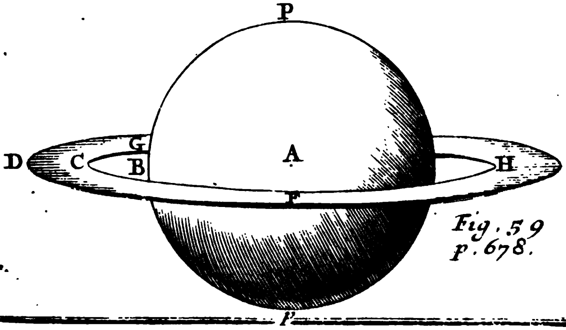


Fig . 59
p . 678 .

Plate 28. Book 4.

Fig 60. p. 681.

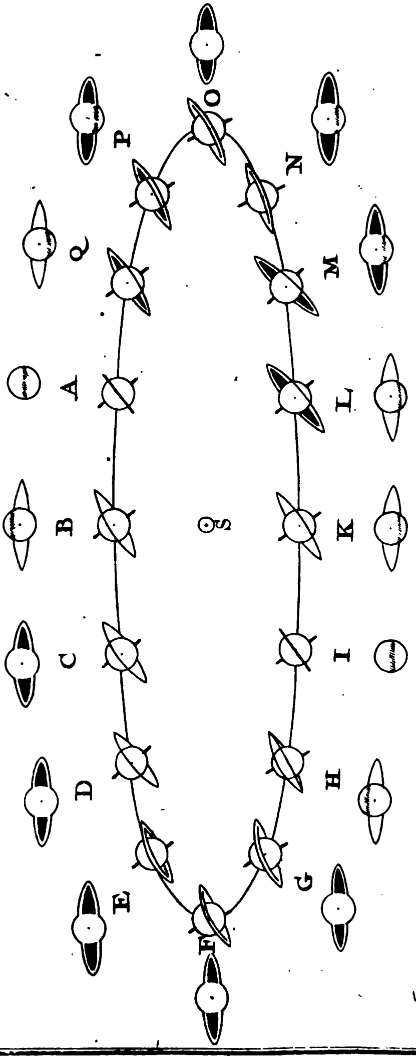


Plate 29. Book 4.

Fig. 61. p. 687.

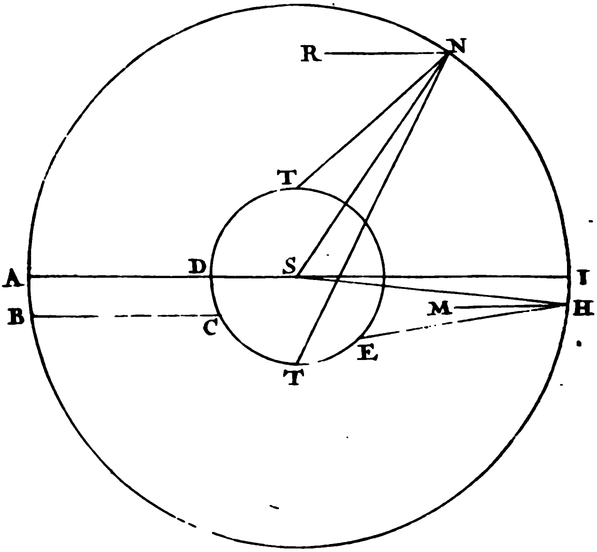


Fig. 1. p. 701.

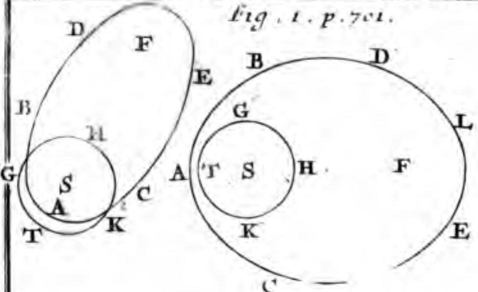


Fig. 2. p. 703

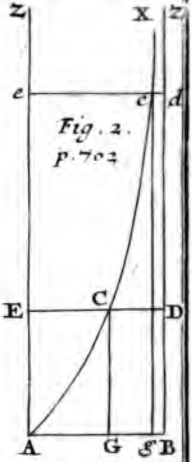


Fig. 3. p. 719.

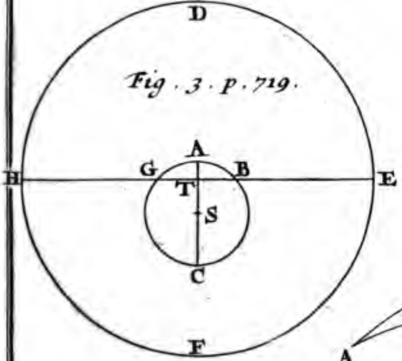


Fig. 4. p. 724.

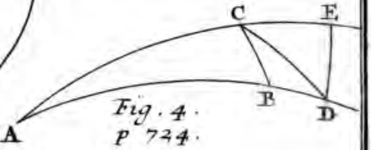
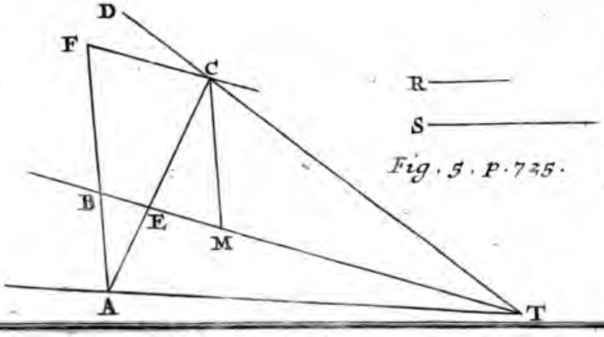


Fig. 5. p. 725.



THE ELEMENTS OF Aſtronomy, Phyſical and Geometrical.

The FIFTH BOOK.

Of Comets.

HAving explain'd the Motions of the Planets as well Primary as Secondary, and the Phænomena thence ariſing, we proceed to explain the Motions of another ſort of Bodies, namely of Comets, a great many kinds of which are mentioned by Authors: For they diſtinguiſh them according to the various colours which they appear to ſhine with, and the Planets which they look like, and the Figures of their Tails. We that treat of Aſtronomy and not Aſtrology, own only one kind of Comets, which have thoſe different Appearances, according to the difference of the Vapours which make their Tails, and the different Situation of the Comet itſelf in reſpect to the Sun.

In treating of them we ſhall firſt take notice of ſome Phyſical Matters, that we may afterwards go on to Geometrical Propoſitions, namely to determine their Orbits. In the Phyſical Things which we ſhall take notice of, it will firſt be neceſſary to mention, and afterwards reject the Opinions of ſome Philoſophers, that we may the better underſtand thoſe Opinions which we muſt embrace. The Reader is deſir'd to pardon our taking no notice of theſe things in the foregoing Books, but reſerving them for this; becauſe the Geometricians have not hitherto had that regard to the Theory of the Comets which they have had to the Theory of the Planets and Fix'd Stars.

SECTION I.

General Propoſitions concerning Comets.

PROPOSITION I.

TO explain the Opinions of Philoſophers concerning the Duration and Places of the Comets, in a few Words.

The moſt ancient Philoſophers and Aſtronomers ſuppos'd Comets to be eternal or conſtant Bodies of the World, which, like Planets, perform'd their Revolutions in ſtated Times: For *Ariſtotle*, ſpeaking of Comets, in his *Firſt Book*

Book of Meteors Chap. 6. ſays, Τῶν δ' Ἰταλικῶν τινὲς καὶ καλουμένων Πυθαγορείων, ἕνα λέγουσιν αὐτὸν εἶναι τῶν πλανήτων ἀστέρων, ἀλλὰ διὰ πολλοῦ τε χρόνου τὴν φαντασίαν αὐτοῦ εἶναι καὶ τὴν ὑπερβολὴν ἐπὶ μικρὸν, ὅπερ συμβαίνει καὶ περὶ τὸν τοῦ Ἑρμοῦ ἀστέρα. *But ſome of the Italians, call'd Pythagoreans, ſay, That a* Comet *is one of the* Planets *or* Wandring Stars, *but that they don't appear unleſs after a long time, and are ſeen but after a ſmall time, which happens alſo to Mercury.* And *Plutarch*, in his *Third Book of Philoſophical Opinions, Chap.*2.ſays, Τῶν ἀπὸ Πυθαγόρου τινὲς μὲν ἀστέρα φασὶν εἶναι τὸν κομήτην τῶν οὐκ ἀεὶ φαινομένων, διά τινος δὲ ὡρισμένου χρόνου περιοδικῶς ἀνατελλόντων. *Some of the* Pythagoreans *ſay that a* Comet *is a Star of the number of thoſe which do not conſtantly appear in the Heavens, but in ſtated Times having gone thro' a determinate Circuit ariſe or appear again*; And *Seneca*, in his *Seventh Book of Nat. Queſtions Chap.* 3. ſays thus, *Democritus, the moſt ſubtile of all the Ancients, ſaid that he ſuſpected that there were ſeveral* Running Stars, (that is, Comets) *but he neither determin'd their number nor names, not having yet rightly underſtood the Courſes of the Five Stars*, that is, of the Planets, except the Sun and Moon. And again, *Seneca*, in the ſame place, ſays, that *Apollonius the Myndian, who was very skilful in natural Sciences, affirm'd, that the Comets were, by the* Chaldæans, *reckon'd among the Planets, and had their Periods or Courſes like them.* This *Apollonius* (as the ſame *Seneca* ſays in the 17th *Chap.*) us'd to ſay, *that a* Comet *was a Star or Celeſtial Body as the Sun or Moon. But that he did not know its Courſe; becauſe it cuts thro' higher parts of the World, and then at laſt appears, when it comes to the bottom of its Courſe*; and that is the very ſame thing which was ſhewn in *Prop.* 10. *B.* 1. Of which Opinion *Seneca* himſelf profeſſes himſelf to be, in the 22d *Chapter*, in theſe words, *I don't think a Comet to* *be a ſudden Fire, but one of the eternal Works of Nature.* In the 2d *Chapter,* he gives the ſole and only Method by which this Queſtion may be ſolv'd, ſaying, *that there ought to be a Collection of former Riſes or Appearances of Comets; becauſe by reaſon of their ſeldom appearing their Courſes cannot yet be underſtood, neither can it be diſcover'd whether they return or no; and that they appear or are produc'd in Order, at their ſettled time.* And at laſt he thus propheſies, in the 25th *Chapter, A time will come when thoſe things which are now hid will at laſt be brought to light, by length of Time and the diligence of Poſterity. One Age is not ſufficient to make ſuch great Diſcoveries. A time will come when thoſe that come after us will wonder that we were ignorant of Things ſo plain.* And in the 26th *Chapter, Some Body will demonſtrate, which way* Comets *wander, why they go ſo far from the reſt of the Celeſtial Bodies, how big, and what ſort of Bodies they are.*

Afterwards the whole School of the *Peripateticks,* to keep up their Doctrine concerning the Heavens, which they ſuppos'd not capable of being generated or corrupted, did not look upon the Comets as eternal or permanent Bodies of the World, but as Bodies newly produc'd, and in a ſhort time to periſh again, and affirm'd them to be Sublunary and made of Exhalations in the Terreſtrial Regions; which Opinion was the more believ'd after ſolid Orbs were introduc'd, becauſe Comets cou'd not paſs thro' them.

But at laſt *Tycho Brahe* and *Kepler* finding by Obſervation, that Comets had no diurnal Parallax, reſtor'd them to Places above the Moon. The eaſieſt and readieſt Way to find the diurnal Parallax of a Comet, if it has any, is the following: By the Obſervations of many Days, made before and after, let the Track be defin'd among the Fix'd Stars, which a Comet wou'd have, if it had no diurnal Parallax. If therefore from its Riſing till it gets to the Meridian, or from thence to its Setting, it has the ſame place among the Fix'd Stars reſpectively, which it wou'd have according to the Track above defin'd, it is a ſign that it has no diurnal Parallax: For when it is near the Horizon, if it had a diurnal Parallax, it wou'd appear lower among the neighbouring Fix'd Stars, which have no Parallax; therefore as the Comet is towards its Riſing it wou'd appear nearer to the Eaſt, and at ſetting nearer to the Weſt, than it is according to the Place agreeable to that Moment of Time, according to former Obſervations. And this Method is no way affected by the uncertainty of the Refraction by which Celeſtial Bodies are ſeen; for the Refraction of the neighbouring Stars is nearly the ſame with that of a Comet. But the readieſt Way that this Obſervation is made, is to find the true Place of a Comet, in a Right line drawn between two Fix'd Stars, ſo that thoſe Fix'd Stars, at that time at a ſmall Height above the Horizon, be in, or nearly in the ſame Almicantarath, which will be eaſily done among ſuch a multitude of Fix'd Stars, at leaſt if we take in ſuch as the Eye perceives with the help of a Teleſcope: For if a Comet has a diurnal Parallax, it will appear out of that Right line and nearer to the Horizon; if not, it will even then appear in the Right line among the Fix'd Stars.

That Comets are above the Moon is farther confirm'd, from the Obſervation which Mr. *Caſſini* made, of that which appear'd in the Year 1680, which he obſerv'd to be but 22 or 23 Degrees diſtant from the Sun, and yet to ſhine at that time with its full Orb; which he had alſo obſerv'd of the Comet which appear'd in *April*, 1665. whence one may certainly conclude, either that thoſe Comets at that time were not only above the Moon, but alſo above the Sun, (ſince Mercury and Venus, at that apparent Diſtance from the Sun do not ſhine with a full Orb, unleſs they be above the Sun;) or that they are lucid Bodies, like the Sun or Fix'd Stars, or at leaſt pellucid and tranſmitting the Sun's Rays, which will hardly be ſuppos'd of ſuch great Bodies, ſince the Sun's Spots, however rare their Texture is, are opake enough. But that Comets are opake Bodies, will be ſhewn below, from the Shadows which they caſt.

PROPOSITION II.

TO give, in a few Words, the Opinions of Philoſophers concerning the Riſe and Orbits of Comets.

Philoſophers are not yet agreed about the Orbits of Comets: *Kepler*, and after him *Carteſius*, *Hevelius*, and others, ſay, that the Trajectories of Comets are Right lines. For *Kepler* did not like *Circles* or other *recurring Curves* for the Motion of Comets, as thinking it *not likely that the Motion of Comets, which never return, ſhou'd be Circular.* But *Kepler*, in his *Phyſiolog. of Com.* believ'd, *That the vaſt depth of Liquid Æther was of ſuch a nature, that it produc'd Comets of itſelf, leſt that Region ſhou'd have any void, and that that was perform'd by a Faculty like the Animal or Vital one, for the refining and purging of the Æthereal Aura, which in ſome places grows thick. And therefore that this groſs fat part of the Air being gather'd together like ſome Excrement into an Impoſthume; it happens, from the nature of the Place, to be enlightened,*

tened, (becauſe the Sun's Light is every where) and to get a Motion like ſome Star: An exact Example of this, how this fat and Celeſtial Matter of Comets has Light and Motion, may be ſeen in Aerial Trajectories (or Star-ſhooting.) But *Hevelius* ſuppos'd Comets to be generated from the moſt ſubtile Exhalations of Planets joining together by chance in the Æther, and retaining the form of a Diſc, one ſide of which is turn'd to the Sun, after that the Comet moving in a Spiral Line, is gone out of the Atmoſphere of the Planet, from which it had its firſt Riſe.

Carteſius ſuppos'd the Body of every Comet permanent like that of a Planet, and made like a Planet, of ſome Fix'd Star or Sun cover'd with Spots and carried away by the next Vortex as it encreas'd. But he wou'd have a Comet be conſtantly carried in a Right line from one Vortex to another.

But thoſe that believe a Comet to be a permanent Body, and to return again in the ſame Track as Planets do, will have its Orbit ſuch as either to contain the Earth in its Circle, tho' very excentric (as Mr. *Caſſini* ſays of the Comet which appear'd in the Year 1577, which he ſuſpects to be the ſame Comet that appear'd again in the Years 1680, and 1681) and run equably thro' the Periphery of ſuch a Circle, contrary to what the Planets do; or not to contain it, ſo that the convexity of the Orbit ſhall be turn'd towards the Earth, as the ſame *Caſſini* ſeems inclinable to believe, in his Theory of the Motion of the Comet of the Year 1664. But it is true that Mr. *Caſſini* only makes uſe of ſuch Orbits chiefly for calculating the Motion of a Comet, being too cautious a Natural Philoſopher to affirm or define any thing concerning that Matter.

From the Obſervation of ſeveral Comets, Mr. *Caſſini* has ſettled the Way of Comets among the Fix'd Stars, within whoſe limits moſt of the Comets, (tho' not all of them) are contain'd as the Planets are, within the limits of the Zodiac. He finds that this Cometary Zone, which is as broad as the Æquator, and like it mark'd with the Figures of Animals, goes thro' *Antinous*, *Pegaſus*, *Andromeda*, the *Bull*, *Orion*, the *leſſer Dog*, the *Hydra*, the *Centaur*, the *Scorpion* and the *Bow of Sagittarius*; and he calls it the *Zodiac of Comets.*

Something like this is the *Syſtem of Comets*, publiſh'd by the ingenious Mr. *James Bernouilli*, in the Year 1682, in which he ſuppoſes that a certain primary Planet revolves about the Sun *at a diſtance from the Sun, equal to* 2583 *Semidiameters of the Magnus Orbis, in the ſpace of* 4 *Years and* 157 *Days, tho' Saturn, who is two hundred and fifty eight times nearer, does not make its Revolution in leſs than* 30 *Years.* About this Primary (*which we can never ſee, as well by reaſon of its ſmallneſs as its immenſe diſtance*) at different diſtances ſeveral Comets revolve, tho none of them come down quite to the Orbit of Saturn; *therefore the Comets*, according to Mr. *Bernouilli*, *are nothing elſe but the Satellites of the ſaid inviſible Primary Planet, which only begin to be viſible when they deſcend towards us in their Perigæum, and ſo are neareſt to us.*

The Opinion concerning the Orbits of Comets, which ſeems moſt probable to us, has been already explain'd in the 35th *Prop.* of *B.* 1. which beſides its being moſt agreeable to the ſimplicity of Nature, may put on the appearance of almoſt every Opinion mention'd before. For if *S* which repreſents the Sun in the firſt firſt Figure of the firſt Scheme, be the common Focus of the Orbit of the Earth *TGHK* which is nearly circular, and the very excentrical Orbit of a Comet be *ABDIEC*; it is plain that the Phænomena of the Comet going thro' the part *EC* or *BD* of its Orbit, will be well enough explain'd by an equable motion in a Right line, becauſe *BD* or *CE*, is but little different from a Right line, and beſides a Comet will move equably in it, becauſe equal Triangles ſtanding upon it, and having *S* for their common Vertex, will alſo have their Baſes nearly equal. But yet it will at firſt ſight hardly appear to be the ſame Comet, which firſt deſcribes *EC* and afterwards *BD*, there being a conſiderable Time between thoſe two Apparitions, namely that in which it deſcribes the Arc *CAB*, and is then hid under the Sun's Rays. Now, when the Earth is at *T* a Curve convex towards *T* will better agree with the Phænomena of a Comet deſcribing that part of the Orbit which is at *A*; for it is ſuch in the 1ſt *Fig.* of *Scheme* 1. But a Curve convex towards the Earth when the Comet is in oppoſition with the Sun, will never agree with the preſent Theory; but if it is concave it will very well agree with it, if (as in the 2d *fig.* of Scheme 1.) *SA* the Perihelion of the Comet be greater than *ST* the Semidiameter of the *Magnus Orbis*: For a curve Concave towards *T* will very well anſwer the Phænomena of a Comet paſſing at *A*; becauſe ſuch a Curve is its real Trajectory.

PROPOSITION III. LEMMA.

IF the Air ſhould expand it ſelf according to the following Law, which is confirm'd by Experiments, namely, that the Spaces in which it is compreſs'd are reciprocally proportionable to the Weights compreſſing; a Globe of our Air of an Inch Diameter, if it ſhou'd become as rare as it wou'd be at the Height of the Semidiameter of the Earth, wou'd fill all the Planetary Regions as far as the Sphere of Saturn, and far beyond. [Fig. 2.]

That the Law of the Expanſion of the Air is the ſame as was laid down here; namely, that the Spaces in which a given Quantity of Air is compreſs'd are reciprocally proportional to the Weights compreſſing, is plain from the Experiments mention'd in the *Philoſophical Tranſactions of* July 1671. N° 73. This Law then being laid down, let *ABZZ* repreſent an upright infinite Tube, full of Air, whoſe Sides *AZ*, *BZ* are parallel to one another, and perpendicular to the Horizon, and the bottom *AB* ſtanding upon the Earth's Surface: It is plain that the Air at *AB* is preſs'd by the Weight of the whole incumbent Air *ABZZ*, and that the Air at *ED* (a Parallel to *AB* drawn at any diſtance from it) is preſs'd by the Weight of the incumbent Air *EDZZ*. Let us conceive the Air expanded along the whole Right line *ED* to be compreſs'd into part of the ſaid Line, as *CD*, ſo that it may have the ſameDenſity as that of the Air *AB*, and that this is alſo done to any other Right line, ſimilarly drawn as *ed*, that is, that all the Air which before was expanded all over the Tube *ABZZ*, is compreſs'd into the Space *ABZX*, ſo that it may be every where of the ſame Denſity. The Right line *BZ* will be the Aſymptota of the Curve *ACcX*, terminating the Space *ACcXZdDBA* on one ſide, becauſe at any Height above the Earth there is ſome Air, which therefore being compreſs'd, till it be equally denſe with the Air *AB*, muſt take up ſome Space.

Since then the ſame Quantity of Air compreſs'd as much as *AB* was compreſs'd (that is, by the incumbent Air *AZZB*) will take up the Space *CD*, but compreſs'd as *ED* will take up the Space *ED*; it is plain, by the forementioned Law of the Compreſſion of Air, that *CD* is to *ED*, that is, to *AB*, as the Weight of the incumbent Air *EDZZ*, to the Weight of the incumbent Air *ABZZ*; that is, (by reaſon of Gravity, which we ſuppos'd the ſame at all Diſtances from the Center) as the incumbent Air *CDZX* to the incumbent Air *ABZX*, or as the unterminated Figure *CDZX*, to the unterminated Figure *ABZX*: But this is the Property of the Logarithmic Curve very well known to Geometricians; therefore the Curve *ACX* is a Logarithmic Curve, whoſe Aſymptote is the Right line *BZ*. When by *Probl.* 3. *Append.* 3. of *Barrow*'s *Twelfth Geometrical Lecture*, (if *CG*, and *eg* be drawn parallel to *BZ* and meet with *AB*, at *G* and *g*,) *BG* or *CD* is in the ſame Order between *BA* and *Bg* or *dc* a Geometrical Mean proportionally as *BD* is an Arithmetical Mean between *Bd* and Nothing; therefore 0 (or Nothing) *BD* and *Bd* are as the Logarithms of the Right lines *BA*, *DC*, and *dc* reſpectively. Therefore *BD*, *Bd*, *BA*, and *DC* being given, it will be eaſy to find *cd*, which, as has been already ſhewn, repreſents the Quantity of Air compreſs'd as much as it is at the Surface of the

Earth, which in the Height *B d* above the Earth will expand itſelf into a Space equal to *B A*; namely making *D B* to *d B* as the differences of Logarithms of *A B* and *CD* to another, this will be the difference of the Logarithms of *A B* and *c d*: and therefore theLogarithm of *A B* being given, the Logarithm of *c d*, and therefore *c d* itſelf will be known.

Let *A B* be 33 foot long, that is, of the Height of a Column of Water whoſe weight is equal to a Column of Air of the ſame thickneſs, extended from the ſurface of the Earth up to the top of the Atmoſphere: And as the Air near the Surface of the Earth takes up a ſpace as it were 850 times greater than Water of the ſame weight, as innumerable Experiments ſhew; a Column of Air 850 Foot high, is of the ſame weight with a Column of Water of equal thickneſs and one Foot high. If therefore we were to riſe 850 Feet above the Surface of the Earth, the Column of Air preſſing upon any Place at that Height, is equal in weight to a Column of Water of the ſame thickneſs and 32 Foot high: If therefore *B D* be taken of 850 Feet, *D C* will be of 32 Feet: For *A B*, *C D*, *c D*, &c. will ſhew the Heights of the Baroſcope (with whatſoever Liquor it is fill'd) in the Places *B*, *D* and *d*; for they are reſpectively proportional to the Weights of the incumbent Air. Therefore to find out *d c* from the Premiſes, ſuppoſing *B d* to be equal to the Semidiameter of the Earth, that is, in a round Number to 20000000 Feet (for according to ſome late Experiments made in *France*, its meaſure is 19615800,) let the Operation be perperform'd by the Rule of Proportion, finding to *D B* 850, *d B* 20000000, and the Difference of the Logarithms of *A B*, *C D* taken out of the Tables, namely, 0,0133639, a fourth Proportional 314,0000000, namely, the difference of the Logarithms of *AB* and *cd*, which being taken from 1,5185139, the Logarithm of *AB* firſt taken from the Tables, and there will remain the Logarithm of *Cd*, which is —312, 0000000, whoſe Characteriſtic (as they call it) is —312. Moreover, from the nature of Logarithms, the Number of the Tables whoſe firſt place is Unity, have Nothing for the Characteriſtic of its Logarithm; but the Number whoſe firſt Place is 2, or the ſecond of Integers, has One for the Characteriſtic of its Logarithm; that whoſe Firſt Place is the Third of the Integers has Two for its Characteriſtic, and ſo on aſcending; and likewiſe the Characteriſtic of a Number whoſe firſt Place is the next below One in a decimal Progreſſion, or the firſt Place of Decimals, is—1, and ſo on in deſcending: Whence converſly, from the Characteriſtic of the Logarithm you may know the Place of the firſt Place of the Number, or of how many Places the Number is; and hence the Number, the Characteriſtic of whoſe Logarithm is —312, is the firſt Place in the 312th Place of Decimals; therefore the Right line *cd* is denoted by a Number whoſe firſt Place is the 312th of Decimals.

Then taking away the Calculation, it is plain that a portion of our Air, denoted by a Number whoſe firſt Place is the 312th of Decimals, at the Height of the Semidiameter of the Earth expands it ſelf to a Magnitude which is denoted by 33; that is, the Space taken up by our Air, is to the Space that it wou'd fill up, if it was as rare as it wou'd be at the diſtance of the Semidiameter of the Earth, as the Number one to a Number expreſs'd by the two Ternaries with 312 Cyphers annex'd. But this Ratio is vaſtly leſs than that which (according to *Ariſtarchus*) a ſmall Grain of Sand has to the Sphere of the Fix'd Stars, as *Archimedes* has demonſtrated it in his *Arenarium*; namely, greater than the Ratio which One has to a Number whoſe Notation is Unity with 63 Cyphers annex'd: But we ſhall preſently ſhew that the ſaid Ratio is much leſs than that which is between a Globe of an Inch Diameter and the Orb of Saturn, much more nearly computed, than the Sphere of the Fix'd Stars was by *Ariſtarchus*.

The Semidiamer of the Earth is, as was ſaid before, in a round Number of 20000000 Feet: Now if the Sun's Parallax be $10''$ (which is the leaſt that Aſtronomers have hitherto attributed to it) whoſe Right Sine is of 485 parts, ſuppoſing the Radius to be of 10000000; the Semidiameter of the *Magnus Orbis* will be of 500000000000 Feet, namely, to the Semidiameter of the Earth as the Radius to the Sine of the Sun's Parallax; and therefore the Semidiameter of the Orb of Saturn (by *Prop.* 41. *B.* 3. ſcarce ten times as great) will be of 5000000000000 Feet, and its Diameter of 10000000000000, or of 120000000000000 Inches. Wherefore the Sphere of Saturn is to a Globe of an Inch Diameter as the Cube of the foregoing Number, or 1728 with 39 Cyphers to Unity, or in a round Number, and at moſt as a Number expreſs'd by 2 and 42 Cyphers is to One or Unity. Therefore ſince a Globe of an Inch Diameter has a much greater Ratio to the Sphere of Saturn, than to the Sphere which Air, like our Air, wou'd fill if it ſhou'd become as rare as the Air at the Height of a Semidiameter of the Earth above the Earth; it is plain that that Globe of Air, whoſe Diameter is one Inch, when it comes to be as rare as it wou'd be at the diſtance of a Semidiameter of the Earth from it, wou'd fill all the Planetary Regions as far as the Sphere of Saturn and much farther. *Q. E. D.*

If the Gravity towards the Earth be ſuppos'd to diminiſh as you recede from the Center, as it really does; it is plain that ſuch a Globe of our Air of an Inch Diameter as we have mention'd, at the Height of a Semidiameter of the Earth above the Earth, wou'd yet take up a greater ſpace.

PROPOSITION IV.

TO explain the Opinions of Philoſophers concerning the Tails of Comets, and confirm that which is the true Opinion.

That a Comet's Tail depends upon the Sun is plain from this, that it is always turn'd from the Sun, which was firſt of all diſcover'd by *Peter Apianus*, in the Comets which appear'd between the Years 1530 and 1540, tho' *Regiomontanus* 60 Years before became almoſt certain of it. As for the Opinion about the manner of the Tails being caus'd by the Sun, it is threefold.

Firſt, *Apianus*, *Cardanus*, *Tycho*, *Snellius*, and others believe that the Sun's Rays being propagated thro' the tranſparent Head or Body of the Comet, are refracted as in a Lens of Glaſs, and look like a Beam behind the Comet, or in thoſe parts which are turn'd from the Sun. But this is contrary to the Laws of Dioptrics: For, beſides the Figure of it not anſwering to this, ſuch a brightneſs propagated thro' the pure Æther will not be perceiv'd by an Eye poſited ſideways; it is requiſite that there ſhou'd be ſome reflecting Subſtance in thoſe Places where the Tail is to make that brightneſs ſenſible, after the ſame manner that there muſt be Particles of Duſt or Smoke flying about in a dark Room that ſuch a brightneſs may be ſeen by an Eye that looks upon a Sunbeam let into the Room, the ſaid Eye not being in the Way of the Beam; and by how much more groſs and denſe the Particles are that fly about in the Medium, by ſo much will the Beam be brighter, and ſo on the contrary. From this there ought to be a more groſs Matter in the Region of the Tail than any where elſe in the Æther about the Tail, otherwiſe the whole Sky round about wou'd be enlighten'd and ſhine as much as the Tail; and therefore the Tail of a Comet does not proceed from the Sun's Rays propagated thro' the tranſparent Body of the Comet. And therefore this Argument will be of as much Force, whether the Rays be ſuppos'd refracted in the Head of a Comet and the Atmoſphere about it, or in the Atmoſphere only, the opake Head hindering the tranſmiſſion of the Rays, as *Hevelius* obſerv'd in the Comet of 1665, which had a Shadow, and the ſame Author ſays, in his *Cometography*, *B.* 8. that ſeveral other Comets have been obſerv'd to caſt a Shadow. But the famous Dr. *Hook* denies ſuch a Shadow, and affirms a Comet to ſhine brighter in that place where the Shadow, if there was any, ought to appear.

The ſecond Opinion is of *Carteſius* and his Followers, who wou'd have the Tail of a Comet to be owing to the refraction of the Light from its Head to the Eye of the Spectator. If this was true, the Rays of the Fix'd Stars and of the Planets, which are propagated thro' the ſame Mediums, wou'd not reach our Eyes ſo free from Refraction as they appear by Obſervation; neither wou'd the Tails of Comets be ſo free from the Colour of the Rain-bow, which always accompany refracted Light. Neither does *Carteſius* take off thoſe difficulties, when he ſays in the 139th *Paragraph* of the third *Book* of his *Principles*, that the twinkling of the Fix'd Stars is owing to this, that their Tails wou'd be longer if the Light was greater; for when we make uſe of Teleſcopes, which encreaſe their Light, their twinkling ceaſes. Beſides, ſeveral of the Planets, as Saturn, Jupiter and Venus, whoſe apparent Diameters are equal to, or even ſurpaſs the Diameters of the Comets that ſend out pretty long Tails, and which have more Light than thoſe Comets that ſend out no Tails; and the Light of the Heads of thoſe Comets that have the greateſt Tails is often very ſmall and dull. If either the Fix'd Stars (or even the Planets) as *Ariſtotle* ſays in his Diſcourſe on Meteors, have ſometimes appear'd Hairy to the *Ægyptians*, this does not happen to them as it does to Comets, but is rather to be aſcrib'd to an accidental Refraction of a Cloud, ſuch an one as that to which *Carteſius* aſcrib'd the hairineſs obſerv'd by *Ariſtotle* himſelf about the Fix'd Star in the Dog's Thigh. Laſtly, If the Tail of a Comet did ariſe from the Refraction of the Celeſtial Medium, it would in the ſame Regions of the Heavens always go towards the ſame parts; which yet does not happen, but is according to the Sun's ſituation thrown towards different parts, as has been ſaid.

There are alſo other Opinions concerning the Tails of Comets to be met with in ſeveral Authors, which we ſhall not here take time to confute; among which is this of Mr. *James*

Bernoulli in the before quoted *Syſtem of Comets.* For he conceives *the Sun to be in the middle of its Vortex as a great Fire kindled in its Hearth, by whoſe Heat the Planets round about are warm'd, whence it happens that a great quantity of very ſubtil Exhalations comes out of the Planets every way, and even from the Sun itſelf, and that from the very rapid motion of the Solar Vortex they are carried upwards towards the Circumference; to which place when theſe Effluvia are come, they are ſtopp'd and repell'd by the Reſiſtance of the neighbouring Vortices, ſo that they can go no farther. Hence, becauſe they have a near addition of Matter, they are by degrees condens'd, and ſtick to the Secondary Planets of that inviſible Primary one which is above Saturn (of which we ſpoke* in Prop. 2.) *when they come near the circumference of the Solar Vortex; juſt as when a pot of Fleſh hangs over a Culinary Fire the ſmoak and ſteam riſing up the Chimney from the Wood and Fleſh ſticks to the Beams, and whatever ſolid ſubſtance it meets in its way ſo as to make Soot: Thus,* he ſays, *the the viſcous Exhalations ſtick to the Planets, and encompaſs them to a conſiderable thickneſs; whence it happens that the Extremity of the Nucleus of a Comet or its Limb, is commonly very irregularly terminated and rough as if it had Soot upon it. Then in the progreſs of the Satellite as it revolves about its Primary, from time to time new Exhalations, (but thinner than thoſe which immediately encompaſs it, which can't be ſo ſtrongly condens'd for want of a ſolid foundation) ſtick to it, and extending themſelves every way round about it, not in the manner of a Globe, but like a broad Diſc, one of whoſe Planes remains turn'd to the Sun, and the Rays of the Sun make Right-angles with the Diſc, whence* (as he ſays) *the reaſon is plain why the Tail always points towards the parts oppoſite to the Sun; for tho' the whole Diſc at once be* *enlighten'd by the Sun, yet no part of the Diſc but that which in reſpect of us is on this ſide of the Nucleus and in the ſame Plane with us and the Sun, reflects Rays to our Eyes, the others being reflected the contrary way and on the other ſide of a Comet.*

The moſt obvious opinion to any one that looks at a Comet is, that the Tail has its Origin from the Head. And this is the Opinion of the moſt ancient Philoſophers, and of *Ariſtotle* himſelf, who ſaid that the Tail was a very thin Vapour ariſing from the Head, but he ſuppos'd it to be on fire. The moſt penetrating Dr. *Hook* from the obſervation of Comets, contriv'd the following manner of explaining the nature of a Comet and its Tail, upon account of the *Comet* which appear'd at *London* in 1678. He ſuppoſes a Comet to ariſe from a Planet, whoſe gravity towards the Center about which it revolv'd before, is deſtroy'd by a certain inteſtine motion of the Parts (by which the Body is ſet on fire and appears in a flame) and which therefore is gone out of its place, moving in a Tangent to that point of its Orbit in which it was when its ſaid change was perfected. Whence carried thro' the Æther in a Right line (except ſo much as that line is bent upon account of the matter of the Comet towards its Center) which not being yet fully chang'd is a little incurvated towards the Sun; the Planet thus chang'd, has its parts diſſolv'd by the Æther and chang'd into it, as Salt is diſſolv'd by Water, or any thing elſe by a proper menſtruum. Moreover, the outer parts ſo chang'd, have not only loſt their gravity, but are become light in reſpect of the Sun, that is, fly from it. Whence as they are abſorb'd in Æther and carried upwards, and at the ſame time fly from the Sun, theſe igneous Particles look like innumerable Projectiles going every way directly from the Nucleus of the Comet and tending towards a diſtant Point oppoſite to the Sun; therefore (by what has been demonſtrated by *Torricelli* in the 30th *Prop. B.* 2d of the *Motion of Projectiles* they have the figure of a Parabolic Conoid, whoſe Axis is in a Right line joyning the Sun and a Comet, having the center of the Nucleus for the Focus of the generating Parabola; Therefore the Tail of the Coment will put on ſuch a ſhape. But ſince the Heavy Bodies that we can come at do not intend or remit, that is, increaſe or diminiſh their Gravity, but their Gravity is proportionable to their Quantity of Matter, whatever inteſtine motion the ſaid Matter has, we muſt not admit a loſs of Graviy or acquſition of Levity in other Bodies.

They that will not allow the Tail to be on fire, and think that to produce the Phænomenon, it is enough for the Vapour that goes from the Comet to be enlighten'd by the Sun, and by reflecting the Rays of the Sun from its Particles, to make that part of the Æther which they take up more enlighten'd than the reſt of the Æther, which tho' it receives the Light cannot reflect them, as wanting a reflecting ſubſtance, are oblig'd to find out the reaſon why the Vapour or reflecting Exhalation goes only on that ſide which is turn'd from the Sun. *Kepler* explains the thing thus: ‘The fat and groſs Matter of the Comet being gather'd into an Orb, from the nature of all things which are united, and the Sun ſtriking with direct Rays and penetrating this Globe, *he imagines* that ſome of the inmoſt Matter of the Comet follows and go out the ſame way as the Sun's

‘Rays go, and therefore by that means the Body of the Comet has parts of it rubb'd, waſh'd, ‘or ſtrain'd off, and thus there is a Quantity of ‘Matter gather'd together on that ſide of the ‘Comet which is turn'd from the Sun, that re-‘flects the Sun's Rays and has the appearance of ‘a Tail. *James Gregory* in ſome Phyſical things ſubjoin'd to his *Univerſ. Parts of Mathemat.* will have *a Comet to be a kind of humid Body emitting its proper Vapours, always wandring about the Æther, and retaining almoſt the ſame ſituation in reſpect to the Sun*; which is alſo the opinion of *Hevelius* in his *Cometography*, pag. 430. who thinks the Tail produc'd much after the ſame manner: namely, *That the thinneſt parts of the Atmoſphere of a Comet are rarified by the force of the Heat, and from the forepart and each ſide of the Comet are driven towards the parts turn'd from the Sun.* And then, according to *James Gregory*'s opinion, the *half of the Comet which is turn'd from the Sun, and never warm'd or enlighten'd by it, has very groſs and very opake Vapours, which being continually ſupplied or fed with the vapour-like matter of the Comet, and no way diſſolv'd (by reaſon of the weak and oblique light of the Sun) grow to an immenſe Height and every way reflect the Sun's Rays, which are not ſtrong enough to expel the Exhalations.*

The incomparable Sir *Iſaac Newton* believes alſo, *That the Tail of a Comet is nothing elſe but a very thin Vapour, which the Head or Nucleus of the Comet ſends out by reaſon of its Heat.*

When a Comet deſcends to its Perihelium, the great quantity of Vapours which encompaſs'd it when it was in thoſe Regions which were the moſt remote from the Sun, and therefore the coldeſt, being rarified by the Sun's Heat aſcend; that is, go towards thoſe parts which are turn'd from the Sun: For all the particles in a medium extended far beyond Saturn, (namely the Ætherial Aura) gravitate towards the Sun. This Aura heated by the warm particles of the Vapour, which mixes with it, becomes rarer and therefore lighter than the reſt of the ambient Aura, and for that reaſon, being driven by it, riſes upwards, and takes with it the reflecting particles of which the Tail is made up; much after the manner that ſmoak riſes up a Chimney by the impulſe of the Air, which being rarified by heat, carries up the particles of the ſmoak which are entangled with it. But becauſe beyond the Atmoſphere of the Comet the æthereal Aura is very rare and next to nothing, or a Vacuum, therefore I ſhall attribute ſomething to the action of the Rays of the Sun carrying along with it the particles of the Atmoſphere of a Comet, tho' *Kepler* is not of this opinion; and altho' I believe, that (beſides the impetus acquir'd in the firſt aſcent of the Vapour) the chief and moſt effectual cauſe of the aſcent of the Vapour which makes the Tail, is the rarity above deſcrib'd of the æthereal Aura, by reaſon of the particles which ariſe duly from the Comet in the neighbourhood of the Sun, where the Aura about the Tail is denſer and heavier. And then the Tail lifted upwards, does, like the Nucleus, gravitate towards the Sun after the manner of another Projectile in a in a Conic Section, and accompanies the Comet in diſtant Regions till it is diſſipated by degrees.

Tho' the Tails of ſome Comets being immenſely extended, ſeem to require a great quantity of Matter to produce them, Experience ſhews us into what a prodigious heap of Smoak a ſmall piece of Wood or Pit-coal is converted, and from the foregoing Lemma, it is plain how ſmall a portion of our Air may be diffuſed into immenſe Spaces by its elaſtic Force. And tho' the Tails are not of ſo rare a Texture, yet that they are extremely rare, in reſpect of Bodies about us, is plain from this, *viz.* that the leaſt Stars ſhine through thoſe immenſly deep Tails notwithſtanding they are enlighten'd by the Sun's Rays, when at the ſame time our Terreſtrial Atmoſphere, whoſe depth is vaſtly leſs, makes even the brighteſt Stars inſenſible when it is enlighten'd: Neither are lucid things ſeen more eaſily thro' a denſe Body when it is fartheſt remov'd from the Eye than when it is neareſt to it, but rather with more difficulty, as daily Experience ſhews us.

COROLLARY I.

Since the Tail of a Comet is rais'd from its Head and the Atmoſphere about it by the heat of the Sun, it follows that it encreaſes as the Comet comes to its Perihelium, and again, that it is diminiſh'd in its receſs from the Sun; therefore it will be greateſt and brighteſt immediately after the Comet has ſuſtain'd a great degree of Heat in its Perihelium, and is juſt got off from the neighbourhood of the Sun. But alſo the Atmoſphere of the Comet (which ſupplies the Tail with moſt of its Matter) is diminiſhed as the Tail encreaſes, and becomes narrower towards the Sun; and therefore *cæteris paribus*, (together with the Comet itſelf, the greater part of which it appears to the naked Eye to make up) it appears leaſt immediately after the Comet has paſs'd the Perihelium, where ſometimes it appears cover'd with a thick and black Smoak, the attendant of a vehement Heat.

COROLLARY 2.

Hence alſo it follows, that if the Tail of a Comet ſhou'd touch the Atmoſphere of our Earth, (or if a part of this Matter ſcatter'd and diffus'd about the Heavens ſhou'd fall into it) the Exhalations of it mix'd with our Atmoſphere (one Fluid with another) may cauſe very ſenſible Changes in our Air, eſpecially in the Animals and Vegetables: For Vapours, as they call 'em, brought from ſtrange and diſtant Regions, and excited by a very intenſe Heat, may be prejudicial to the Inhabitants or Products of the Earth; wherefore thoſe things which have been obſerv'd by all Nations, and in all Ages, to follow the Apparition of Comets, may happen; and it is a thing unworthy a Philoſopher to look upon them as falſe and ridiculous.

COROLLARY 3.

The Tail riſing like a Vapour directly upwards from a Comet and its Atmoſphere, that is towards thoſe parts turn'd from the Sun, will remain in the Plane of the Orbit of a Comet, (that is, a Plane drawn thro' the Sun and the Track of a Comet) becauſe there is no reaſon, why in the quiet Æther it ſhou'd be ſo diſturb'd as to be carried more on one ſide than another.

Moreover, ſince the Vapour going from the Head joins two Motions, and afterwards continues to go on with that Motion, compounded of the Motion of a Right Aſcent near the Sun, and of the Motion of the Progreſſion of the Head; the Tail won't be directly turn'd from the Sun, but a little declining from it towards thoſe parts left by the Comet; nay, the Tail will not be ſtreight but bent, being convex towards the forepart and concave towards the back part. And tho' the riſing Vapour partakes of the Motion of the Head, yet by reaſon of ſome reſiſtance of the Æther it will not go on ſo faſt as the Head. And by how much higher the Vapour riſes which makes up the Tail, that is, by how much longer the Tail is, by ſo much more will it be reſiſted by the Æther; when the foreſide of the Tail, or the Side towards the parts where the Comet is to go will be convex, but the following one concave, and therefore the Tail of the Comet it ſelf will not be preciſely turn'd from the Sun, and even not direct but bent. And this deviation from the Sun's Oppoſition is ſo much the leſs, as a Right line joining the Sun and a Comet is more oblique to the Track of a Comet: For if a Comet ſhou'd go directly towards or from the Sun, the Tail wou'd be directly turn'd from the Sun and ſtreight; whence it happens that in the very Perihelium the Tail very much deflects from the Sun's Oppoſition, and is alſo very Curve, becauſe a Right line joining the Sun and the Comet will then be perpendicular to its Orbit. But by reaſon of the ſaid reſiſtance of the Æther, however ſmall, the convex part of the Tail, (that is, the part that runs into the Æther) will be ſomething thicker, and conſequently will appear brighter and better terminated than the concave one, by which the parts reflecting the Light, being as it were left, are diffus'd about the Æther. Moreover, this deviation of the Tail and Incurvation from the Sun's Oppopoſition, will, *cæteris paribus*, be greater, for both the aforeſaid cauſes, when the Comet's Orbit is moſt Curve: For that the Tail might be conſtantly turn'd from the Sun in ſuch a caſe, its Extremity ſhou'd deſcribe a Circumference greater than, and exterior to, that deſcrib'd by the Head, which it cou'd not do by the Motion which it receives from the Head, tho' there ſhou'd be no reſiſtance of the Æther to diminiſh it. Therefore the Extremity of the Tail which is the moſt remote from the Comet, becauſe it does not deſcribe that greater Circumference in ſo ſhort a time as the Head does the leſſer, (and that for the two reaſons that we mention'd) will in this caſe be more Curve. But as this happens to a Comet at its Perihelium, where the Tail, which before was lifted up, almoſt vaniſhes, and the Vapour which now riſes directly up chiefly makes up the Tail; for that reaſon the Curvity will not be ſo ſenſible.

COROLLARY 4.

The Figure of the Tail which we have deſcrib'd in the foregoing Corollary, is that into which the Tail is really extended from the Head of the Comet; notwithſtanding which, if the Earth and the Spectator together be in the Plane of the Comet's Orbit, the curvity of the Tail will not be perceiv'd at all, becauſe it is all in that Plane: And either the whole Tail will diſappear when the Head is between the Eye and the Tail, or if the Head be leſs and the Tail broader towards the other End, the Comet will appear Hairy, that is, encompaſs'd round with the appearance of Hairs, (and either of theſe may happen when the Sun and the Comet are in oppoſition) or laſtly, the Tail will preciſely appear to be in oppoſition with the Sun. But if the Spectator be out of the Plane of the Comet's Orbit, the curvity of the Tail will be perceivable; and will be by ſo much the more ſenſible as a Right line drawn from the Earth to the Tail of the Comet becomes more perpendicular, or nearer to a perpendicular to the Orbit of the Comet (and conſequently to the Orbit of the Tail) and all the Symptoms or Signs of the Curvature of the Tail deſcrib'd in the following Corollary will become more ſenſible. And theſe are the ſettled and conſtant Changes of the Tail of a Comet: For as to the uncertain and ſudden ones which Authors mention, they ſeem to have had their Riſe from the Vapours of our Atmoſphere, which by an accidental Refraction ſeemingly alter'd the Figure of the Tail; or from the Motion of the Clouds darkening ſome parts of the Tail; or from ſome neighbouring Stars, or a heap of them, or the Milky Way, ſometimes taken for part of the Tail.

PROPOSITION V.

MOre Comets are ſeen in the Hemiſphere towards the Sun, than in that which is oppoſite to it, [Fig. 3.]

Let *S* denote the Sun, *T* the Earth; the Circle *DEFH* deſcrib'd about the Center *T*, the Sphere of the Fix'd Stars, or rather that which bounds the ſight. Becauſe Comets ſhine by the reflected Light of the Sun, they will not be ſeen before they are ſo enlighten'd by the Sun as to be able to affect our Eyes by that Light: And beſides, as Comets are very conſpicuous upon account of their Tails, and do not emit them till they are ſomewhat heated by the Sun; it is plain, that to have either the Comet or its Tail viſible, it is neceſſary for the Comet to come within a defin'd diſtance of the Sun. Let *ABCG* be a Sphere concentric to the Sun, de-

deſcrib'd at the ſaid diſtance from it; namely, ſuch an one, that no Comet, for want of being enlighten'd, ſhall appear, before it comes within the Surface of that Sphere: having join'd *S* and *T* produce the Line *ST* every way, till it meets with the ſaid Surface in *A* and *C*: Thro' *T* draw the Plane *HE*, to which *AC* is perpendicular, then it will divide the Sphere *DEFH* into two Hemiſpheres, one of which, *HFE* is towards the Sun, and *HDE* oppoſite to the Sun. All the Comets which are in the Segment of the Sphere *BCG*, will appear in the Hemiſphere towards the Sun; but all thoſe which are in the Segment *BAG* will appear in the Hemiſphere oppoſite to the Sun. But it is plain that the Segment *BCG* is greater than *BAG*, becauſe the Center of the Sphere is in it; therefore more Comets will be ſeen in the Hemiſphere towards the Sun, than in that which is oppoſite to it. *Q. E. D.*

But more of them will be firſt ſeen or diſcover'd in the Hemiſphere towards the Sun, than in the other, becauſe the part *BCG* of the Surface of the Sphere is greater than the remaining part *BAG*; but they are ſuppos'd to be diſcover'd as ſoon as they touch that Surface.

SCHOLIUM.

By how much leſs the Sphere *ABCG* is ſuppos'd, that is, by how much nearer a Comet muſt be to the Sun before it becomes viſible, by ſo much does the number of the Comets, ſeen in the Hemiſphere towards the Sun, exceed the number of thoſe which appear in the oppoſite Hemiſphere; becauſe the Segment *BCG* does ſo much exceed the Segment *BAG*. And for that reaſon the number of thoſe which are firſt diſcover'd is greater in the former than in the latter.

If *ABCG* be ſuppos'd to be a little leſs than the Sphere of Jupiter, or if the Comets are not viſible before they come nearer to the Sun than Jupiter, that is, if *SA* be ſuppos'd five times longer than *ST* (for the diſtance of Jupiter from the Sun is more than five times the diſtance of the Earth from it,) the Segment *BCG* will be almoſt twice the Segment *BAG*, and therefore almoſt twice as many Comets will be ſeen in the Hemiſphere towards the Sun than in the oppoſite Hemiſphere: But the Surface *BCG* is only once and a half the Surface *BAG*, and therefore half as many more will be firſt diſcover'd in that than in this. If the Sphere *ABCG* be leſs, as for Example, if *SA* be once and a half the diſtance of Mars from the Sun, four times more Comets (and a greater number) will be ſeen in the Hemiſphere towards the Sun, than in the oppoſite one; but the number of thoſe which are firſt diſcover'd in the former will be three times greater than the number of thoſe diſcover'd in this latter. We are to take notice that from this number we muſt take thoſe which cannot be ſeen by reaſon of their too great nearneſs to the Sun; and on the other hand, thoſe which are not too near the Sun are better ſeen in the Hemiſphere towards the Sun, than thoſe which are in the oppoſite Hemiſphere, becauſe they are more ſtrongly enlighten'd.

PROPOSITION VI.

TO make an Eſtimate of a Comet's diſtance from the Sun, by comparing the Light of a Comet with the Light of a Planet.

Since a Comet has no diurnal Parallax, and no obſervations depending upon it can make its diſtance known, we may make an Eſtimate of by comparing the Light of a Comet with that of a Planet. And becauſe (by *Prop.* 57. *B.* 3.) the enlightening of the Comet (or the ſtrength of its Light) at different diſtances from the Sun, are in a duplicate reciprocal Ratio of the ſaid diſtances, and therefore (for reaſons alledg'd in the ſaid Prop.) by how much more remov'd the lucid Body is, by ſo much fewer Rays coming from it enter the Pupil of the Eye, and that in a duplicate Ratio of the encreaſs'd diſtance, or in a duplicate Ratio of its diminiſh'd apparent Diameter; that is, the brightneſs of equal lucid Bodies are in a duplicate Ratio of their apparent Diameters; therefore (in this leſs accurate way of making an Eſtimate, by abſtracting from the different real Magnitude of the Planet and Comet) the Light of a Comet is to the Light of a Planet in a Ratio made up of the duplicate Ratio of the apparent Diameter of a Comet, to the apparent Diameter of a Planet, and the duplicate Ratio of the diſtance of a Planet from the Sun, to the diſtance of a Comet from it: And therefore the Square of the diſtance of a Comet from the Sun, is to the Square of the diſtance of a Planet from it, in a Ratio compounded of the duplicate Ratio of the apparent Diameter of the Comet to the apparent Diameter of the Planet, and a Ratio of the Light or Brightneſs of the Planet to the Light of the Comet. Whence the diſtance of a Comet from the Sun, is to the diſtance of a Planet from it, in a Ratio compounded of the Ratio of the apparent Diameter of a Comet, to the apparent Diameter of the Planet, and a ſubduplicate Ratio of the Light of the Planet to the Light of the Comet. But each compounding Ratio is given by Obſervation; therefore the compound Ratio is given, namely, that between the diſtances of a Comet and a given Planet from the Sun. And from *Prop.* 45. *B.* 3. we have the diſtance of a Planet from the Sun; and therefore we ſhall alſo have the diſtance of a Comet from the Sun.

By reaſon of the thick Smoke and the Atmoſphere it ſelf of the Comet which darkens its Nucleus, the Nucleus (or ſolid Body of the Comet which we mean here) does not appear ſo bright; and therefore its diſtance from the Sun, found after the above mention'd manner, muſt be diminiſh'd to become true.

SECTION II.

To determine very nearly the Way and apparent Place of a Comet.

HAving premis'd ſome Phyſical Conſiderations concerning Comets and their Tails, we ſhall go on to determine their Orbits by Obſervation, which is the chief deſign of this 5th Book. And firſt we ſhall ſhew how, by the Method of that penetrating Aſtronomer Mr. *Caſſini*, from ſome Obſervations made at the beginning of the appearance of any Comet, the Way which it ſhall afterwards go among the Fix'd Stars, and the Places which it will be in every Day may nearly be determin'd: Not that we would have thoſe be look'd upon as accurate, but that from thence it may be foretold in what place a Comet muſt be look'd for at a given

Time;

Time; whether or no it will in a little time be hid under the Sun's Rays, when it will be neareſt to the Earth, and move ſwifteſt, and other things referring to this.

PROPOSITION VII.

KNowing *by Obſervation Two Places of a Comet ſeen from the Earth, to determine its Way among the Fix'd Stars, that is, its Node and Inclination to the Ecliptic.* [Fig. 4.]

Let *CE* repreſent the Ecliptic, and let *B* and *D* be two Places of a Comet found by Obſervation. Let a great Circle be ſuppos'd to be drawn thro' *B* and *D* cutting the Ecliptic at *A*, which by what has been ſhewn in *Prop.* 10. *B.* 1. will be the Way of a Comet, whoſe Situation in reſpect of the Ecliptic is thus determin'd: Thro' *B* and *D* draw the Circles of Latitude *BC* and *DE*, and alſo the great Circle *DC*. In the Spherical Triangle *DCE* rectangular at *E*, you have given the Sides *GE* and *ED*, this being the Comet's Latitude when at *D*, and that the difference of Longitude of the Places of the Comet *B* and *D*; therefore the Side *DC* will be found. Again, in the Triangle *DBC* you have all the Sides given; namely, *DC* lately found, *BC* the Latitude of *B*, the Place obſerv'd and *BD* the obſerv'd diſtance of the Places; and therefore the Angle *CBD* will be known, and therefore its Complement *CBA*. Laſtly, in the Triangle *BAC* you have, beſides the Right Angle at *C*, the Side *BC*, and the Angle *ABC*; therefore the Side *CA* and the Angle at *A* will be known, the Angle being the Inclination of the Comet's Way to the Ecliptic, and the Side, the Interval of the Node, and the Point *C* known in the Ecliptic.

PROPOSITION VIII. LEMMA.

THree Right lines TA, TB, TD, *being given in Poſition, drawn from the ſame Point and lying in the ſame Plane, from the Point* A *given in any one of them, (as in* T A) *to draw the Right line* AC, *whoſe parts* AE, EC, *intercepted between the ſaid Right lines, will have a given Ratio, namely, that of* R *to* S. [Fig. 5.]

Let the Right line *AB* be drawn any how from *A*, to meet with *TB* at the Point *B*; in this Line (produc'd if there be occaſion) on the part of it towards *TD*, take (by *Prop.* 12. *El.* 6.) *BF*, ſo that it may be to *AB* as *S* to *R*; thro' *F* draw *FC* parallel to *BT* meeting with *TD* at *C*: and you will have *AC* requir'd. For (by *Prop.* 2. *El.* 6.) *AE* will be to *EC*, as *AB* to *BF*, that is, (by conſtruction) as *R* to *S*.

SCHOLIUM.

The Poſition and Magnitude of the Right line *AC*, are eaſily defin'd, by Trigonometrical Calculation: For in the Triangle *ATB* you have given the Side *AT*, the Angle *ATB* and the Angle *TAB* known by drawing the Line *AB*; whence the Angle *ABT* and the Side *AB* will be known; therefore *BF* having a Ratio to the given *AB* is given, and for that reaſon its parallel, and conſequently equal *CM*. Again, in the Triangle *TMC*, knowing the Angle *CTM*, (given at firſt) and *CMT* equal to *FBM* (that is, to the Complement of the before found *ABT* to two Right Angles) and the Side *CM*, you will alſo have *CT*. Laſtly, in the Triangle *ACT*, having the Sides *TA*, *CA* and the Angle between them *ATC*, you will find the third Side *CA* and the Angles *TAC* and *TCA*; that is, the Magnitude and Poſition of the Right line *AC*.

PROPOSITION IX.

From three Obſervations made at the beginning of the Appearance of a Comet, to determine the Place of the Perigæum, at what time a Comet will be in it, and to find the Place of the Comet for a given Time. [Fig. 6.]

Let *T* repreſent the Earth, (which here, where we go not to the greateſt accuracy, we ſhall conſider as at reſt, abſtracting from its Motion) *ABDG* a Circle among the Fix'd Stars, in which the Comet ſeems to go, determin'd by *Prop.* 7. in which the Three obſerv'd Places of a Comet are, *A*, *B* and *D*. Let *R* be put for the Time between the Obſervations at *A* and *B*, and *S* for the Time between the Obſervations at *B* and *D*: Let the Right lines *TA*, *TB* and *TD* be ſuppos'd to be drawn, and, by the foregoing Lemma, let the Right line *AO* be drawn thro' *A*, whoſe parts *AE*, *EC*, intercepted between the two Right lines *TA*, *TB*, and the two *TB*, *TD*, are as *R* to *S*: This will be nearly the Trajectory of the Comet, eſpecially in ſome remarkable diſtance of the Comet from the Perihelium. For in that caſe the Way of the Comet is nearly ſtrait, and the Comet goes equably thro' it; therefore the Spaces gone thro' are proportionable to the Times: But *AE* is to *EC* as the Time in which the Comet is ſeen to move from *A* to *B* among the Fix'd Stars to the Time in which it is gone from *B* to *D*; and that agrees to no Right line drawn from *A* to the ſame parts of *T*, except *AO*: This is therefore nearly the Comets Trajectory.

You have (by the foregoing Lem.) the Situation of the Right line *AO* and the Magnitude of *AC* in reſpect to *AT*; that is, in the Triangle *ACT* you have all the Angles and all the Sides given. Let *TP* be drawn from *T*, perpendicular to *AO* the Right-lin'd Trajectory, and let it be produc'd till it meets with the Circle *ABD* at *G*. It is plain that the Comet moving in the Trajectory *AO*, is in its Perigæum at *P*; for the Right line *TP* is the ſhorteſt that can be drawn from *T* to *AO*: And *G* will be the Place among the Fix'd Stars, which a Comet in its Perigæum will appear to be in. To find out which, in the Triangle *APT*, rectangular at *P*, you have the Side *AT* and the Angle *PAT*; from which you will know the Sides *AP*, *TP*, and the Angle *ATP*, the Complement of *PAT* to a Right Angle, and its Meaſure the Arc *AG*, and therefore the Place of the Perigæum in a great Circle between the Fix'd Stars which the Comet deſcribes, when the Point *A* is known by Obſervation. This Place *G* of the Perigæum is thus very eaſily reduc'd to the Ecliptic *NKL*: Thro' *G* draw the Circle of Latitude *GL*, meeting with the Ecliptic at *L*: And in the Triangle *NLG*, beſides the Right Angle at *L*, you have the Angle *N*, (found by *Prop.* 7.) and *NG* the ſum (or difference) of the known Arcs *NA*, *AG*; whence will be known *NL*, the Longitude of the Perigæum from the Node, and *GL* the Latitude of the Comet in the Perigæum. Moreover, knowing the Ratio of the Right lines *AC* and *AP*, to the ſame *AT*, you will alſo know the Ratio between *AC* and *AP*: But becauſe a Comet moves equably, the Ratio between the Times in which the Comet moves thro' *AC*, *AP* is the ſame with the Ratio of

AC to *AP*; and the Time in which it has moved from *A* to *C* is known by Obſervation, being the ſame which is ſpent between the firſt Obſervation in *A* and the third in *D*; and therefore the Time will be known in which a Comet does move really from *A* to *P*, and apparently from *A* to *G*; that is, between a known Point of Time and the Time in which the Comet will come to the Perigæum. Therefore the Time will be known in which a Comet will be in its Perigæum.

Beſides, let the Interval between the given Time for which the Place is ſought, and the Time of any Obſervotion (as for Example, of the firſt in *A*) be requir'd. Let the Line *AQ* be taken in the Line *AO*, ſo that it may be to *AC* as the lately found Interval of Time, to the known Interval of the Obſervations of the Comet at *A* and *D*: draw *TQ* and produce it till it meets with the Circle *NAD* in the Point *V*, thro' which, we muſt ſuppoſe the Circle of Latitude *VX* to be drawn. It is plain from what has been ſaid before, that the Comet is in the Point *Q* of the Trajectory *AO* at the given Time. In the Triangle *QAT*, from the given Sides *AT*, *AQ*, and the intercepted Angle *TAQ*, you will find the Angle *ATQ*, and therefore its Meaſure (namely, the Arc *AV*) will be known: But the Point *A* is known; therefore alſo the Point *V*, or the requir'd Place of the Comet in the apparent Way at the Time given. But the Situation of this Place in reſpect to the Ecliptic is found, by having already, in the Triangle *NXV*, rectangular at *X*, the Angle *VNX* and Side *NU*, by having found *NX*, the Longitude of the Comet from the

Node, and *XU* in Latitude agreeable to a given Time.

Inſtead of the firſt and ſecond Obſervations, any two Obſervations may be taken in this practice, or any one of them with the Place and Time of the Perigæum above found. And that way *Ephemeris's* for Comets may be made.

SCHOLIUM.

In a Matter of ſo little accuracy, it will perhaps be ſufficient to deſcribe the Way of a Comet, at its firſt appearance on the Surface of the Celeſtial Globe, and to mark its Places for every Midnight of an Obſerver's Habitation by protraction. For if at the beginning of the appearance of a Comet, its Situation among the Fix'd Stars be obſerv'd, namely, by looking at it with a ſtretch'd Thread, and ſo knowing when it lies in a Right line with two known Fix'd Stars, and at the ſame time finding two other Fix'd Stars, with which alſo it lies in a Right line, which may eaſily be done among ſuch a multitude of Fix'd Stars; a great Circle going through two (or more) Places obſerv'd, mark'd in the Globe (which the farther they are from one another, the more exact the practice will be) will be the future Way of a Comet. But a great Circle going thro' them will eaſily be found, by raiſing or depreſſing one of the Poles, and at the ſame time turning the Globe till the Places mark'd are at the ſame time found in the Horizon; for then the Circle which the Horizon at that time denotes on the Surface of the Globe, will be the Circle requir'd. Therefore its Interſections with the Ecliptic will be the Nodes of the Orbit of the Comet, and the Angle which the Ecliptic makes with the Horizon (meaſur'd by the Altitude of the Nonageſim Degree) will be the Inclination of the Orbit of a Comet to the Ecliptic. Let a Circle be deſcrib'd with its Diameter equal to that of the Globe, as for Example *ABD*, whoſe Center is *T*, in whoſe Circumference, if you take the Point *A*, it will repreſent the Place among the Fix'd Stars, in which the Comet was firſt obſerv'd: Let the Arcs *AB*, *AD* be taken equal to the diſtances of the Places of the Comet, mark'd on the Surface of the Globe, from the Place firſt obſerv'd, and let *TB*, *TD* be drawn: And (by *Prop.* 8.) let the Right line *AO* be drawn in ſuch manner thro' *A*, that *AE* may be to *EC* as *R* to *S*, that is, (by conſtruction) as the Time between the firſt Obſervation and the ſecond, to the Time between the ſecond and the third. From *T* letting fall *TP* perpendicular to *AO*, and producing it (if need requires) as far as the Circumference at *G*; an Arc equal to *AG* being transfer'd from the Place firſt obſerv'd, to the Way of the Comet deſcrib'd above on the Surface of the Globe, the Point *G* will ſhew the Place among the Fix'd Stars which the Comet will be in, in its Perigæum.

Moreover, if the Right line *AC* be divided (either actually, or by help of *Galilæus*'s Compaſſes of proportion) into ſo many equal parts, as there are Hours (or Minutes, if the Scheme be large enough) between the firſt Obſervation in *A*, and the third in *C*; each of thoſe parts will be the Way of a Comet moving equably in *AO* for the ſpace of an Hour, and twenty four of them will be the Way of it in the ſpace of a natural Day, and ſo on. Therefore the Place of a Comet among the Fix'd Stars, for a given Time, is determin'd upon the Globe, if ſo many of the foreſaid parts be taken from *A* towards O, in the Right line *AO*, as Hours are paſs'd between the Time of the firſt Obſervation and the given Time; as for Example, *AQ*, and let the Right line *TQ*, joining the Center *T* and the End *Q* of the taken parts, be produced to *V*, and let an Arc be taken equal to *AV*, from a Point anſwering to *A* in the Comet's Way, one Extremity of it will ſhew the Place of the Comet among the Fix'd Stars, agreeable to the ſaid Time. The Place of a Comet for every Midnight will after the ſame manner be protracted upon the Surface of the Globe: Whence the Longitude, Latitude and other things belonging to a Comet, will (by the Operations taught in *B.* 2.) be eaſily defin'd. Theſe things which are done on a Globe, may likewiſe be done in any Planiſphere in which the Fix'd Stars are drawn.

The Way of a Comet being firſt drawn, and the Places agreeable to different Times will be corrected by the following Obſervations: For this Operation and Calculation is not to be expected to be without any Error.

PROPOSITION X.

TO *define in what Time a Vapour going out of a Comet, riſes from the Head to the Extremity of the Tail.* [Fig. 7.]

Let S denote the Sun; *AB* part of the Trajectory of a Comet, in which let *N* be the Nucleus of a Comet moving from *A* towards *B*; C the Extremity of the Tail. Now ſince (by *Corol.* 3. *Prop.* 4.) the Tail *NFC* is not exactly turn'd from the Sun, that is, *NC* does not lye *in directum* with *S N*, but turning towards *A*, the parts which the Comet has left, and whoſe Inclination is known by Obſervation; let SE be drawn parallel to NC, cutting the Trajectory at the Point D behind the Comet. The Vapour which now makes the Extremity C of the Tail, began to riſe from the Nucleus when the Comet was in the Point D; becauſe a Vapour with a Motion of aſcent, or directly from the Sun, is compounded of the progreſſive Motion of the Comet impreſs'd to it from the Head before its ſeparation. But by the foregoing *Prop.* will be defin'd (exactly enough for ſuch an enquiry as this, which is not very accurate) the Time that a Comet was at the Point D, before the Time propos'd of the Obſervation, as great as is requiſite for a Comet to go thro' the Right line DN, given in length. Therefore the Time is found in which the Vapour riſes from the Head to the End of the Tail.

PROPOSITION XI. LEMMA.

FOur Right lines given in Poſition, lying in the ſame Plane, to draw a fifth which ſhall be cut by thoſe four, in three parts, having a given Ratio and a given Order among themſelves. [Fig. 8.]

Let there be four Right lines given in Poſition, namely, AE, AC, BC, and BD produc'd *in infinitum*, and three Quantities given, R, S, and V: a Right line is to be drawn cutting the four firſt given Lines at M, O, N and K, ſo that MO may be to ON, as R to S, and ON to NK as S to V.

Let CG be taken, ſo that it may be to CE as S and V together to R; take alſo FC to CD, as R and S together to V: Draw the Right lines AG, BF and produce them till they meet; thro' their concourſe H draw HK, parallel to DC, and *HM* parallel to *BC*, with reſpective Right lines concurring as in the Scheme: Draw *MK* and it will be the Line requir'd.

For by reaſon of the Parallels *HM* and *TN*, *KN* is to *NM* as *KT* to *TH*. But becauſe *HK* is parallel to *FD*, *KT* is to *TH* as *DC* to *CF*, that is, by conſtruction, as *V* to *R* and *S* together; and therefore *KN* is to *NM* as *V* to *R* and *S* together. Again, by reaſon of the parallels *HK* and *OX*, *MO* is to *OK* as *MX* to *XH*: But becauſe *HM* is parallel to *EG*, *MX* is to *XH* as *EC* to *CG*, that is, by conſtruction, as *R* to *S* and *V* together; and therefore *MO* is to *OK* as *R* to *S* and *V* together. But it has already been ſhewn, that *NK* is to *MN*, as *V* to *R* and *S* together. Therefore, by diviſion, the three Right lines *MO*, *ON* and *NK* have the ſame Ratio to one another as the three Quantities *R*, *S* and *V*. *Q. E. F.*

We do not ſay any thing here of the Limitations and different Caſes of this Problem, as being manifeſt enough to any one that conſiders this Matter.

This Conſtruction is the Invention of the famous Geometrician Sir *Chriſtopher Wren*.

SCHOLIUM.

The Poſition of the Right line *MK* and of the Points *M*, *O*, *N* and *K*, of the diſtance from *A*, *B*, *C*, *D* or *E*, are thus eaſily determin'd by Calculation. All the Interſections of the four Right lines firſt given, *AE*, *AC*, *BC*, and *BD*, as the Points *A*, *B*, *C*, *D* and *E*, are given (by the 25th *Datum*, and the Angles at them, by the inverſe of *Dat*. 29. wherefore the Right lines *AC*, *BC*, *CB*, *CE* and *BD* (by the 26th *Datum*) are given in Magnitude. But (by the 2d *Dat.*)

CF and *CG* are alſo given, having a given Ratio to *CD* and *CE* reſpectively. In the Triangle *BCF* the Sides *CB*, *CF*, and the Angle between them, *BCF* being given, you have the Side *BF* and the Angle *BFC*, and conſequently the Angle on the other ſide of it *AFH*. Likewiſe, in the Triangle *ACG*, from the given Sides *AC* and *CG*, with the Angle *ACG*, you will have the Angle *CAG*. Therefore in the Triangle *AFH* two Angles, *AFH* and *FAH*, which were before found, being given, with the Side between them *AF*, (the ſum or difference of the known Right lines *AC* and *CF*) the Side *AH* will be known. Laſtly, in the Triangle *AHM*, having the Angle *HAM*, (the ſum or difference of the known Angle *CAE*, *HAC*) and the Angle *AMH*, equal to the given *AEC*, as alſo the Side *AH*, you will know the Sides *HM* and *AM*, the diſtance of the Point *M* from the known Point *A*. You may, by the ſame Steps, find the Sides *BK* and *HK* in the Triangle *BKH*. Therefore in the Triangle *MHK*, knowing the Sides *HM*, *HK*, and the Angle between them *MHK* (equal to *ECA* before known, you will know the Angles *HMK*, *HKM* and the Baſe *MK*: But the Angles *HMA*, *HMK* being given, you will have their ſum or difference *AMK*; that is, the Poſition of the Right line *MK*, *AM* being given in Poſition. After the ſame manner *AO*, or *BN*, or *BK*, and the Angles which *MK* makes with them, are found by Trigonometry.

PRO-

PROPOSITION XII.

Four obſerv'd Places of a Comet being given, to determine its Trajectory, if rectilineal. [Fig. 9.]

Let *S* repreſent the Sun; *ABCD* the *Magnus Orbis*; and let the Points *A*, *B*, *C* and *D* be known Places of the Earth at the Times of the Obſervations; *AE*, *BF*, *CG* and *DH*; four Longitudes of an obſerv'd Comet. By *Prop.* 11. draw the Right line *EH* in ſuch manner that its part *EF* intercepted between *AE* and *BF*, may be to *FG*, its part intercepted between *BF* and *CG*, as the Time between the firſt and ſecond Obſervation, to the Time between the ſecond and third; and that the ſaid *FG* may be to *GH*, its part intercepted between *GC* and *DH*, as the Time between the ſecond and third Obſervation, to the Time between the third and fourth; and *EH* thus deſcrib'd will be the Mark of the Orbit of a Comet, in the Plane of the Ecliptic; that is, if thro' a Right line, which is the Trajectory of a Comet, a Plane be ſuppos'd to be drawn perpendicular to the Ecliptic, this Right line *EH* will be the common Section of thoſe Planes. For ſince, according to the preſent Hypotheſis, a Comet moves equably in a Right line, if from the Points of that Right line, in which the Comet was at the time of the four Obſervations, Perpendiculars be let fall to the Plane of the Ecliptic, they will fall on the Right lines *AE*, *BF*, *CG*, and *DH* reſpectively, becauſe theſe denote the Places of the Comet in the Ecliptic; and theſe four Points will be in the ſame Right line, namely, the common Section of a Plane thro' the Trajectory perpendicular to the Plane of the Ecliptic with the ſaid Plane of the Ecliptic; and beſides (by

Prop. 2. *El* 6.) the diſtances between thoſe Points will be as the diſtances of the Points in which the Comet was found in its proper Trajectory, and therefore as the Times between the Obſervations: But *EH* is the only Right line whoſe parts lying between the Right lines *AB*, *BF*, *CG* and *DH* are as the Times between the four Obſervations reſpectively; and therefore the Right line *EH* is the Mark of the Orbit of a Comet on the Plane of the Ecliptic.

But to compare the diſtance of a given Point of this Right line with the mean diſtance of the Earth from the Sun, for a time given, we muſt work thus: In the Triangle *ASB* you have (by *Prop.* 18. *B.* 3.) the Sides *SA*, *SB*, the diſtances of the Earth from the Sun for given Times, and the intercepted Angle *ASB* which is the difference of the Places of the Earth ſeen from the Sun; therefore you will have the Angles *SAB*, *SBA*, and the Right line *AB* the Chord of the Arc gone thro' by the Earth in that Time; that is, the Ratio of that Chord, to the Semidiameter of the *Magnus Orbis.* Again, in the Triangle *KAB* all the Angles are given, becauſe you have the two *KAB*, *KBA* (namely, this laſt being the ſum or difference of the known Angles *SBA*, *SBK*, and the firſt likewiſe the ſum or difference of the known Angles *SAB*, *SAK*;) wherefore you have the Ratio of the Sides *AK* and *AB*: But the Ratio of *SA* and *AB* was given before; therefore the Ratio of *SA* to *KA* will be known. Juſt after the ſame manner, from two Places of the Earth, *A* and *D*, the Ratio between *SA* and *AO* will be found; wherefore the Ratio between *SA* and *KO* will be known, being the difference between *AO* and *KO*: But it has been ſhewn in the foregoing

going Propoſition how to find the Ratio between *K O* and *K E*, wherefore the Ratio between *S A* and *K E* will be known, and therefore alſo the Ratio between *S A* and *A E*. Moreover in the foregoing Propoſition, the Angle *A E H* is alſo found; therefore *A E* is found, being the Diſtance (in a right line given in poſition) of the mark of a Comet's Trajectory from the Earth, expreſs'd in parts of the Semidiameter of the *Magnus Orbis*, and its inclination to the ſaid right line. After the ſame manner *B F*, or *C G*, or *D H*, may be compar'd with the Semidiameter of the *Magnus Orbis*, and the Angles at *F*, *G* and *H* will be found.

To determine this Trajectory, from *E* to the Plane of the Ecliptic, (which is the ſame with the Plane of this Scheme) let *E M* be ſuppos'd to be the erected Tangent of the Angle of the Comet's Latitude in the firſt Obſervation, ſuppoſing *A E* the Radius; and *M* will be the place of the Comet in the Mundane Space for the firſt Obſervation, becauſe the poſition of the Right line *A E* is its Longitude, and the Angle *M A E* its Latitude. Likewiſe at *H* let the Right line *HL* be rais'd perpendicular to the Plane of the Ecliptic, and equal to the Tangent of the Latitude obſerv'd the fourth time for the Radius *DH*; and *L* will be the place of a Comet in the Mundane Space for the fourth time: And therefore *M L* being drawn is the requir'd rectilineal Trajectory of a Comet, in which a perpendicular let fall from *S* will ſhew the Perihelium. It is plain, that the diſtance of the Point *M* from *A*, or the Right line *A M* is to the Right line *A E* as the ſecant of the Latitude in the firſt Obſervation to the Radius, and

that

that *D L* is to *D H* as the ſecant of the Latitude obſerv'd the fourth time to the Radius; but *L M* will be found in the Triangle *L P M*, *L P* being drawn thro' *L* parallel to *H E*, meeting with *M E* at *P*: For beſides the Right Angle at *P*, the ſides *L P*, *P M* are given, the firſt of them equal to the before known *H E*, and the laſt equal to the difference of the known *M E*, *H L*; wherefore *L M* will be known, being a Right line equally gone through by a Comet during a known Time, namely, whilſt the Earth moves from *A* to *D*.

Moreover, for finding the Node, the Time when the Comet will be in the Node, the inclination of the Comet's Way to the Ecliptic, and other things of this nature, let *M L* be produc'd till it meets with *E H*, as *for Example* at *N*; *N H* will be to *H L* as *H E* to *P M*, and therefore known; alſo *L N* will be to *L H* as *L M* to *M P*, and therefore alſo given: Let a Time be taken which is to the Time between the firſt and fourth Obſervation as *N L* to *L M*, and it will be the Time between the fourth Obſervation and the moment when the Comet will be in the Node; for in an equable motion (ſuch as we ſuppoſe this of a Comet) the Times are as the Spaces gone thro': But the place of the Comet in the Node will be given: For ſince the point *N* is given, and (by reaſon of the Time known between the fourth Obſervation and the moment when the Comet is in the Node) the Place alſo of the Earth is given for that moment; you will have (by *Dat.* 26.) the poſition of a Right line joining thoſe Points, that is, the Longitude of the Comet when it is in the Node. Moreover, becauſe you have the diſtance of the Node from the point *H* (the place of the Comet at the fourth Obſervation) and the Latitude of the Comet at the ſaid point *H*, (namely the two ſides about the Right Angle in a rectangular ſpherical Triangle;) you will have the Inclination of the Hypothenuſe or of the Comet's Way to the Ecliptic.

PROPOSITION XIII.

T*Hings being as before, to find the Place of a Comet ſeen from the Earth for a given Time*, [Fig. 10.]

Let the mark or ſection of the Orbit of a Comet be, by the foregoing *Prop.* determin'd on the Plane of the Ecliptic, namely *EHNR*, and *MLN* the Orbit itſelf in the Mundane Space, the Points *N*, *H*, *E*, *M* and *L*, denoting the ſame things as before: Let *ER* be taken to *EH* as the ſpace between the firſt Obſervation and the Time given, for which the Comet's Place is requir'd, to the Time between the firſt and fourth Obſervation: Moreover, find (by *Prop.* 18. *B.*3.) the place of the Earth for the Time propos'd, *viz.* the point *Q*. The poſition of the Right line *QR* whoſe two Extremities *Q* and *R* are known, (being given by *Dat.* 26.) gives the requir'd Longitude of the Comet. Moreover, let *RV* be taken to *RN* as *ME* to *EN*, and the Latitude of the Comet will be at the given Time an Angle whoſe Radius is *QR*, and the Tangent the Right line *RV*.

After this manner may you make Ephemeris's of the motion of a Comet, either graphically or by calculation.

SCHOLIUM.

After the ſame manner you may find the diſtance of a Comet from the Earth, either in its Perigæum, or for a Time given, as well ſhorten'd as in its Trajectory, expreſs'd in parts of the Diameter of the *Magnus Orbis*; as alſo the Angle which the Trajectory of a Comet makes with a Right line joyning a Comet and the Earth, and which the mark or ſection of the Orbit of a Comet in the Plane of the Ecliptic contains with a Right line drawn from the Earth to the place of the Comet in the Ecliptic.

COROLLARY.

Hence the true Magnitude as well of the Nucleus of a Comet, as of the Peruke or Hairineſs about it like an Atmoſphere, will be determin'd; for when once you know the Diſtance of a Comet and the apparent Diameter of the Nucleus, and of the Atmoſphere about the Nucleus by obſervation, their Magnitude will be known after the ſame manner that the Magnitude of the Planets is known from the like *Data* in *Prop.* 46. *B.* 3. For by knowing the Places of the Sun and of a Comet, the Angle which the Tail makes with a Right line joyning the Earth and a Comet will be known, and the Angle at the Earth which the Tail of a Comet ſubtends is known by Obſervation; from which things being known and from the given diſtance of a Comet from the Earth (namely the ſide between the ſaid Angles) the length of the Tail will be known: Its thickneſs will be found the ſame way as that of a Comet.

SECTION III.

To determine the true Trajectory of a Comet.

WE have in ſome of the foregoing Propoſitions ſhewn how to find ſuch a Trajectory of a Comet as does not much diſagree with the Phænomena, not only becauſe ſuch a Trajectory will ſuffice for nearly determining the Places of ſeveral Comets, and becauſe a pretty many of the chief Philoſophers did believe that a Comet was really carried in a Right line; but alſo becauſe by that means we have laid a Foundation for finding the true Parabolical or Elliptical Trajectory to be deſcrib'd from Obſervations, which we ſhall do in the following Pages, after we have premiſed ſome Propoſitions concerning the motion of a Comet in a Conic Section for the better underſtanding of it, and laid down neceſſary Lemmas.

PROPOSITION XIV. LEMMA.

If the Right line GAB *touches the Parabola* V A, *whoſe Vertex is* V *and Axis* VD *in the Point* A; *and from* F *the Focus of the Parabola* F C *be let fall perpendicularly on the ſaid line, it will be a Mean Proportional between the Right lines* FA *and* F V. [Fig. 11.]

Let the right line touching the Parabola and the Axis of the Parabola be produc'd till they meet together, as for *Example* at *B*: Thro' *A* let *AE* be drawn parallel and *AD* perpendicular to the Axis; and *BV* will (by *Prop.* 35. *B.* 1. *El. Conic.*) be equal to *VD*, and by the well-known catoptrical property of the Parabola) the Angle *GAE* is equal to the Angle *FAB*; But *GAE* is equal to *ABF*; whence *FAB* is equal to *FBA*, and therefore *FB* is equal to *FA*, and *AC* equal to *CB*; therefore (by *Prop.* 2. *El.* 6.) *VC* being drawn is parallel to *AD*, that is, perpendicular to *BF*. Whence (by *Prop.* 8. *El.* 6.) *CF* is a mean proportional between *BF* and *VF*: But *AF* has been ſhewn equal to *BF*, and therefore *CF* is a mean proportional between *FA* and *FV*. *Q. E. D.*

COROLLARY 1.

Hence it follows, that in the ſame Parabola *FC* is in a ſubduplicate ratio of *FA*, or that this *FC* is to any other *FC* in a ſubduplicate ratio of this *FA* to another *FA* agreeable to another ſaid *FC*; or ſince *VF* is conſtant and unchangable, FC^q will be as *FA* itſelf, and therefore the right line *FC* itſelf in a *ſubduplicate* ratio of the right line *FA*.

COROLLARY 2.

If in *FV* produc'd be taken *VH* equal to *VF*, and the infinite Line *HK* be drawn parallel to *VC*, on which from any point *A* of the Parabola *AK* be let fall perpendicular, it will be equal to *FA* the diſtance of the point *A* from the Focus. And converſly if *AK*, let fall perpendicularly from *A* on the right line *HK*, be equal to the right line *AF*; the point *A* will be in the Parabola above deſcrib'd, whoſe Focus is *F* and Vertex *V*.

The Firſt it appears from hence: That *BV* is equal to *VD* (by *Prop.* 33. *B.* 1. *Conic.*) and *HV* by conſtruction is equal to *VF*, and therefore *BF* is equal to *HD*: But it has been before ſhewn, that *AF* is equal to *BF* and by (*Prop.* 34. *El.* 1.) *KA* is equal to *HD*; whence *KA* and *FA* are equal.

But the ſecond from hence: By ſuppoſition *AK* is equal to *AF*, wherefore the point *A* is in a right line perpendicular to *FK* rais'd from its middle point. Now, if a Parabola whoſe Vertex is *V* and Focus *F*, does not go thro' *A*, let it go thro' any other Point of the right line *KA*, (or produc'd if occaſion require) as for *Example* α: Becauſe (by what has been already ſhewn, *F*α is equal to *KA*; whence the Point α is in the foreſaid perpendicular to *FK* rais'd from its middle Point. Therefore the ſaid perpendicular will meet with the two Points *A* and α of the right line *KA*, which is abſurd; and therefore a Parabola whoſe Focus is *F* and Vertex *V*, will go thro' the point *A*, if *AK* and *AF* be equal.

PROPOSITION XV.

If a Body moves in a Parabola, (and at the ſame time be urg'd by Forces tending to the Focus of the Parabola, which Forces are reciprocally proportionable to the Square of the diſtance of the Focus) its Velocity in any point of the Parabola is reciprocally in a ſubduplicate ratio of the diſtance from the Focus of the Parabola. [Fig. 12.]

Let *DC* be the Parabolical Trajectory of a Body, whoſe Focus is *S*; it is plain (from *Corol.* 2. *Prop.* 41. *B.* 1.) that its Velocity in the point *C* is reciprocally as *SE* drawn perpendicularly from the Focus *S* on the right line *CE*, which touches the Parabola at *C*, or that the Velocity of the Body in *C* is to its Velocity in *c* as *Se* to *SE*: But (from *Corol.* 1. of the foregoing *Prop.*) *Se* is to *SE* in a ſubduplicate ratio of the right line *Sc* to the right line *SC*; and therefore the velocity of the Body in *C* is to its velocity in *c* in a ſubduplicate ratio of the right line *Sc* to the right line *SC*; that is, the velocity of the Body is inverſly in a ſubduplicate ratio of its diſtance from the Focus. *Q. E. D.*

PROPOSITION XVI.

THE ſame things being ſuppos'd as before, the velocity of a Body in any point of a Parabola is to the velocity of a Body revolving in a Circle, about the ſame center of Forces deſcrib'd at the ſame diſtance, as the Diameter of a Square to its Side.

For (by *Cor.* 1. *Prop.* 41. *B.* 1.) the velocity of a Body moving in a Parabola at the leaſt diſtance from the Focus (that is, in the Vertex of the Parabola) is to the velocity of a Body moving in a Circle at the ſame diſtance about the ſame center of Forces, in a ſubduplicate ratio of the principal *Latus rectum* of the Parabola to twice the diſtance of the Vertex from the Focus: But the principal *Latus rectum* of the *Parabola* is equal to four times the diſtance between the Focus of the Parabola and its Vertex, and therefore the velocity of a Body moving in a Parabola after the manner mention'd, whilſt it is in its Vertex, is to the velocity of a Body moving in a Circle about the ſame center of Forces at the ſame diſtance, in a ſubduplicate ratio of Four to Two, or of Two to One; that is, in the ſame ratio which the Diameter of a Square has to its Side. Again, (by the foregoing *Prop.*) the velocity of a Body in any other Point taken at pleaſure, is to its velocity in the Vertex in a ſubduplicate ratio of the diſtance of the Vertex from the Focus to the diſtance of the ſaid Point taken at pleaſure from it: And (by *Prop.* 27. *B.* 1.) if Bodies, which are urg'd by Forces reciprocally proportional to the ſquare of the diſtance from the Center, deſcribe about a given Center Circles, Circle, at the ſame diſtances as the forementi-on'd Points of the Parabola, the velocity of a Body revolving in a Circle paſſing thro' the ſaid Point, will be to the velocity of a Body in a Circle paſſing thro' the Vertex of the Parabola in a ſubduplicate Ratio of the diſtance of the Vertex from the Focus, to the diſtance of this other taken Point from it; that is, in the ſame Ratio that the velocities of a Body carried in a Parabola at the ſame Diſtances, have been ſhewn to be to each other. And therefore by Tranſpoſition the velocity of a Body moving in a Parabola at a Point taken at pleaſure is to the velocity of a Body moving in a Circle at the ſame diſtance, as the velocity of a Body moving in a Parabola at the Vertex, to the velocity of it moving in a Circle at the diſtance of the ſaid Vertex; that is, as was before ſhewn, in a Ratio of the Diameter of the Square to its ſide. *Q. E. D.*

PROPOSITION XVII.

THe velocity of any Comet is to the velocity of any Primary Planet in a ſubduplicate Ratio of twice the diſtance of a Planet from the Sun to the diſtance of a Comet from it.

It is plain from the Firſt Book, that tho' the Way of a Comet about the Sun be Elliptical, yet by reaſon of the great Excentricity of ſuch an Ellipſis, it may be look'd upon as a Parabola in whoſe Focus the Sun is; at leaſt in that part of it which being in our Planetary Region is ſubject to Obſervation, it does not ſenſibly differ from a Parabola. And it is plain from the Third Book, that the Orbits of the Primary Planets, by reaſon of their ſmall Excentricity, differ very little from Circles concentric to the Sun.

Whence, from the foregoing *Prop.* it is plain, that the velocity of a Comet is to the velocity of a Primary Planet revolving at the ſame diſtance as the Comet in a Subduplicate Ratio of Two to One. And the velocity of ſuch a ſuppos'd Planet is to the velocity of any other Primary Planet revolving at any diſtance (by *Prop.* 27. *B.* 1.) in a ſubduplicate Ratio of the diſtance of this ſaid laſt real Primary Planet's diſtance from the Sun, to the diſtance of the ſuppos'd one, that is, to the diſtance of the Comet itſelf from it: And therefore the Ratio of the velocity of a Comet, to the velocity of any Primary Planet, made up of the Two aforeſaid, is equal to a ſubduplicate Ratio of twice the diſtance of the Planet from the Sun to the diſtance of the Comet from it. *Q. E. D.*

COROLLARY.

Hence the velocity of a Comet may be compar'd with the velocity of any Primary (as for example of the Earth) carried about the Sun, if the Ratio be known between the diſtance of a Comet from the Sun, and the mean diſtance of the ſaid Planet from it.

PROPOSITION XVIII.

LEt PAB *be any Parabola whoſe Focus is* S, *and any Arc of it* ACB, *whoſe Chord* AC *is biſſected in the Point* M, *thro' which the right line* MV *is drawn parallel to the Axis of the Parabola meeting the Parabola in* V, *and produc'd to* G, *ſo that* MV *be the double of* VG; *let* GS *be drawn cutting* AB *in* Q, *and be produc'd to* E, *ſo that* SE *m*ᵛ*y be the double of* SG: *If a Comet deſcribes the Parabolic Arc* ACB *about the Sun* S (*as in the* 35th Prop. B. 1) *the right line* EC, *joining the Comet when it is any where*

where in the ſaid Arc (as at C) *and the Point* E *determin'd before, will cut the Chord* AB *in* D, *in ſuch manner that* AD *will be nearly to* DB *as the Time in which a Comet deſcribes the Arc* A C, *to the Time that it deſcribes the Arc* C B. [Fig. 13.]

Draw the right lines *SB*, *SD*, *SC*, *GA* and *AV*; draw alſo *GD* meeting with the Parabola at *K*. Becauſe *MV* is parallel to *SX* it will (by *Prop.* 46. *B* 1. *El. Conic.*) cut all the right lines parallel to *AB* terminated by the Parabola into two equal parts, and therefore it will divide the Parabolical Segment comprehended under the Curve *AVB*, and the right line *AB* into two equal parts or half Segments *AHVM* and *BCVM*. Thro' *V* draw *VF* parallel to *AB*, (which by *Prop.* 17. *B.* 1. *El. Conic.*) will touch the Parabola in *V*) meeting the right line *GD* at *F*, and let us ſuppoſe the right lines *SF*, *CF* to be drawn.

Becauſe by conſtruction *MV* is the double of *VG*, and *ES* alſo the double of *SG*, *MV* will be to *VG* as *ES* to *SG*. But by reaſon of the Parallels *VF* and *MD*, *DF* is (by *Prop.* 2. *El.* 6.) to *FG* as *MV* to *VG*; and therefore *EF* is to *FG* as *ES* to *SG*. Therefore the right line *SF* will be parallel to the right line *EC* (by *Prop.* 2. *El.* 6.) and therefore the Triangles *DSC*, *DFC* (by *Prop.* 37. *El.* 1.) will be equal: But the Triangle *DFC* is nearly equal to the Triangle *DKC*, by reaſon of the nearneſs of the Points *F* and *K*; therefore the Triangle *DSC* will be nearly equal to the Triangle *DKC*. Moreover, the Triangle *MAG* is (by *Prop.* 1. *El.* 6.) to the Triangle *MAV* in a ſeſquialteral Ratio, becauſe they have the ſame Vertex *A* and the Baſe *GM* is to the Baſe *MV* as one and a half to one. But the Triangle *AVM* (by the 24th *Prop.* of *Archimedes of the Quadrature of the Parabola*) is ſubſeſquiſeſquitertial to the Segment *AHVM*; therefore the Triangle *AGM* is eight times and a half the Semiſegment *AHVM*, namely, in a Ratio compounded of a ſeſquialteral and ſubſeſquitertial Ratio. Again, the Triangle *MGD* is nine times its ſimilar *VGF*, the ſide *MG* being three times greater than the homologous ſide *VG*; therefore the Triangle *MGD* is eight times and a half the Trapezium *MVFD*: therefore the Triangle *AGM* is to the Semiſegment *AHVM* as the Triangle *MGD* to the *Trapezium MVFD*; and again, the half Segment *AHVM* is to the Trapezium *MVFD* as the Triangle *AGM* to the Triangle *MGD*, that is (by *Prop.*1. *El.* 6. (as the right line *AM* to the right line *MD*. But the Trapezium *MVFD* is nearly equal to a protion of the Parabolical Segment *MVKD* contain'd under the right lines *KD*, *DM*, *MV* and the ſmall part *VK* of the Parabolic Curve; and therefore the Semiſegment *AHVM* is nearly to the portion *MVKD* as the right line *AM* to the right line *MD*. Wherefore the half Segment *AHVM* encreas'd by *MVKD* (that is, the Parabolical Area *AHVKDA*) is to the Semiſegment when the ſaid part is taken from it (that is, the part *BCKDB*) as *AM* and *MD* together to *MB* when *MD* is taken from it, or as *AD* to *DB*; and the Triangle *ASD* is alſo to the Triangle *BSD* (by *Prop.* 1. *El.* 6.) as *AD* to *DB*. Therefore the ſpace *SAHVKDS* is to the ſpace *SBCKDS* as *AD* to *DB*. Now if to the firſt ſpace you add the Triangle *DKC*, and from the Sum take away the Triangle *DSC*, equal to the foreſaid Triangle, by what has been already ſhewn, you will have the ſpace *SAHKCS* equal to the ſaid firſt ſpace. Likewiſe, if to the laſt ſpace be added the Triangle *DSC*, and from the

Sum be taken the Triangle *DKC* equal to it, you will have the ſpace *SBCS* equal to the ſaid laſt ſpace. Therefore the right line *AD* is to the right line *DB* nearly as the ſpace *SAHVCS* to the ſpace *SCBS*. But the ſpace *SAHVCS* is the Area which a Comet by a Radius drawn to the Sun deſcribes whilſt it goes through the Parabolic Arc *AC*, and the ſpace *SBCS* is the Area which the ſame Comet deſcribes after the ſame manner as it goes thro' the Arc *CB*; and theſe Areas (by *Prop.* 11. *B.* 1.) are as the Times in which a Comet deſcribes the Arcs *AC* and *CB*: therefore the right line *AD* is to *DB* nearly as the Time in which a Comet deſcribes the Parbolic Arc *AC* to the Time that it deſcribes the Arc *BC*. But the point C (the Comet's place) is taken at pleaſure: Therefore univerſally the right line *EC* does ſo divide the right line *AB* at D, that *AD* is nearly to *DB* as the Time in which a Comet deſcribes the Arc *AC*, to the time in which it deſcribes the Arc *CB*. *Q. E. D.*

SCHOLIUM.

If the Point C coincides with the Vertex *V* of the Parabolic Arc, then *AD* will be to *DB* accurately as the Time in which a Comet deſcribes the Arc *AC*, to the time in which it deſcribes the Arc *CB*. For [*Fig.*14.] the Triangle *ASD* is (as it was before) to the Triangle *BSD* as *AD* to *BD*, and alſo the Semi-ſegment *AHVM* when the Trapezium *MVFD* is taken from it, is to the Semi-ſegment *BVM* encreas'd by the ſaid Trapezium *MVFD* as *AD* to *BD*: But the Trapezium *MVFD* is made up of the two Triangles *DMV* and *DFV*; and therefore the Triangle $ASD+$ the Semi-ſegment $AHVM-$, the Triangle $DMV-$ the Triangle *DFV* is to the Triangle *BSD*+ the Semi-ſegment *B V M*+ the Triangle *DMV* + the Triangle *D FV* as *AD* to *DB*. Inſtead of the Triangle *DFV*, let us make the Triangle *DSC* equal to it; and the Triangle *ASD*+, the Semi-ſequent *AHVM*— the Triangle *DMV*— the Triangle *DSC* will be to the Triangle *BSD*+ the Semi-ſegment *BVM*+ the Triangle *DMV* + the Triangle *DSC* as *AD* to *DB*; that is, the Area *ASCHA* is to the Area *CSBC*, or (by *Prop.* 11.*B*.1.) the Time in which the Comet deſcribes the Arc *AC*, to the Time in which it deſcribes the Arc *CB*, as the right line *AD* to the right line *DB*; and that accurately; for the Curvilinear ſpace has not been taken nearly equal to the Rectilinear one, as it was done in the Propoſition, when the Point C was different from the Point *V*.

COROLLARY 1.

Hence it follows, that the nearer the Point C be to *V* the Vertex of the Segment, ſo much more accurately is the Chord *AB* divided by the right line *EC* into the two Segments *AD BD*, having to one another the Ratio of the Times in which the Comet goes thro' the Arcs *AC*, *CB*: For if it falls in with *V*, that is exactly ſo; if not, then it is nearly ſo. But we muſt obſerve that the Error will be leſs if C is diſtant from *V* towards *B*, than if it ſhou'd be as far diſtant from it towards *A*; becauſe the Portion *VA* of the Parabola nearer to the principal Vertex *X* is more, curve, and declines ſooner from the right line *VF* Tangent to it, than the portion *VB* remoter from *X*. And hence it happens, that if three moments of time be requir'd at which the Comet is in three Points of the Parabola (ſuch as are A, C and B) ſo diſpos'd that the right line EC may cut the Chord AB in D, ſo that AD may be to DB accurately as the two Times between the three intercepted Moments, the ſaid Times muſt be nearly equal to one another. For in ſuch parts of the parabolic Trajectory as we can obſerve, the Point D (where the right line joyning the Vertex V and the Point E cuts the Chord AB) is not very far diſtant from M the middle point of the Chord. But that interval of Time where the Comet mov'd ſloweſt muſt be ſomething greater than the other; for it is plain that when the Comet is in V, where the thing requir'd is exactly found, that the right line DB is greater than AD; that is, the time in which the Comet (then moving ſloweſt by *Prop.* 15.) deſcribes the Arc BC, is greater than the Time in which the ſaid Comet by a ſwifter Motion deſcribes the Arc AC. But it is ſafer to take Times that are not very unequal, that the point D may rather decline towards B than A, upon account of the reaſon before alledg'd; namely that the Error may be leſs if C is diſtant from V towards B than towards A.

COROLLARY 2. [Fig. 15.]

If V, the Vertex of the Segment, is not far diſtant from X the principal Vertex, nor the Point C from V, the right line SC, drawn from S the true Focus of the Parabola, will divide the Chord AB in γ, nearly in a Ratio of the Times in which the Comet deſcribes the Arcs AC and CB. If SV is very great in reſpect of MV, MQ will be the triple of $Q\gamma$: For in that caſe the right lines SQG, $S\gamma V$ are to be look'd upon as parallel; therefore (by *Prop.* 2. *El.* 6.) MQ is to $Q\gamma$ as MG to GV; that is, by conſtruction, as 3 to 1.

COROLLARY 3.

Things remaining as before, $\mathcal{Q}E$ will be equal to $3\mathcal{Q}S+3MV$: For by conſtruction SE is equal to $2SG$, therefore GE is equal to $3SG$, that is, $GE=3S\mathcal{Q}+3\mathcal{Q}G$: if therefore from each ſide be taken $\mathcal{Q}G$, $\mathcal{Q}E$ will be equal to $3S\mathcal{Q}+2\mathcal{Q}G$: but by reaſon of the Parallels $\mathcal{Q}G$, γV, $\mathcal{Q}G$ is to γV as MG to MV, that is as 3 to 2; wherefore $2\mathcal{Q}G=3\gamma V$: and γV is equal to MV, becauſe the Angles γMV and $M\gamma V$ are equal, by reaſon of SV and MV, which make equal Angles with a right line touching the Parabola at V, which (by *Prop.* 46. *B.* 1. of *Conics*) is parallel to the Chord AB; therefore $3MV=2\mathcal{Q}G$; and therefore $\mathcal{Q}E=3\mathcal{Q}S+3MV$. If the poſition of the right line AB be a little chang'd, all the foregoing things will notwithſtanding be nearly true.

PROPOSITION XIX. LEMMA.

IF in the Parabola BAP, *whoſe Focus is* S, *from the Chord drawn* AB *be cut off the Segment* AVB, *whoſe Vertex is* V, *and Diameter* VM *meeting with the Chord in its middle Point* M, *and from* M *to* AB *be drawn perpendicularly* MI *meeting at* I *with the right line* LV *parallel to* AB, *and the Parallelogram* VIMK *be fill'd up; as the Points* V *and* M *come together, or as the Segment* AVB *is naſcent, the Diagonal* IK *produc'd, falls upon* S. [Fig. 16.]

For (by *Prop.* 17. *B.* 1. *Elem. Conic.*) the right line LI touches the Parabola at V, whence the Angle SVL is equal to the Angle MVI: But becauſe the Parallelogram $MKVI$ is rectangular, the Angle KIV is equal to the Angle MVI; and therefore the Angle KIV is equal to the Angle SVL, therefore (by *Prop.* 28. *El.* 1.) IK is parallel to VS, and it is always ſo; therefore when the Points *I* and *V* come together; that is, when *M* and *V* come together, the right line *IK* produc'd goes thro' the Focus *S*. *Q. E. D.*

COROLLARY [Fig. 17.]

Hence it follows, that if from any Point C of the Parabola, near the Vertex of the Segment *ACB*, be drawn the right line *C I* parallel to the Chord *A B* meeting with the right line *M I* at *I*, and *S I* be drawn, which cuts the Chord in *K*, and the Parallelogram *I M K V* be fill'd up; the Point *V* will be nearly the Vertex of the Segment, if the diſtance of the Focus be immenſly greater than either of the right lines *CI*, *IM*, or *MK*. For it is well known to Geometricians, that the Immenſity of the right line *SK*, in reſpect of the right lines *C I*, *IM*, *M K*, brings the matter to the ſame as if we make the Segment *A B* to vaniſh; in which caſe the Propoſition is plain from what has been already ſaid.

PROPOSITION XX.

LET A B *be any Parabola whoſe Focus is* S; AVB *any Arc of the Parabola, whoſe Chord is* A B *and vertex* V *and Diameter* MV; *let the right line* S V *be drawn and produc'd at* L *and* R, *ſo that* V L *may be the third part of* M V, *and* S R *a third proportional to* S V, S L; *if a Comet, at the ſame time that it deſcribes the Parabolic Arc* A V B *about the Sun* S, *ſhou'd go forward equably with the ſame velocity which it has at a diſtance from the Sun equal to* S R, *it wou'd deſcribe a Line equal to the Chord* AB. [Fig. 18.]

Let the right line *S V* meet with the Chord *A B* at *Z*: now ſince the Angles *FVM*, *G V Z*, are equal, therefore alſo their alternate Angles on the two Parallels *A B*, *G F*, namely, *Z MV*, *MZV*,

MZV will be equal, and the right lines *VZ*, *VM* are equal. Moreover, (by *Corol.* 5. *Prop.* 41. *B.* 1.) if a Comet deſcribing the Parabola *PAB*, at the ſame time that it deſcribes the Arc *AVB*, ſhou'd go on with the velocity that it has in *V* equably in the Tangent *VF*, the Arc, which it wou'd deſcribe by a Radius drawn to the Sun, wou'd be equal to the Parabolic Area *AVBSA*: And as *SV* is inclin'd to the Tangent *VF* in the ſame angle that *SZ* is inclin'd to *AB*, the ſpace contain'd in length by the Tangent deſcrib'd (as has been ſaid) and *SV* wou'd be to the ſpace contain'd under the right line *AB* and *SZ* as the Parabolic Area *AVBSA* to the Triangle *ASB*; that is, as the Triangle *ASB* + the Parabolical Segment *AVB* to the Triangle *ASB*; that is, as the Triangle $ASB+\frac{2}{3}$ of the Parallelogram *AGFB* to the Triangle *ASB*; that is, as $AB\times\frac{1}{2}ZS+AB\times\frac{2}{3}MV$ to $AB\times\frac{1}{2}ZS$; that is (by applying it to *AB*) as $\frac{1}{2}ZS+\frac{2}{3}MV$ to $\frac{1}{2}ZS$, or as $ZS+\frac{4}{3}MV$ to *ZS*. But *VL* is made equal to $\frac{1}{3}MV$; and it has before been ſhewn that *ZV* is equal to *MV*; therefore *ZL* is equal to $\frac{4}{3}MV$. And therefore the ſpace contain'd in length by the Tangent deſcrib'd, and *SV* wou'd be to the ſpace contain'd by *AB* and *ZS*, as $SZ+ZL$ to *SZ*; that is, *SL* to *SZ*. And by applying the antecedents of this Analogy to *SV*, and the conſequents to *SZ*, the length in the Tangent deſcrib'd will be to the Chord *AB*, as $\frac{SL}{SV}$ to $\frac{SZ}{SZ}$ or Unity; that is as *SL* to *SV*. Moreover (by *Prop.* 15.) the velocity of a Comet deſcribing a Parabola, in the height or diſtance from the Sun *SV*, is the velocity of it in the height or altitude *SK*, in a ſubduplicate Ratio of *S R* to *S V*; that is, as *S L* to *S V*: And the Lengths gone thro' in the ſame time by moving Bodies carried equably, are as the Velocities; therefore the length in the deſcribed Tangent is to the length deſcrib'd in an equal Time (namely the time in which the Comet goes thro' the Parabolic arc *A V B*) by the Comet equalby carried, with the velocity that it has in the altitude *S R*, as *S L* to *S V*. Since therefore the ſaid length deſcrib'd along the Tangent has the ſame Ratio, (namely that of *S L* to *S V*) to the Chord *A B* as it has to the length which the Comet wou'd go thro' moving with the velocity that it has at the altitude *S R*, during a Time equal to that in which the ſame Comet deſcribes the arc *A V B*; theſe lengths will be equal to one another, namely the Chord *A B*, and the length gone thro' by a Comet with the ſame velocity that it has at the altitude *S R*, and in the ſame time that the ſame Comet deſcribes the arc *A V B*. *Q. E. D.*

COROLLARY.

If *S V* be very great in reſpect of *V L*, the three Geometrical Proportionals *SV*, *SL* and *SR*, will be alſo nearly Arithmetically Proportional; that is, *LR* will be equal to *V L*, or to the third part of *MV*; therefore *VR* will be nearly equal to half *MV*. And therefore equably mov'd with the velocity that it has in an altitude or diſtance from the Sun equal to $SV+\frac{1}{3}MV$, and in the ſame Time that it wou'd deſcribe the Parabolic arc *A V B*, wou'd go thro' a length equal to *A B*, the Chord of the ſaid arc.

PROPOSITION XXI.

IN the ſame Figure as before [*viz.* Fig. 18.] *if a Comet from reſt ſhou'd be let fall from the Height* L S *toward the Sun* S, *and afterward ſhou'd be urg'd by the centripetal Force which it has at* L *neither encreas'd nor diminiſh'd; ſuch a Comet in half the Time that it wou'd deſcribe the Parabolic Arc* A V B, *woud deſcend along a right line equal to half the length of the right line* M V.

By the foregoing Propoſition in the Time that the Comet deſcribes the Parabolic arc *A V B*, if it ſhou'd move equably with the velocity that it has in the Parabola, at an altitude or diſtance from the Sun equal to *S R*, it wou'd deſcribe a length equal to the Chord *A B*: But the velocity of a Comet in a Parobola at the altitude *S R* (by *Prop.* 16.) is to the velocity of a Body going round in a Circle at the ſame diſtance from the Sun, in the circumference of which Circle it is retain'd by the force of its Gravity, in a ſubduplicate ratio of Two to One; therefore the arc which a Body carried in a Circle whoſe Semidiameter is *S R*, goes thro' in the time that a Comet deſcribes the Parabolic arc *A V B*, is to *A B* the Chord of a Parabolic arc, as 1 to $\surd 2$, and therefore equal to $\frac{AB}{\surd 2}$. but (by *Corol.* 1. *Prop.* 25. *B.* 1.) any Body let fall from reſt towards the Sun, the ſame centripetal Force always acting (as is here ſuppos'd) in half the time that it goes thro' the arc of a Circle deſcrib'd with the diſtance from the Sun whence it had fallen as a Radius, it falls thro' a ſpace equal to the Square of half the ſaid circular arc, (that is to the ſquare of half $\frac{AB}{\sqrt{2}}$ applied to the Diameter of the Circle or $2SR$; that is, to the Square of $\frac{AM}{\sqrt{2}}$ applied to $2SR$ or $\frac{AM^q}{4SR}$. Therefore the ſpace thro' which the Body let fall from reſt from the Height RS goes thro' in its fall, in half the time that a Comet will deſcribe the Parabolic arc AVB, is equal to $\frac{AM^q}{4SR}$. But (by *Prop.* 42. *B.* 1.) the centripetal Force, or the accelerating Gravity towards the Sun at the diſtance SL is to the accelerating Gravity towards it at the diſtance SR as SR^q to SL^q; that is, by reaſon of the Proportionals SR, SL, SV, as SR to SV: and therefore ſince a Comet depriv'd of all motion and let fall from R, in half the time that it wou'd deſcribe theParabolic arc AVB falls toward the Sun tho' a ſpace equal to $\frac{AM^q}{4SR}$; it will in the ſame Time, being after the ſame manner let fall from L towards the Sun, go thro' a ſpace which is to $\frac{AM^q}{4SR}$ as SR to SV; that is, a ſpace equal to $\frac{AM^q}{4SV}$. Moreover from the nature of a Parabola, $4SV$ is the *Latus rectum* of the Diameter VM; that is, the Rectangle contain'd under $4SV$ and VM is equal to the Square of AM, therefore $\frac{AM^q}{4SV}$ is equal to the Right line VM: Therefore a Comet depriv'd of all Motion and let fall from the Height LS

towards the Sun *S*, if it be urg'd by an accelerating Force or uniform Gravity equal to that by which it was urg'd at the ſaid altitude *S L*, in half the time that it wou'd deſcribe the Parabolic arc *AVB*, will fall along a Right line equal to the Right line *VM*. *Q. E. D.*

PROPOSITION XXII. LEMMA.

If from the Point S, *which is the Focus of the Parabola* AVB, *and the center of the Circle* Ttτ, *be drawn the equal right lines* SV *and* St *and there be alſo drawn the right lines* AB, Tτ, *Chords of the arcs whoſe Vertices are* V *and* t, *cutting the right lines* SV, St. *in the Points* D *and* N, *ſo that* VD *and* tN *may be equal and very ſmall in reſpect of the right lines* SV *and* St; *the Chord* AB *will be to the Diameter of the ſquare, whoſe ſide is the Chord* Tτ, *in a ſubduplicate ratio of the right line* St *to the right line* SD. [Fig. 19.]

Thro' *V* draw *VM* parallel to the axis of the Parabola, meeting with the Chord at *M*, and which (by *Prop.* 46. *B.* 1. *Conic.*) will be the Diameter of the Segment *AVB*, whoſe Parameter is the quadruple of the right line *SV*; therefore $AM^q = 4SV \times VM$. And as (by *Prop.* 35. *El.* 3.) $TN^q = tS + SN \times tN$, equal to $2St \times tN$ becauſe $St = SN$; then $AM^q : 4SV \times VM :: TN^q : 2St \times tN$, therefore (by taking the quadruples of the antecedents and by tranſpoſition) the Analogy will be this $AB^q : T\tau^q :: 4SV \times VM : 2St \times tN$. But (by what has been ſhewn in the demonſtration of *Prop.* 20.) $VM = VD$, and by ſuppoſition $VD = tN$; therefore by applying the third Term of the foregoing Analogy to *VM* and the fourth to *tN*, $AB^q : T\tau^q :: 4SV : 2St$; that is, (by reaſon that *SV* and *St* are equal by ſuppoſition) $AB^q : T\tau^q :: 2St : SV$; that is, as $2ST$ to *SV* or *SD*: and by doubling the

Conſequents, $AB^q : 2\times T\tau^q :: 2St : 2SD$; that is, as DS to St: Therefore AB is to $\sqrt{2\times T\tau}$ in a ſubduplicate Ratio of St to SD. Q. E. D.

PROPOSITION XXIII. LEMMA.

IF *about the Focus* S *be drawn any two Parabola's,* ACB, EGB, *in which the Chords* AB *and* EF *being drawn, cut off Segments whoſe vertices are* C *and* G, *in ſuch manner that the right lines* SC *and* SG *cutting the foreſaid Chords in* D *and* H, CD *is equal to* GH; *I ſay that* AB *is to* EF *in a ſubduplicate Ratio of the right line* SC *to the right line* SG. [*Fig.* 20.]

Thro' C draw CM the Diameter of the Segment ACB, and thro' G the right line GK, the Diameter of the Segment VGF, meeting with the reſpective Chords at M and K. The Parameter of thoſe Diameters are equal to the quadruples of SC, SG, ſhewn as before; and (by *Prop.* 49. *B.* 1. *Conic.*) AM^q will be $= 4SC\times CD$ and $EK^q = 4SG\times GH$: Whence $AM^q =$ is to EK^q as $4SC\times CD$ to $4SG\times GH$, that is, (by reaſon of the equality of CD and GH) as as SC to SG; and therefore AM is to EK in a ſubduplicate Ratio of SC to SG. Whence AB (the double of AM) into EF (the double of EK) in a ſubduplicate Ratio of SC to SG.

PROPOSITION XXIV.

TO *deſcribe a Parabola about a given Focus, in ſuch manner that it may paſs thro' two given Points.* [Fig. 21.]

Let S be the given Focus, and AB, the two given Points thro' which the Parabola muſt paſs: with the Centers A and B, and the diſtances AS and BS draw two Circles, and (by the Conſtruction of the 118th *Prop.* of the 7th *Book* of *Pappus's Mathemat. Collect.*) draw the the right line *CD* touching each of them, on which from the Point *S* let fall the perpendicular *SE*, which muſt be biſected at *V*. With the Vertex *V*, and the Axis *VSX*, and a *Latus rectum* equal to the quadruple of the right line *VS* (by *Prop.* 52. *B* 1. *Conic.*) draw a Parabola, and it will be the Parabola requir'd.

That *S* is the Axis of the deſcrib'd Parabola is plain from this, that in the Axis of the Parabola it is diſtant from the Vertex of it the fourth part of the *Latus rectum*, which is the moſt known property of the Focus of a Parabola. And that it goes thro' the points *A* and *B* is prov'd thus: The Axis *VX* of the Parabola is produc'd beyond the Vertex to *E*, ſo that *VE* may be equal to *VS*, the diſtance of the Focus from the Vertex; and thro' *E* the Right line *CED* is drawn perpendicular to the Axis of the Parbola, and the point *A* is ſuch, that *AS* its *diſtance* from the Focus, is equal to *AC*, which (by *Prop.* 18. *El.* 3.) is a perpendicular drawn from *A* to the right line *CD*: Whence (by *Corol.* 2. *Prop.* 14.) the before deſcrib'd Parabola goes thro' *A*, and for the ſame reaſons it alſo goes thro' *B*. *Q. E. F.*

This Problem might have been look'd upon as ſolv'd by the 26th Propoſition of the Third Book, as was there obſerv'd; but by reaſon of its great uſe we have put it in this place. Two Points given in a Parabola do the ſame here as three Points given there in an Ellipſis, becauſe the Ratio given in every Parabola between the *Latus rectum* and the *Latus tranſverſum* is equivalent to one Point. It is not neceſſary to make uſe of a Trigonometrical Calculation to determine the Parabola's Vertex, Parameter, *&c.* for they will be eaſily found by the latter part of the 26th Propoſition of the 3d Book.

PROPOSITION XXV.

From ſome Places of a Comet obſerv'd, to find its Place for any intermediate Time given ; and to determine the Time in which it ſhall be in any intermediate Place. [Fig. 22.]

Let the line ♈ 123 be the Way of a Comet, and the obſerv'd Places 1, 2,3,4,5,6,7, in ſuch manner that an equal ſpace of Time may be ſpent between the moments that a Comet was obſerv'd in the Points 1 and 2, and that during which it mov'd from 2 to 3, or from 3 to 4, or from 4 to 5, and ſo on. Let ſuch a Time be call'd *T.* Let the firſt differences of thoſe Places from the fix'd Point ♈, *viz.* ♈ 1, ♈ 2, ♈ 3, ♈ 4, ♈ 5, *&c.* be taken (as is uſual with Aſtronomers) that is, the arcs 12, 23, 34, 45, *&c.* which we ſhall call *A,B,C,D,E,F*: Again, take their ſecond Differences, or the Differences of the firſt Differences, namely, *G,H,K,L,M*; and then the Differences of thoſe ſecond Differences, or of the Arcs ♈ 1, ♈ 2, ♈ 3, *&c.* the third Differences *P,Q,R,S*; and ſo on, till we come to Differences equal to Nothing, namely the differences of equal Differences immediately preceeding. Every one of the equal Times ſpent in going thro' the Arcs *A, B, C, &c.* (namely *T*) muſt be ſuppos'd divided into any number of equal parts, (the more of them the more accurate the thing will be) and let an Arc gone thro' by a Comet in any Time *T* be divided into as many parts as *T* is divided, in ſuch manner that the Differences of thoſe parts reckon'd from ♈ ſimilar to the Differences of ♈ 1,♈ 2,♈ 3,♈ 4, *&c.* may be equal to nothing; and let them, being encreas'd or diminiſh'd, go on in the ſame order towards the ſame parts, and ſo one of them being in the middle, may be contain'd as often in the whole Arc as the Time which belongs to it here is contain'd in the whole Time, which will be eaſy for any one that underſtands Aſtronomical Calculations; As for *Example*, if the third Differences of the given Arcs ♈1, ♈2, ♈3, ♈4, *&c.* are found to be equal to nothing; then from ♈ to the Diviſions of *A*, *B*, *C*, *&c.* of the computed Arcs, the third Differences be alſo equal to nothing; if in the other caſe only, the fourth Differences are equal to nothing; here the fourth Differences will alſo vaniſh, and ſo of the reſt; and the points of diviſion of the Arcs will be the places of a Comet analogous to the moments of Time: But the intermediate Arcs may ſafely be reckon'd to be gone thro' equably by a Comet.

For thus it will happen, that every Arc 12, 23, 34, or 45, will be gone thro' by the Comet mov'd after the ſame Law that it moves thro' the whole Arc 17: For from the conſtruction every one of the foreſaid Arcs, and every part of them containing as many parts as the Arc 17 contains of the parts 12, 23, 34, *&c.* is divided after the ſame manner as the Arc itſelf in the Points 2, 3, 4, 5, 6; and as the Times between the moments, at which a Comet comes to thoſe Points, are equal, as alſo thoſe in which the Arcs 12, 23, 34, 45, *&c.* are deſcrib'd; it is plain that a Comet will come to the found Points in Times agreeable to the known Times, becauſe a Comet is mov'd after the ſame tenour when it deſcribes any of thoſe Arcs, as it was mov'd when it deſcribed the preceeding or following Arcs, which plainly happens in every natural motion. But the moments of Time being known, that a Comet comes to known

Points, and it being beſides ſuppos'd that a Comet goes equably thro' the little intermediate Arcs, which may ſafely be allow'd if theſe Arcs (namely the parts of the Arcs 12, 23, 34, 45, *&c.*) be ſmall enough, that is, if *T* be ſuppos'd divided into a number of parts great enough; any intermediate Time being given between the moments that a Comet was at the Points 1 and 7, the Place which the Comet is in will be known: And on the other hand the Time will be known when the Comet comes to a given Place between 1 and 7.

SCHOLIUM.

Aſtronomers commonly make the foreſaid Time *T* a natural Day. But if a Comet cannot be exactly obſerv'd after 24 Hours are paſs'd, they reduce the place that it is obſerved to be in one or two Hours before or after to the end of the natural Day, ſuppoſing the motion made in the mean time to be equable, which in ſo ſmall a Time can cauſe no ſenſible Error. In this Practiſe they divide the natural Day, or the Time expreſs'd by *T* in the Propoſition, into four parts, or parts of ſix Hours, which they ſuppoſe enough, unleſs the motion of the Comet be very much accelerated or retarded, and they wou'd proceed very exactly; in which caſe they divide that Time, (at leaſt thoſe ſix Hours in which the given Time is found, for which the Comet's Place is requir'd) into Hours, and at laſt very ſafely ſuppoſe that the motion of a Comet during one Hour is equable.

It often happens that in the moſt exact obſervations of a Comet, the third Differences of the Places reckon'd from a fix'd beginning, which are denoted in the ſaid *Fig.* 24, by

P, *Q*, *R*, *S*, are equal to nothing, or that their ſecond Differences *G*, *H*, *K*, *L*, *M*, that is, the firſt differences of the Arcs, *A*,*B*,*C*,*D*,*E*,*F*, are equal to each other: Therefore Aſtronomers divide the Arc which a Comet goes thro in 24 Hours, into 24 parts, ſuch that the Arc which belongs to the 12th Hour ſhall be the 24th part of the Arc which the Comet is obſerv'd to go thro' in the ſpace of a natural Day; or rather (ſince this arc is not preciſely in the middle, but together with the arc of the following Hour takes up the middle place) the diviſion is made in ſuch manner that the two arcs belonging to thoſe two Hours taken together, ſhall make a twelfth part of the arc deſcribed in the ſpace of a natural Day, and ſo that the firſt differences of thoſe parts ſhall alſo be equal to each other, or there ſhall be no ſecond Differences; and at laſt the greater parts ſhall be on the ſame ſide in reſpect of the leſſer, as they are in the arcs *A*,*B*,*C*,*D*,*E*,*F*, and to every Hour they give one of thoſe parts in the ſame order. For it will by that means happen, that a Comet being after the ſame tenour ſimilarly accelerated or retarded, ſhall deſcribe any arc in the ſpace of one Day, as it deſcrib'd the arc 15 in the ſpace of four Days: For there is only need of the Obſervations of four Days near one another to know whether the Differences of the arcs, 12, 23, 34, 45, are equal to each other. And if the propos'd Time falls between four other Days, they again inveſtigate what Differences become equal; and ſo go on as before.

This Method is not only made uſe of to determine the place of a Comet in its apparent Way for an intermediate Time, but alſo (if you divide the motion of the Comet into that of Longitude and that of Latitude, Right Aſcenſion and Declination, as is uſual, or any way elſe) that from ſome of its Places obſerv'd according to any of theſe Circumſtances, its Place may be found according to the ſame for an intermediate Time given: For here it is only ſuppos'd that the Comet's motion, conſider'd according to any Direction, goes on after the ſame tenour for any one ſingle Day, as it does for ſeveral Days before and after it.

This way of proceeding by Differences is very uſeful alſo in many other things, (as for *Example*, for calculating Tables to ſmaller Diviſions) which have been treated of at large by the Reverend Father *Gabriel Mouton* of *Lyons*, who teaches the manner of working by theſe Differences in his Book of *Obſervations of the apparent Diameters of the Sun and Moon.*

PROPOSITION XXVI.

TO determine by Obſervation the Trajectory of a Comet moving in a Parabola. [Fig. 23.]

Let S denote the Sun, $Tt\tau$ the Orbit of the Earth: Chooſe three Obſervations, diſtant from one another almoſt equal intervals of time, yet ſo that the Time may be greater where the Comet moves the ſloweſt: Let the Points T, t and τ be Places of the Earth for the ſaid Times; that is, T a Place of the Earth at the firſt Obſervation, t is Place at the ſecond Obſervation, and τ at the third. If ſuch Obſervations are not given, the Place of the Comet is (by *Prop.* 25.) to be found for One Time in thoſe circumſtances in reſpect of Two other Times at which the Places agreeable are obſerv'd. Joyn $T\tau$, and from t let tN fall perpendicular to it: Let the right lines TA, tC, τB, be three obſerv'd Longitudes; therefore theſe three right lines are given in poſition. In that of them tC which is in the middle, take any where the Point C for the mark of a Comet's Track on the Plane of the Ecliptic: From C to the Plane of the Ecliptic draw perpendicularly the Right line $C\gamma$ equal to the Tangent of the known Latitude of the Comet in the ſecond Obſervation, tC being ſuppos'd the Radius: Joyn γS, CS, in the laſt of which towards S take CD in ſuch manner that it may be to the known tN (which is the vers'd Sine of the known arc Tt) as a Solid, whoſe Baſe is the Square of St and Altitude a Right line SC, to the Cube of the Right line $S\gamma$: Thro' the Point D draw (by *Prop.* 8.) the Right line ADB meeting with TA, tB at A and B, ſo that AD may be to DB as the Time between the firſt and ſecond Obſervation is to the Time between the ſecond and third; and A and B will be nearly the Mark of the Comet's Way on the Plane of the Ecliptic in the firſt and third Obſervation, if ſo be that the Point C rightly taken be its Place in the ſecond Obſervation. Becauſe by ſuppoſition C is the Mark of the Comet in the Plane of the Ecliptic in the ſecond Obſervation, and $C\gamma$ the Tangent of the Latitude obſerv'd from t the Radius being tC; it is plain, that γ is the Comet's place in the Mundane ſpace, and γS the diſtance of a Comet from the Sun in the ſecond Obſervation. Thro' D draw $D\delta$ parallel to $C\gamma$, which (by *Prop.* 8. *El.* 11.) is perpendicular to the Plane of the Ecliptic, and is found in the Plane of the Triangle $SC\gamma$: Let it meet $S\gamma$ at δ. The ratio of the Right line $\gamma\delta$ to the Right line tN, is compounded of the Ratio of $\gamma\delta$ to CD and the Ratio of CD to tN: But (by *Prop.* 2. *El.* 6.) $\gamma\delta$

is to CD as γS to CS, and CD is to tN as $St^q \times SC$ to the Cube of the Right line $S\gamma$; whence $\gamma\delta$ is to tN in a Ratio compounded of $S\gamma$ to CS and the Ratio of $St^q \times SC$ to the Cube of the Riight line $S\gamma$: And the Ratio compounded of theſe two, is the ſame with the Ratio of St^q to $S\gamma^q$; and therefore $\gamma\delta$ is to tN as St^q to $S\gamma^q$. But (by *Corol.* 1. *Prop.* 25. *B* 1.) tN is nearly the Space thro' which the Earth being let fall from reſt wou'd by the Force of its gravity fall towards the Sun during the time that it deſcribes half the Arc $T\tau$, if it was every where urg'd with the ſame accelerating Gravity uniformly continu'd which acted upon it in the Place t, becauſe tN is nearly equal to the ſquare of the arc Tt applied to the Diameter of the *Magnus Orbis*: And from *B.*1. the accelerating gravity towards the Sun in the Place t is to the accelerating Gravity towards it in the Place γ, as $S\gamma^q$ to St^q, namely in a duplicate reciprocal Ratio of the Diſtances from the Sun, and the Spaces which are gone thro' in the ſame time by the action of thoſe Forces, in deſcent towards the Sun, are in the ſame Ratio as theſe Forces; and therefore the Right line $\gamma\delta$ is the ſpace thro' which the Comet let fall from reſt at γ towards the Sun wou'd fall, in half the time that the Earth deſcribes the arc $T\tau$, if it was every way acted upon by the ſame accelerating Gravity uniformly continu'd as it is urg'd by in the Place γ; then $\gamma\delta$, will be nearly the ſpace which a Comet by its fall towards the Sun would really deſcribe in half the time that the Earth deſcribes the arc $T\tau$ of its Orbit; that is, in half the time that a Comet deſcribes an arc of its Trajectory intercepted between the two Longitudes TA and τB, and therefore the Point δ is in the

Chord of that Arc. And therefore if as well the marks of $\alpha\gamma\beta$ the arc aforeſaid of the Trajectory, intercepted between the ends of the obſerv'd Lengths *TA* and τB, as the marks of the point δ be conceiv'd in a Plane by letting fall perpendiculars to the Plane of the Ecliptic, namely *ACB* and *D*; the point *D* wou'd be in the Chord of the arc *ACB*. But the Chord of the arc *ACB* is ſuch (by *Corol.* 2. *Prop.* 18.) as to be divided by *SC* almoſt in a ratio of the Times in which the Comet reduc'd to the Ecliptic deſcribes the arc *AC*, *CB*; and by conſtruction the right line *ADB* is divided in the ſame ratio, and it is the only Line which can be drawn with theſe conditions. Since then the Chord of the arc, which is the mark of part of a Comet's Trajectory intercepted between the ends of the Lengths *TA*, τB, muſt be terminated by the right lines *TA*, τB, and go thro' *D*, and be divided at *D* in a ratio of the two Times betwen the three Obſervations; and ſince the right line *ADB* has all theſe three Conditions, and is the only line which has theſe Properties; it is plain that the right line *ADB* (drawn as before mention'd) is the Chord of the ſaid arc, therefore the points *A* and *B* are nearly the mark of the Way of a Comet in the Plane of the Ecliptic in the firſt and third Obſervations, if *C* be its true mark in the ſecond Obſervation.

But becauſe the point *C* is not taken near enough, and tho' it had been the true Point, yet the Points *A* and *B* deduc'd from it are not accurately enough defin'd; we muſt from theſe, however irregularly determin'd, correct all at laſt. For which end, in *Fig.* 29. (where the ſame conſtuction is ſuppos'd as in the foregoing, leaving out only ſuch things as were neceſſary to prove the things above, but now left out to avoid confuſion) at *M* the middle point of the right line *AB* raiſe *MI* perpendicular to *AB*, which the right line *CI* will meet with in *I*, being drawn through *C* parallel to *BA*; draw *SI* cutting *AB* in *K*, and fill up the Parallelogram *MIVK*, and take *MQ* the triple of *MK*: Draw *QS* and produce it to *E* till *QE* be equal to $3QS+3IK$. The point *E* being determin'd, let the Signs or Letters *A*,*M*,*D* and *B*, be ſuppos'd blotted out; let *CE* be joyn'd, and in it towards *E* take the new *CD* in a duplicate Ratio of the right line *SC* to a right line equal to $SV+\frac{1}{3}IK$, and thro' the point *D* lately found (by *Prop.* 8.) draw a new line *AB* terminated at *TA* and τ*B*, in ſuch manner that *AD* may be to *DB* as the Time between the firſt and ſecond Obſervation, to the Time between the ſecond and third Obſervation, the Point *A* and *B* will be more accurately than before the Marks of the Comet in the Plane of the Ecliptic at the firſt and third Obſervation, if the point *C* be ſo rightly taken as to be its Mark in the ſecond Obſervation. Let us imagine from every point of the Comet's Trajectory perpendiculars to be let fall on the Plane of the Ecliptic, or (what is the ſame) let us imagine that a Cylindric Surface is rais'd perpendicularly upon the plane of the Ecliptic, and goes thro' the Comet's Trajectory; by the points mark'd on the Plane of the Ecliptic by the aforeſaid perpendiculars will be drawn (or the common Section of the ſaid cylindric Surface with the plane of the Ecliptic will be) a parabolic Curve *ACB*, whoſe Focus is *S*, or a Curve very little different from a Parabola: The Chord of that Arc *ACB* comprehended

(as

(as we have before ſhewn) between the right lines TA, τB, is nearly the before drawn AB, whoſe middle Point is M. And as in the ſaid arc the point C is taken not far from the Vertex of the Segment ACB,(for, for that reaſon the Times of theſe Obſervations are taken at almoſt equal invervals from one another, yet in ſuch manner that that Time may be ſomething greater where the Comet moves ſlower) and a right line is drawn parallel to BA meeting with a perpendicular drawn from M to AB at I, and SI drawn cutting AB in K, and the Parallelogram $IMKV$ fill'd up; and ſince moreover upon account of the immenſe diſtance of the Sun, the diſtance of the points M and V vaniſhes in compariſon of it, that is the ſaid Points come together; (by *Corol. Prop.* 19.) V is the Vertex of the Segment ACB. If VS be drawn cutting the Chord at Υ (*by reaſon* of the immenſe diſtance of the point S) $V\Upsilon$ will be nearly parallel to IK, therefore $K\Upsilon$ equal to IV, and therefore to MK; but by conſtruction $M\mathcal{Q}$ is the triple of MK, whence it is alſo the triple of $K\Upsilon$, and therefore alſo of the remaining $\Upsilon\mathcal{Q}$; for which reaſon (by *Corol.* 2. *Prop.* 18.) $\mathcal{Q}S$ being drawn is the ſame as $\mathcal{Q}S$ in *Prop.* 18. and its *Corollaries*, that is, in $\mathcal{Q}S$ produc'd towards S you find the point E, from which any right line being drawn cutting the Chord AB, and the Arc ACB, cuts the Chord into Segments that have to one another the ratio of the Times, in which the adjacent arcs are deſcrib'd by a Comet. Moreover by conſtruction $\mathcal{Q}E$ is equal to $3\mathcal{Q}S + 3IK$, and IK equal to MV (they being the Diameters of the ſame rectangular Parallelogram;) whence $\mathcal{Q}E$ is equal to $3\mathcal{Q}S + 3MV$. And therefore (by

Corol. 3. *Prop.* 18.) the point E above determin'd is the ſame, from which a right line being drawn any how divides the Chord AB into Segments having the ſame ratio as the Times in which the two parts of the Arc AB mark'd out by the ſame right line produc'd are deſcrib'd by a Comet, which before we had falſly (tho' not too erroneouſly) ſuppos'd of the Focus S. Having therefore, to avoid confuſion, rubbed out the former CD drawn towards S, a new line muſt be drawn towards E, which is to the former as the ſquare of SC to the ſquare of $SV+\frac{1}{3}IK$, that is, (by reaſon of the equal lines IK and MV) to the Square of $SV+\frac{1}{3}MV$; and SC is nearly equal to SV: Wherefore the new CD is to the former CD as the ſquare of SV to the ſquare of SL, ſuppoſing VL to be the third part of MV or IK, as in the conſtruction of *Prop.* 20, and *Prop.* 21. But (by *Prop.* 42. *B.* 1.) the accelerating Gravity towards the Sun in the place L is to the accelerating Gravity towards it in the place C (or V) as the ſquare of SC (or SV) to the ſquare of SL: And as the accelerating Gravities towards the Sun in different Diſtances, ſo are (the ſaid Force remaining) the Spaces run thro' by falling during the ſame Time towards the Sun; and therefore the new CD is to the former CD as the ſpace gone thro' by falling towards the Sun (the ſame accelerating Force acting which acted at the place L,) during half the Time that the Comet deſcribes the Arc intercepted between the length TA and τB, is to the ſpace gone thro' in falling towards the Sun, the ſame accelerating Gravity acting which acted at C. But the Conſequents of this Analogy are equal, therefore the Antecedents are alſo equal; that is, the new drawn CD is equal to a ſpace gone thro' by a heavy Body falling towards the Sun half the Time that a Comet deſcribes the Arc *ACB* in the Ecliptic, by the action of the ſame accelerating Gravity, which obtain'd at the diſtance *SL* from the Sun uniformly continu'd. But (by *Prop.* 21.) the ſpace thro' which a Body falls towards the Sun, in half the time that a Comet deſcribes the Arc *AVB*, (for now we conſider the motion of a Comet as made in the Plane of the Ecliptic, looking at the mark of a Comet's Trajectory in the Ecliptic inſtead of the Trajectory itſelf) when it is urg'd by the ſame accelerating Gravity towards the Sun which acted at *L* uniformly continu'd, is equal to *VΥ* the Segment of the right line *VS*, intercepted between the Vertex *V* and the Chord *AB*; therefore nearly equal to the Segment *CE* of *CD* intercepted between the Point C (near to *V*) and the Chord *AB*. Whence the point *D* is in the Chord *AB* much more accurately than before; that is, the chord of an Arc, which in the plane of the Ecliptic, is the mark of a Comet's Trajectory intercepted between the lengths *TA* and *τB* goes nearly thro' the point *D* determin'd laſt. Moreover (by *Prop.* 18.) the foreſaid Chord drawn thro' *D* muſt have ſuch a ſituation between *TA* and *τB* that *AD* may be to *DB* as the Time in which a Comet deſcribes an arc of the Ecliptic between the lengths *TA* and *tC*, to the Time in which it deſcribes the Arc intercepted between *tC* and *τB*: But *AB* is (by *Prop.* 8.) drawn in ſuch manner thro' *D* that *AD* is to *DB* in that ſame ratio, namely as the Time between the firſt and ſecond Obſervation is to the Time between the ſecond and third; therefore *AB* is rightly drawn and as it ought to be, namely going thro *D*, and divided at *D* as is ſaid before.

fore. And therefore, if the point *C* is rightly taken for the mark of the Comet's Place at the ſecond Obſervation, the points *A* and *B* are nearly the marks of its Place at the firſt and third Obſervations.

Now what remains, is to know whether the point *C* in the middle of the Longitude be the mark of the Comet rightly taken; if not, the error ariſing from it muſt be corrected, the other things hitherto done remaining unalter'd. Having rubb'd out the former Parallelogram *IMKV* made on the former leſs accurate Chord *AB*, make anotheron the laſt and more accurate Chord *AB* with the ſame conſtruction as before. From *A*, *M* and *B* draw $A\alpha$, $M\mu$ and $B\beta$, of which let $A\alpha$ be the Tangent of the known Latitude in the firſtObſervation for the Radius *TA*, and $B\beta$ the Tangent of the Latitude at the third Obſervation for the Radius τB: Draw $\alpha\beta$, with which $M\mu$ meets at μ. If theTrapezium $AB\beta\alpha$ be erected perpendicularly to the plane of the Ecliptic,the right line *AB* remaining, the points α and β (as was before ſhewn) are the places of a Comet in the Mundane Space,if the point *C* is its Mark in the Plane of the Ecliptic in the ſecond Obſervation, and the Plane going thro' the three points *S*, α and β, be the Plane of the Comet's Trajectory: Therefore the right line $\alpha\beta$ is the Chord of the Arc of the Parabolical Trajectory gone thro' by the Comet between the firſt and third Obſervation,and $S\alpha$ $S\beta$ are reſpectively the diſtances of a Cometfrom the Sun at the firſt and third Obſervation, that is,the true diſtance of every point of the Comet's Trajectory from the Sun, is the Hypothenuſe of the rectangular Triangle, one ſide of which is the diſtance from the Sun of the Mark of that point in the Plane of the Ecliptic,and the other a perpendicular drawn from that point on the Plane of the Ecliptic to the point which terminates the Trajectory. As ſome of thoſe perpendiculars are longer, as $B\beta$, and ſome ſhorter, as $A\alpha$, take a middle one between thoſe, as for Example, $M\mu$; and commonly ſpeaking, the diſtance of any point of the Comet's Trajectory from the Sun, will be nearly the the Hypothenuſe of a rectangular Triangle, one ſide of which is the diſtance of the analogous point deſcrib'd in the Mark of the Trajectory, and the other the right line $M\mu$ itſelf. Which being done in MA (or MA produc'd) take MH equal to $SV + \frac{2}{3} IK$, that is, equal to SR, ſuppoſing LR equal to LV, and draw $H\mu$; this line will be nearly equal to the diſtance from the Sun of a point of the Trajectory (whoſe mark is V) encreas'd by the double of the right line intercepted between the point and the Chord of the Arc of the Trajectory, namely $\alpha\beta$ in the erected Trapezium $AB\beta\alpha$, that is, the right line $H\mu$ is equal to a right line in the Plane of the Comet's Trajectory, analogous to SR in its Mark in the Plane of the Ecliptic; that is, $H\mu$ is equal to the right line SR in the Parabola of *Prop.* 20. By the Corollary of the 17th Propoſition, let the velocity of a Comet when it moves in its Parabolical Trajectory at a diſtance from the Sun equal to the right line $H\mu$, be compar'd wiih the velocity of the Earth towards the Sun, and the line which the Comet moving equably with the ſaid Velocity, wou'd go thro' in the whole time that the Earth deſcribes the Arc $Tt\tau$, or in the whole Time that a Comet goes thro' the Arc ACB in the Ecliptic, be defin'd in parts of the Arc $Tt\tau$ deſcrib'd by the Earth at the ſame time; which may be eaſily done in the following manner. By Calculation ſubtract the length of the Arc $Tt\tau$ deſcrib'd by the Earth between the firſt and third Obſervation, ſuppoſing any round number for the mean diſtance of the Earth from the Sun; a length (as for *Example* Z,) which is to this Arc in a ſubduplicate ratio of the Diameter of the *Magnus Orbis* to the known right line $H\mu$, and which therefore is given, is the length requir'd; namely that which a Comet equably carried with the ſame velocity that it has when it deſcribes its Parabolic Trajectory at a diſtance from the Sun equal to $H\mu$ wou'd go thro' in the time that a Comet does really go thro' an Arc whoſe Chord is $\alpha\beta$: For (by *Prop.* 17.) the velocity of a Comet at the ſaid diſtance $H\mu$ is to the velocity of the Earth in the above-mention'd Ratio. But (by *Prop.* 20.) the ſaid Length Z is equal to the Chord of an Arc which a Comet does really deſcribe in that time: Therefore if Z be found equal to the Chord $\alpha\beta$, it is certain that the point C in the Longitude obſerv'd the ſecond time was rightly taken for the Comet's place; therefore *A*, C and *B*, are the three places or Marks of the Comet in the Plane of the Ecliptic.

If Z be not found to be equal to the right line $\alpha\beta$, that is, if the point C be not rightly taken in t C the Longitude obſerv'd the ſecond time; in $\alpha\beta$ (or $\alpha\beta$ produc'd if occaſion require) take $\alpha\varphi$, $B\pi$ equal to Z, take alſo AF, BP, equal to $\alpha\varphi$, $B\pi$, in ſuch manner that F and φ may be on the ſame ſide of the right line $A\alpha$, and P and π on the ſame ſide of the right line $B\beta$. Then by the ſame method, that from the aſſum'd point C were found the points *A*,*B*,*D*,*F*,*P*, from other aſſum'd points 2 C and 3 C, find the new *2A*,*2B*,*2D*,*2F*,*2P*, and *3A*,*3B*,*3D*,*3F*,*3P*, as in the 26th Figure; where the ſame things remaining, and the ſame letters being us'd as before, thoſe things are only left out of which we have no more occaſion to ſpeak. If *Z* be leſs than $\alpha\beta$, or *AF*, or *BP* leſs than *AB* (as it happens in this 26th *fig.*) the point 2 *C* muſt be taken nearer to the point σ (in which *TA* and and τ *B* meet;) and ſo on, if at leaſt 3 *A*, 3 *F*, be greater than 3 *A* 3 *B*: But if on the contrary *AF* be greater than *AB*, τ2 *C* muſt be taken greater than τ*C*; and ſo on if at leaſt 3 *A* 3 *F* be leſs than 3 *A*, 3 *B*. Thro' the points *F*, 2 *F* and 3 *F* draw a Circle, which cuts the right line τ *B* between *B* and 3 *B*; as for *Example* in *O*, if the new points 2 *C*, 3 *C* be taken in the manner before ſhewn. Likewiſe thro' the points *P*, 2 *P* and 3 *P*, draw a Circle cuting the right line *TA* in *X*; the points *O* and *X will be* the Places of a Comet reduc'd to the Ecliptic, or the Marks of the Comet at the firſt and third Obſervation. For it has been above demonſtrated, that the points *A* and *B* are the Marks of the Comet in the Plane of the Ecliptic at the firſt and third Obſervation, if *C* be its Mark in the ſecond: The ſame does likewiſe obtain in 2 *A*, 2 *B* and 2 *C*; and alſo in 3 *A*, 3 *B* and 3 *C*. It is alſo demonſtrated that *C* is truly the Mark of the Comet's Place in the ſecond Obſervation, if *Z* be equal to $\alpha\beta$; that is, if either of the right lines *AF* and *BP* be equal to *AB*; that is laſtly, if the points *B* and *F* coincide, and *A* and *P*. And therefore, if (things remain as before) the points *B* and *F* coincide, ſuch a point *B* will be the Mark of the Comet at the third Obſervation: Likewiſe when *A* and *P* coincide, ſuch a point *A* will be its Mark at the firſt Obſervation.

ſervation. And to make theſe coincide, thro' the three rightly found Points *F*, 2 *F* and 3 *F*, a Circle is drawn, that is, a very ſimple line going thro' three Points, cutting the right line ♈ *B* in *O*. Since then the point *O* be as well in the place of the point *B*, namely in the right line ♈ *B*, as in the place of the point *F*, namely in a Circle; when the point *B* happens to be at *O*, the point *F* will alſo be there, that is, in that caſe the Points *B* and *F* will coincide; and therefore the point *O* is the true Mark of the Comet in the plane of the Ecliptic at the third Obſervation, all the requiſite conditions agreeing with it. Likewiſe for the ſame reaſons the point *X* is the true Mark of a Comet in the firſt Obſervation. Therefore, if from the point *O* to the plane of this Figure, which repreſents the Plane of the Ecliptic, be drawn the perpendicular *O* ω, equal to the the Tangent of the known Latitude in the third Obſervation for the Radius ♈ *O*; ω will be the true place of a Comet in its Trajectory. Likewiſe *X* ξ perpendicular to the Plane of the Ecliptic rais'd at the point *X*, equal to the Tangent of the Latitude at the firſt Obſervation of the Radius *T X*, will ſhew the point ξ the other place of the Comet in its proper Trajectory. With the Focus *S* thro' the points ω and ξ (by *Prop.* 24.) deſcribe a Parabola; and it will be the Comet's requir'd Trajectory. But becauſe there are two Parabolas deſcrib'd about the Focus *S* paſſing thro' the two Points ω and ξ, which of theſe is the Trajectory of our Comet will appear from any obſervation of the Comet: For its Place deduc'd from one of thoſe Parabola's will agree to the obſerv'd Place, but taken from the other will not. Therefore we have from Obſervations determin'd the Trajectory of a Comet moving in a Parabola. *Q. E. F.*

For one may in TA or τB, chooſe Points at pleaſure, as was done here in tC, and work with the Points deduc'd from hence in the two remaining lines; after the ſame manner that we work'd with the Points in TA, τB.

In the foregoing Inveſtigation, whether Arithmetical or Graphical, or compounded of both, there is no regard had to the Monthly Parallax of the Sun ariſing from this, *viz.* That it is not the Earth's Center, but the common Center of Gravity of the Earth and Moon which is carried in the *Magnus Orbis*; for this Monthly Parallax of the Sun, three times greater than the Diurnal, is ſenſible in the defining of the Orbits of ſome Comets: Therefore we muſt by it correct the Comet's obſerv'd Places.

If the Elliptic Orbit of a Comet be found to agree better let it be thus deſcrib'd: Find the Mark of a Comet in the Plane of the Ecliptic in the ſecond Obſervation; and it will be the ſame in order and ſituation between the points C, $2C$, $3C$, as the point O was between B, $2B$, $3B$, or as X between A, $2A$, $3A$: from the Mark thus found, raiſe perpendicularly to the Plane of the Ecliptic a line which is the Tangent of the Latitude in the ſecond Obſervation for a Radius equal to the diſtance between t and the ſaid Mark; the upper end of the ſaid Perpendicular, or the end of it above the Plane of the Ecliptic, is the Place of the Comet in its Trajectory obſerv'd the ſecond time. An Ellipſe with the Focus S drawn thro' the points ω, ξ and the lately found Point (by *Prop.* 29. *B.* 3.) will be the requir'd Orbit of the Comet.

SCHOLIUM.

The foregoing Method for determining the point O or the point *X* is univerſal, and of great uſe in ſeveral Phyſical Aproximations. For the moſt part three Points, as here *A*, 2 *A* and 3 *A*, found according to Art from the three aſſum'd ones, *viz.* C, 2 C and 3 C, muſt be ſo poſited that the requir'd Point may be between their Extremities *A* and 3 *A* which is the reaſon why from the three Points, *P* alone muſt be plac'd on the other ſide of the correſponding point *A*:) For thus will the requir'd Point be more certainly and nearly determin'd by the interſection of a Circle with the right line *A* 2 *A*, in which it muſt neceſſarily be, than if the requir'd Point being beyond *A* and 3 *A* was to be determin'd by the Interſection of a right line with the Circle beyond the points *A* and 3 *P*; for the ſame reaſon from ſome obſerv'd place of one Comet (or of any other mov'd according to any Rule) its Place and its intermediate Time whatever is more certainly defin'd, than its Place or Time before the firſt or after the laſt Obſervation.

PROPOSITION XXVII.

TO *determine the Point that is fitteſt to be taken, the firſt, ſecond and third time for the Comet's Mark* (*or the Mark of its Place*) *on the Plane of the Ecliptic.* [Fig. 27.]

The Conſtruction of the foregoing Problem is univerſal: But in practice we muſt not take any points C, 2 C for the Mark of the Comet in the ſecond Obſervation, but we muſt take thoſe Points nearly true, if we can by any means know whereabout the true ones ſhou'd be. And that may be done by help of the 12th *Prop.* by four obſervations of a Comet. For a right line being drawn, whoſe three parts intercepted between four right lines given in poſition, have the ſame ratio as three Intervals of Time between four obſervations, will not differ very much from that part of a Comet's Trajectory which is then run through. Therefore its Interſection with a right line that repreſents the Longitude in the ſecond obſervation, will very fitly be taken for the mark of a Comet in that ſecond Obſervation.

If you know within a little the quantity of the Angle which the mark of the Orbit of a Comet contains with a right line joyning the Earth and the Comet in the ſecond Obſervation, or its equal (by *Prop.* 17. *B.* 1. *Conic*) namely, $A\delta t$, which the Chord AB contains with the right line tC, which may be done by help of the 9th *Prop.* or more accurately by the Schol. of *Prop.* 13; then the point C to be taken at firſt will be determin'd after the ſame manner: Draw the right line AB terminated at each end by the right lines TA, τB given in poſition, and cutting the right line tC, which is alſo given in poſition at δ, in an Angle equal to the given Angle; in ſuch manner that it may be to $\sqrt{2} \times T\tau$ the Chord (that is, nearly to $\frac{7}{5}$ of $T\tau$) in a ſubduplicate ratio of St to $S\delta$; and draw thro' S, the right line SDC, whoſe part DC interjected between the Legs of the Angle $A\delta C$ muſt be equal to the right line tN, which is an univerſal Problem of the Ancients *concerning Inclinations*, and which is done by their Method, or by the 7th Section of the 5th *Optical Lecture*, or in the 3d Section of the 6th *Geometrical Lect.* of Dr. *Barrow*. The Point C thus determin'd is that very ſame which is conveniently taken the firſt time for the mark of the Comet in the Plane of the Ecliptic. Suppoſe that C is the mark of the the Comet in the Plane of the Ecliptic, and a Parabolic Arc going through A, B and C is the mark of an Arc of the Trajectory deſcribed between the firſt and third Obſervation: Therefore ſuppoſing that the accelerating Gravity towards the Sun is the ſame or equal at the diſtance of the Earth from the Sun, and at the diſtance of a Comet from it, (that is, in *Galilæus*'s Suppoſition of Gravity, which is nearly true,) CD and tN are equal, as being the ſpaces in falling toward the Sun gone thro' at the ſame time by a Comet and by the Earth; AB the the Chord of the Parabola will (by *Prop.* 22.) be to $\sqrt{2} \times T\tau$ the Chord of the Circle, whoſe Center agrees with the Focus of a Parabola, in a ſubduplicate Ratio of the right line St to the right line SD; But AB is drawn to $\sqrt{2} \times T\tau$ in a ſubduplicate Ratio of St to $S\delta$, and alſo AB cuts the right line tC in a due Angle, and therefore CD is equal to tN: therefore the right line AB has nearly all the requir'd Conditions to become the Chord of the Arc, which is the Mark of the Comet's Trajectory, intercepted between the firſt Longitude TA and the third τB; and therefore the point C has all the Conditions requir'd to be nearly the Mark of the Comet at the ſecond Obſervation. Therefore the point C is rightly taken for the Mark of the Comet (tho' it muſt afterwards be corrected) at the ſecond Obſervation.

The Point C therefore, which is to be taken the firſt time, being determin'd, the Points which are afterwards taken (namely, $2C$ and $3C$ [*Fig.* 28.]) are not to be taken without conſideration or allowances; neither does the Caution

tion given in the foregoing Propoſition, namely, that the point $2C$ be taken nearer the point σ when the Line Z is leſs than AB, and ſo on the contrary, ſuffice; but when we have proceeded ſo far that AB rightly drawn be equal to the Longitude Z agreeable to it; in τC take the Point $2C$ in ſuch manner that the diſtance $\sigma 2C$ may be to the diſtance σC in a Ratio compounded of the Ratio of Z to AB, and a ſubduplicate Ratio of SC to $S2C$. The cauſe why the firſt compounding Ratio is made uſe of, is manifeſt from what is ſaid before: For ſince AB is to be found ſo as to be equal to Z agreeable to it, if it be greater or leſs it muſt be diminiſh'd or encreas'd till they become equal; now they wou'd be equal by the firſt compounding Ratio alone, if Z was conſtant and invariable; but as the Longitude Z is always different as AB is different, therefore the ſecond compounding Ratio is neceſſary. Now it is plain (from *Prop.* 23.) that in Parabolas the Chords of the Arcs ACB, $2A$, $2C$, $2B$ (in which CD, $2C$, $2D$ tending to the Focus and intercepted between the Vertex and the Chord, are equal) are in a ſubduplicate Ratio of the right lines SC, $S2C$; and it has been ſhewn above that the right lines CD, $2C$, $2D$ are nearly equal (namely being the Spaces which a Comet falling towards the Sun wou'd deſcribe at different diſtances from it in the ſame time,) and alſo that the Mark of the Comet in the ſecond Obſervation is near to the Vertex of the Arc ACB, that is, of a part of the Mark of the Trajectory of a Comet deſcrib'd between the firſt and third Obſervation. But alſo the Point $2C$ found after that manner is not ſaid to be the accurate Mark of the Comet in the Plane of the Ecliptic, but it is nearly true, and but very little remov'd from a Point truly accurate.

After the ſame manner may be determin'd the Point 3 *C* from 2 *C*, as 2 *C* was determin'd from *C* if we find three Points, as was done in the foregoing Propoſition; but if we work this way, it will hardly be needful to make uſe of a ſecond operation: and a right line joyning the two Points *F* and 2 *F*, determin'd from the two aſſum'd points *C* and 2 *C* by the foregoing, will cut *τ B* in the point *O* (to which they are very near) the accurate Mark of the Comet in the third Obſervation, and the right line *P* 2 *P* will likewiſe cut *TA* in the point *X*, the Mark of the ſame Comet at the firſt Obſervation, as we have ſhewn before, that it muſt happen when *C* and 2 *C* are near to the true Point. Therefore we have determin'd the Point which (in the operation of the foregoing *Prop.*) is moſt conveniently taken for the Mark of the Comet in the plane of the Ecliptic the firſt, ſecond and third time.

In nearly determining the points *C* and 2 *C*, after the aforeſaid manner, in Graphical Operations we muſt often proceed by Tryals, where a more accurate or a Geometrical method is wanting; neither is it needful here to make uſe of Curves very difficult to be drawn, which the conſtruction wou'd require.

PROPOSITION XXVIII.

TO determine the the poſition of the line of Nodes of the Trajectory of a Comet, and the Time when a Comet is in the Nodes. [*fig.* 29.]

Things remaining the ſame, and being denoted by the ſame Letters as in the foregoing Propoſitions, the right lines *OX*, *ωξ* muſt be produc'd till they meet in ☊: Draw *SO*, *SX*, *S*☊.

All the Points were before given, namely S, O and X, and therefore as well the ſides as the angles of the Triangle SOX; and therefore the Angle SX☊, which is an Angle next on the the other ſide of one of them on the ſame line. Moreover, if thro' ξ you draw ξo parallel to XO meeting with the right line $O\omega$ at o, the Triangles $\omega o\xi$ and ξX☊ will be equiangular; and therefore from the known ſides $o\xi$ (equal to OX) and $o\omega$ (the difference of the known right lines ωO and ξX) together with the right angle $\omega o\xi$, the remaining Sides and Angle will be known, and alſo the angles of the Triangle ξX☊: But in this Triangle, one ſide $X\xi$ is given, therefore the other Sides will alſo be given; namely, X☊ and ξ☊. Then in the Triangle ☊XS you have the Sides XS, X☊, with the Angle between them SX☊; therefore you will alſo know the Angle XS☊. *But as was* ſaid before, the poſition of the right line SX is given, or the Angle which it makes with TX, whoſe poſition is known by Obſervation, (for in the Triangle XTS, the Sides TS and TX, are given and alſo the Angle XTS, which is the diſtance between the known Place of the Sun and the Place of the Comet obſerv'd the firſt time, whence TXS is known) and therefore the poſition of the Right line ☊S☋ will be known, or the Places of the Nodes ſeen from the Sun. If the Right line $O\omega$, $X\xi$ are equal, the Line of the Nodes is parallel to OX, and therefore known in poſition.

To determine the time that a Comet is in the Node, let $\omega\xi$ [*Fig.* 30.] be the Trajectory of a Comet deſcrib'd by *Prop.* 26. thro' the Points ω and ξ, in whoſe Plane are found the right lines $S\omega$, $S\xi$, and let the before found Line of Nodes

☊S☋

☊ *S* ☋ meet with the Trajectory in ☊ and ☋; and (by *Prop.* 14. *B.* 1.) the Interval of the Time between the firſt Obſervation and the moment that the Comet comes to the Node ☊ will be to the Interval of Time between the firſt and third Obſervation as the Area ξ ☊ *S* to the Area ω ξ *S*; but the ratio of the Areas ξ ☊ *S* and ω ξ *S* is known; (for the Areas themſelves are known by the 24th *Prop. of Archimedes of the Quadrature of the Parabola*) therefore alſo the Time at which a Comet is in the Node ☊. After the ſame manner will be known the Time that a Comet comes to the other Node ☋. *Q. E. F.*

PROPOSITION XXIX.

TO *determine the Inclination of the Plane of the Trajectory of a Comet to the Plane of the Ecliptic.* [Fig. 31.]

Things remaining as before, from the point O to ☋ *S* ☋ the Line of Nodes, draw the perpendicular O ☋, and join ω ☋. In the Triangle O *S* ☋ beſides the right Angle at ☋ you have (by the foregoing *Prop.*) the Angle O *S* ☋, and the ſide O *S*; therefore O ☋ is known. Then in the rectangular Triangle ω O ☋ the ſides about the right Angle, namely, O ω and O ☋ are known, and therefore the Angle O ☋ ω will be known, which (by *Def.* 6. *El.* 11.) is the Inclination of the Plane of the Trajectory, to the Plane of the Ecliptic.

PROPOSITION XXX.

TO *determine the principal* Latus rectum *of a Comet's Trajectory, the Comet's Perihelium, and the Time that a Comet is in the Perihelium.* [Fig. 32.]

With the Focus *S*, thro' the Points ω and ξ, deſcribe

deſcribe the Trajectory of a Comet by *Prop.* 24. with S for your Center, thro' either of the ſaid Points (as for Ex.) draw a Circle meeting again with the Trajectory at V; draw ωV, to which from S draw the perpendicular $S\phi$, which produce till it meets with the Parabola at π; π will be the Perihelium of the Trajectory, and therefore the Quadruple of $S\pi$ is the principal *Latus rectum* of it.

The firſt is plain; for ſince the Focus S is found in the Axis of the Parabola, a Circle deſcrib'd about the Center S will cut the Parabola (if it cuts it at all) in two points equally diſtant from the Axis, and therefore the right line ωV joining the interſections, will be perpendicular to the Axis, and π the Vertex of the Parabola, or the Perihelium of the Trajectory, and the Quadruple of $S\pi$, the Parameter of the Diameter whoſe Vertex is π (that is, of the Axis itſelf,) or the principal *Latus rectum.*

Take a Time whoſe Interval from the firſt Obſervation when the Comet was in ξ, is to the Interval of the Time between the firſt and third Obſervation as the Area $\xi\pi S$ to the Area $\omega\xi S$; and that will be the Time that the Comet is in the Perihelium.

We leave the variety of Cauſes, namely, when the Time that the Comet is in the Perihelium is before the firſt Obſervation, and when it is after it, to the Judgment of the Aſtronomer, as alſo innumerable other things relating to this Problem, about defining a Comet's Trajectory and the parts of it.

SCHOLIUM.

Hence alſo will a Comet's Perigæum and the Time of it be determined: For ſince the Time is given (namely that between the firſt and third Obſervation) that the given Area $\xi\omega S$ is deſcrib'd by a radius drawn from the Comet to the Sun, you will alſo have the Area deſcribed in one Day; and ſince ξ is given, which is the Place of a Comet in the firſt Obſervation, the Places of a Comet for each Day will be given in its proper Trajectory. But the Place of the Earth in its Orbit is given for every Day; and (from what has been ſaid before) the Situation of the Earth's Orbit and a Comet's Trajectory in reſpect to each other; whence the Place where the Comet is neareſt to the Earth, that is, the Comet Perigæum will eaſily be known, and alſo the Time when it happens.

PROPOSITION XXXI.

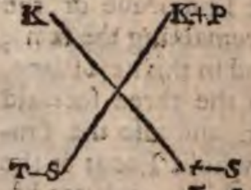
TO correct the Trajectory of a Comet which was found by Prop. 26.

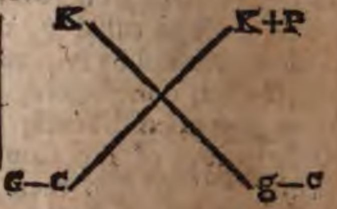
Let the poſition of the Plane of a Comet's Trajectory, determin'd by *Prop.* 26. be aſſum'd, then chooſe three Places of the Comet nicely obſerv'd, different from thoſe three by which the Trajectory was firſt determin'd, and very far diſtant from one another: It will be convenient for one of them to not be far diſtant from the Comet's Perihelium. Let the Time between the firſt and third of thoſe Obſervations be called S: Take a number C which may be to 1 as the Time between the firſt and ſecond Obſervation, to the Time between the ſecond and third: From theſe apparent Places, by a trigonometrical Calculation, after the method us'd before, find three Places of the Comet in the foreſaid Plane of the Trajectory, taken as very accurate; that is, find three Points of the before defin'd Plane, in which the Comet has the ſame Longitude and Latitude, as it is really found to have; and (by *Prop.* 29. *B.* 3. or perhaps by *Prop.* 24 *B.* 5.) determine by Calculation what Conic Section goes thro' thoſe three points, whoſe Focus coincides with the Sun, and whoſe Areas terminated by right lines drawn to the Sun and portions of the Curve are determin'd, and therefore their ratio will be alſo determin'd: As for Example, let its Area between the firſt and ſecond Obſervation, be to its Area between the ſecond and third Obſervation as *G* to 1; and that ſame will be the ratio between the Times in which theſe Areas wou'd be deſcrib'd by a Comet, from which a radius is drawn to the Sun. Moreover, from the known magnitude of this Track of the Comet, and the magnitude of all its parts, in reſpect of the Track of the Earth and its parts, may (by *Prop.* 41. *B.* 1.) be known the Velocity with which a Comet deſcribes it; *therefore* the Time that a Comet wou'd deſcribe the two above found Areas; let it be call'd *T.* Let $T=S$, and $G=C$, the poſition of the Plane of the Trajectory hitherto found, will be true and accurate, and want no correction; but if it ſhou'd, then $T-S$ will be the Error in the whole Time between the firſt and third Obſervation, ariſing from the leſs accurate poſition of the Plane of the Trajectory, and $G-C$ will alſo be the Error ariſing from it in the ratio of the Time between the ſecond and third Obſervation; which appears to be ſo, becauſe in each caſe Unity is taken for the Conſequent of two Terms, which expreſs the ratio between the two Times.

Let the Longitude of either Node of the Trajectory (found by *Prop.* 28.) call'd *K*, be encreas'd by 20 or 30 Minutes, which Minutes call *P*, keeping the forementioned Inclination of the Plane of the Trajectory to the Plane of the Ecliptic; and after the manner before made uſe of, find in this new Place three Points, which may accurately anſwer to the three foreſaid moſt accurate Obſervations of the Comet, and the Trajectory going thro' them, whoſe Focus muſt coincide with the Sun's Center, and let its two Areas between the three Obſervations deſcrib'd by a right line drawn to the Sun, whoſe ratio is expreſs'd by that of g to L, and alſo the whole Times which the Comet will take up in deſcribing each of thoſe Areas, be expreſs'd by t. If the Time t be found to be equal to S and g equal to C, this aſſum'd poſition of the Plane will be the true one; if not, $t-S$ will be the Error in the whole Time between the firſt and third Obſervation, and $g-C$ the Error in the ratio of the Time between the firſt and ſecond Obſervation, to the Time between the ſecond and third, each ariſing from the leſs accurate poſition of the Plane of the Trajectory to the Plane of the Ecliptic.

Let the found Inclination of the Plane of the Trajectory to the Plane of the Ecliptic call'd I, be (by *Prop.* 29.) encreas'd 20 or 30 Minutes, which muſt be call'd $\mathcal{Q}$, the Longitude of the Node firſt found by *Prop.* 28. remaining the ſame; and, as was done before, find in this new Plane, three Points anſwerable to the three foreſaid Obſervations of the Comet, and alſo the Trajectory going thro' them, whoſe Focus is the ſame as the Sun's Center, and its two Areas between the three Places obſerv'd, deſcrib'd by a right line drawn to the Sun, whoſe ratio is the ſame as that of γ to I: Find alſo the whole Time τ, which the Comet wou'd take up in deſcribing each Area at the ſame Time, by a right line drawn to the Sun. If τ be found to be equal to S, and γ alſo equal to C, this aſſum'd poſition of the Plane of the Trajectory will be the true one, and as it ſhou'd be; if not, $\tau-S$ will be the Error in the whole Time, between the firſt and third Obſervation, and $\gamma-C$ likewiſe the Error in the ratio of the Time between the firſt and ſecond Obſervation, to the Time between the ſecond and third Obſervation.

Let the Operation therefore be perform'd by the Rule of Falſe Poſition, the Scheme of whoſe Operation is here annex'd, where K is one poſition of the Longitude of the Node, and $T-S$ the Error ariſing from thence in the whole Time between the firſt and third Obſervation; $K+P$ the other poſition of the Longitude of the Node, and $t-S$ the Error of the ſaid whole Time ariſing from thence. According to the ſaid Rule, as the difference of Errors to the difference of Poſitions, ſo let either of the Errors be to a fourth Quantity; and this will be the correction of the Poſition, from which flow'd the aſſum'd Error.

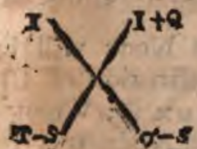




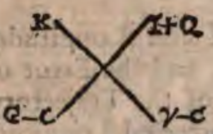
$T-t : T-S :: P : \frac{T-S}{T-t} \times P$ $G-g : G-C :: P : \frac{G-C}{G-g} \times P$

After that manner the Error of the poſited Longitude of the Node, that is, of K itſelf, in the whole Time between the firſt and third obſervation, is $\frac{T-S}{T-t} \times P$; and the Error P of the ſame Longitude K in the ratio between the two Times between the three Obſervations is $\frac{G-C}{G-g}\times P$; therefore the true and corrected Longitude of the Node is $K+\frac{T-S}{T-t}\times P$, or $K+\frac{G-C}{G-g}\times P$.

Likewiſe the Error of the poſited Inclination of the Plane of the Trajectory to the Plane of the Ecliptic, *viz.* I, from the firſt Head is $\frac{T-S}{T-\tau}\times\mathcal{Q}$; and from the ſecond it is $\frac{G-C}{G-\gamma}\times\mathcal{Q}$; and therefore the true and corrected Inclination of the Plane of the Trajectory to the Plane of the Ecliptic is $I+\frac{T-S}{T-\tau}\times\mathcal{Q}$, or $I+\frac{G-C}{G-\gamma}\times\mathcal{Q}$. And the corrected Longitude of the Node, and Inclination of the Plane of the Trajectory to the Plane of the Ecliptic, will become the ſame, if in the operation we make uſe of the Errors ariſing from the ſecond Poſition.



$T-\tau : T-S :: \mathcal{Q} : \frac{T-S}{T-\tau}\times\mathcal{Q}$



$G-\gamma : G-C :: \mathcal{Q} : \frac{G-C}{G-\gamma}\times\mathcal{Q}$

And becauſe both the Errors come under conſideration, as well that in the whole Time, as that in the ratio between the two Times between the three Obſervations, let us ſuppoſe as well $\frac{T-S}{T-t}\times P$, as $\frac{G-C}{G-g}\times P$, to be ſeparately equal to $m \times P$; that is, $\frac{T-S}{T-t} = m$ and $\frac{G-C}{G-g} = m$: Likewiſe let us ſuppoſe as well $\frac{T-S}{T-\tau} \times \mathcal{Q}$ as $\frac{G-C}{G-\gamma} \times \mathcal{Q} = n \times \mathcal{Q}$; that is, $\frac{T-S}{T-\tau} = n$, and $\frac{G-C}{G-\gamma} = n$. Whence $mT - mt$ will $= T - S$, and $mG - mg$ will $= G - C$; and alſo $nT - n\tau = T - S$, and $nG - n\gamma = G - C$: Whence the double of $T - S = mT - mt + nT - n\tau$, and the double of $G - C = mG - mg + nG - n\gamma$; and therefore going back, if m and n be taken in ſuch manner that the double of $T - S$ may be $= mT - mt + nT - n\tau$, and the double of $G - C = mG - mg + nG - n\gamma$ then as well $\frac{T-S}{T-t} \times P$, as $\frac{G-C}{G-g} \times P$ will be $= m \times P$, and likewiſe as well $\frac{T-S}{T-\tau} \times \mathcal{Q}$ as $\frac{G-C}{G-\gamma} \times \mathcal{Q}$ will be $= n \times \mathcal{Q}$; therefore the Error of the Longitude of the Node will be $m \times P$, and the Error of the Inclination of the Plane of the Trajectory will be $n \times \mathcal{Q}$: Whence the corrected Longitude of the Node will be $K + mP$, and the corrected Inclination of the Plane of the Trajectory to the Plane of the Ecliptic is $I + n\mathcal{Q}$.

The Plane of the Trajectory being corrected as well in reſpect to its Interſection with the Plane of the Ecliptic (that is, the Longitude of either Node) as its Inclination with the Plane of the Ecliptic; find (as before) in the Plane thus corrected and now moſt accurately poſited, three Points, which may agree with the three aforeſaid moſt accurate Obſervations, and (as before) taking the Sun's Center for your Focus, thro' the three ſaid Points draw a Conic Section: And this being corrected will be the Comet's requir'd Trajectory.

SCHOLIUM.

Having found the correct and accurate Trajectory, its Axis, *Latus rectum* agreeable to the Axis, the Time in which the Comet is in the Perihelium, the other things belonging to it will alſo be accurately defin'd, according to the Rules foregoing. If it happens that an Ellipſis deſcrib'd about the Sun's Center as its Focus (and not a Parabola) goes thro' thoſe three Points: that is, if that part of the Comet's Trajectory, which comes under our Obſervation, does evidently ſhew that the Comet returns in an Orbit, and does not run out *in infinitum*, then from knowing the greater Axis of this Orbit, you will know the Comet's Periodical Time, for (by *Prop.* 40 and 42. *B.* 1.) the Square of the Periodical Time of a Comet is to the Square of the Periodical Time of the Earth about the Sun, as the Cube of the greater Axis of the Orbit of the Comet to the Cube of the greater Axis of the Orbit which the Earth deſcribes about the Sun, or to the double of the mean diſtance of the Earth from the Sun.

If from the known principal *Latus rectum*, as for Example of the three Trajectories deſcrib'd in the three aſſum'd Planes before the full correction, *viz.* R the firſt, r the ſecond, and ρ the third, you wou'd find the principal *Latus rectum* of this laſt Trajectory deſcrib'd in the Plane accurately poſited; it may be found by the *Rule of Falſe* according to the Method of the things above; and it will be equal to $R + mr - mR + n\rho - nR$. For in the true and accurate Longitude of either of the Nodes you add to its Longitude in the firſt Plane (determin'd by *Prop.* 28.) the exceſs of the aſſum'd Longitude in the ſecond Plane above the other Longitude multiplied into m; and in the accurate Inclination of the Plane of the Trajectory you add to the Inclination of the firſt Plane (determin'd by *Prop.* 29.) the exceſs of the aſſum'd Inclination in the third Plane above the foregoing Inclination multiplied into n. And ſince the Trajectory and its *Latus rectum* are chang'd, as well upon account of the chang'd Longitude of the Node, as of the chang'd Inclination of the Plane to the Plane of the Ecliptic; to the *Latus rectum* of the Trajectory in the firſt Plane, or to R muſt be added $mr - mR$, that is, the exceſs of the *Latus rectum* in the ſecond Plane above the *Latus rectum* in the firſt Plane multiplied into m; and alſo you muſt ſuperadd $n\rho - nR$, namely, the exceſs of the *Latus rectum* in the third Plane above the *Latus rectum* in the firſt drawn into n. And after that manner you will have the true *Latus rectum*, viz. $R + mr - mR + n\rho - nR$. Much after the ſame manner will be corrected the *Latus tranſverſum*, if it be defin'd in magnitude.

SECTION IV.

Of the Way of finding a Comet's Heliocentric or Geocentric Place in its Trajectory before determin'd for a given Time; and of the Tables neceſſary for readily defining the Comet's Motions.

PROPOSITION XXXII. LEMMA.

F*Rom the Focus* S *of the given Parabola* VP, *whoſe Vertex is* V *and Axis* VE, *to draw the Right line* SA *in ſuch manner that the Parabolic Area* VRAS *comprehended under the right lines* VS, SA *and the Parabolic Curve* VRA *may be equal to a given rectilineal Figure, as for Example, to the Square of the right line* B; *and to define by computation the length and Inclination to the Axis* VE *of the right line* SA *ſo drawn.* [Fig. 33.]

Suppoſe it done, and thro' *V* draw the right line *VF* perpendicular to *VE*, and let the point C be in it, thro' which *CA* being drawn parallel to *VE* meets with the Parabola in the Point *A*, ſuch as *SA* being drawn may give a Solution to the Problem. Thro' *A* draw *AD* perpendicular to *VE*, meeting with the Axis at *D*. Let *VS* the diſtance of the Focus from the Vertex be call'd f; and (as is commonly known) the principal *Latus rectum* of the Parabola will be $4f$. Let the unknown *VC* be call'd x; and *AD* will be alſo $= x$, therefore (from *El. Conic.*) *VD* or its equal *CA* will be $\frac{x^2}{4f}$: Whence the rectangular parallelogram *VCAD* is equal to $\frac{x^3}{4f}$, and the

Parabolic Space *VRAD* (which, from *Archimedes*'s *Dimenſion of the Parabola* is ſubſeſquialteral to the foregoing) will be $\frac{x^3}{6f}$, namely, $\frac{2}{3}$ of $\frac{x^3}{4f}$. Moreover, ſince *VD* is $=\frac{x^2}{4f}$ and *VS* $=f$, *SD* will be $=\frac{x^2}{4f}-f$, if *S* be between *V* and *D*, as in the 1ſt *Fig.* 33; or if *D* be between *V* and *S*, as in the 2d *Fig.* 33, *SD* will be $=f-\frac{x^2}{4f}$; that is, in the firſt caſe, $SD=\frac{x^2-4f^2}{4f}$ and in the ſecond $SD=\frac{4f^2-x^2}{4f}$. Whence the Rectangle under *AD* and *DS* will be $=\frac{x^3-4f^2x}{4f}$ or $\frac{4f^2x-x^3}{4f}$; and therefore half of it, namely, the Triangle *ASD* will be $\frac{x^3-4f^2x}{8f}$ or $\frac{4f^2x-x^3}{8f}$. And therefore the ſpace *VRAS* (for it is in the firſt caſe equal to *VRAD* the Triangle *ASD* being taken from it, in the ſecond equal to *VRAD*, encreas'd by *ABD*) will be $\frac{x^3}{6f}-\frac{x^3-4f^2x}{8f}$ in the firſt caſe, and $\frac{x^3}{6f}+\frac{4f^2x-x^3}{8f}$ in the ſecond; and therefore in every caſe *VRAS* is equal to $\frac{x^3+12f^2x}{24f}$. And this muſt be equal to b^2, therefore $24fb^2 = x^3+12f^2x$. And having plac'd the Æquation as is uſual, $x^3=-12f^2x+24fb^2$. Therefore, by the *Cardan*'s Rule the Value of x or *AD*, expreſs'd in known Quantities, will be $\sqrt{C:\,12fb^2+\sqrt{144f^2b^4+64f^6}}-\sqrt{C:\,12fb^2-\sqrt{144f^2b^4+64f^6}}$. But when *AD* is known, *VD* will alſo be known; for it is equal to the Square of *AD* apply'd to the quadruple of the known *VS*. Therefore (by the 3d *Datum*) you have the compound of *DV* and *VS*, to which the Right line *SA* is equal, by *Corol.* 2. *Prop.* 14, and therefore this *SA* will be alſo given in magnitude; but *DA* is alſo given, therefore (by *Dat.* 1.) the Ratio between *SA* and *AD*, that is, between the Radius and the Right Sine of the Angle *ASD*, which *SA* contains with the Axis. Therefore that Angle *ASD* or *VSA* is known: And *SA* was before known in length. *Q. E. F.*

Converſly alſo in the given Parabola you will find the area *VRAS*, which will make the right line *SA* equal to the given right line; that is, you will find the Ratio of the given area *VRAS* to the Square of the right line *VS*, which will make *SA* to be in a given Ratio to the right line *VS*.

SCHOLIUM. 1. (*Fig.* 34.)

We have before ſhew'd the method of the Calculation made uſe of in Aſtronomy, for determining the magnitude and poſition of the right line *AS*. But if you like a Geometrical Conſtruction better, it may thus be eaſily had, if according to the univerſal method of Geometricians the before found Cubic Equation $x^3+12f^2x-24fb^2=0$ be conſtructed by the propos'd Parabola *RVP* and a Circle. Let the *Latus rectum* of the Parabola propos'd, (which is equal to $4f$) be call'd L; $12f^2=q$, and $24fb^2=r$, that the propos'd Equation may be in this uſual form $x^3+qx-r=0$. Take (according to the commonly known precepts of this conſtruction) $VG=\frac{1}{2}L$ in the Axis of the Parabola, and $GH=\frac{q}{2L}$ (and that backward towards the Vertex, becauſe it is $+q$) that is, in our Equation $GH=\frac{12f^2}{8f}$ or $\frac{2}{3}f$, whence it happens that the Point H will come to be in the the middle Point between V and S: For becauſe $VG=\frac{1}{2}L$, VG will be the double of VS, and taking from hence $GH=\frac{3}{2}VS$, $VH=\frac{1}{2}VS$. Moreover, ſuppoſe HK perpendicular to $VS=\frac{r}{2L^2}$; that is, in our caſe $=\frac{24fb^2}{32f^2}=\frac{3b^2}{4f}$; that is, equal to $\frac{1}{3}m$, if $m=\frac{b^2}{4f}$; (or to the right line found by this *Prop.*) if (by *Prop.* 45. *El.* 1.) the propos'd rectilineal Figure be apply'd to the *Latus rectum* of the Parabola) and towards thoſe parts of the right line VS, towards which AS is to be drawn. And a Circle deſcrib'd with the Center K thro' V, will cut the Parabola RVP in the requir'd point A, and AD, which is perpendicular to VS, will be x, or the Root of the foregoing Equation. This conſtruction has no need of a demonſtration, becauſe of the common conſtruction of the Equation $x^3+12f^2x-24fb^2=0$ ariſing from the nature of the Problem: But it is eaſie to conſtruct it running over the ſteps of the foregoing Analyſis.

After the ſame manner may be defin'd the magnitude and poſition of the right line AS drawn from the Focus S, ſuch as the Parabolic ſpace $QRASQ$, comprehended between that Line and another SQ given in poſition different from SV, and that part of the Parabolic Curve between them QRA, may be equal to the given rectilineal Figure B^q. For the Axis SV being drawn, you muſt draw the right line SA in ſuch manner that the Space $VRAS$ may be equal to the ſum or difference of the Spaces B^q and QVS, as SQ lies toward the ſame parts of SV towards which SA muſt be drawn, or towards contrary parts.

SCHOLIUM 2. [*Fig.* 35.]

Suppoſing VP and $\upsilon\pi$ any two Parabolas, whoſe Foci are S and σ and Vertices V and υ, and ſuppoſing the Areas VAS, $\upsilon\alpha\sigma$ equal to the reſpective Squares of B and β; if B be to VS as β to $\upsilon\sigma$, then AS, $\alpha\sigma$, VS, $\upsilon\sigma$ will be proportional, and the Angles VSA, $\upsilon\sigma\alpha$ equal. For, becauſe all Parabolas are ſimilar Figures, and the Foci and Vertices are Points ſimilarly poſited in them; and beſides, as the Figures VAS, $\upsilon\alpha\sigma$ are in a duplicate Ratio of the homologous ſides VS, $\upsilon\sigma$ (for their equals B^q and β^q are in a duplicate Ratio of B to β, that is, by ſuppoſition, in a duplicate Ratio of VS to $\upsilon\sigma$;) it is alſo plain that theſe Lines are ſimilar to one another, and therefore the Angles VSA, $\upsilon\sigma\alpha$ equal, and that AS is to $\alpha\sigma$ as VS to $\upsilon\sigma$. And therefore from hence, when you have laid down SV the diſtance of the Vertex of the Parabola from its Focus, expreſs'd in any round number, you will find the Angle VSA and the length of the right line SA, according to the different ratio between the rectilineal Figure B^q to which VSA is ſuppos'd equal, and the Square of VS. And the lengths of the right lines AS and of the Angles

VSA diſpos'd in Tables, ſerve for any Parabola, ſince (as was before ſhewn) the right line *AS* will in every Parabola ſtill have the ſame ratio and poſition to *VS*, the ſame ratio remaining between *B* and *VS*.

PROPOSITION XXXIII.

TO *find a propos'd Comet's Heliocentric Place in Longitude and Latitude, and its Diſtance from the Sun for a given Time.* [Fig. 36.]

Let ☊ *V* be a Comet's Trajectory, deſcrib'd by *Prop.* 26. and corrected by *Prop.* 31. in whoſe Focus *S* let the Sun be plac'd; and let ω be a Point in it in which the Comet is at any one of the Obſervations by which the Trajectory was determin'd. Let *VX* be the Axis of this Trajectory, whoſe poſition is known by *Prop.* 30: Thro' the ſame *Prop.* will alſo be known the Time that the Comet is in the Perihelium *V*; and let ☊ ☋ be the line of Nodes known in poſition by *Prop.* 28. If ω*VR* the corrected Trajectory of the Comet, be found to be Parabolical, take ſuch a Space as may be to the Space ω*VS* (known from *Archimedes's Quadrature of the Parabola*) as the Interval between the given Time and the beforefound Moment that the Comet is in the Perihelium, to the interval between the ſaid moment and the obſervation of the Comet in ω; call ſuch a Space B^q. By the foregoing *Prop.* Let the Parabolic Space *VRAS* be ſuppos'd equal to B^q, and find as well the poſition as the length of the right line *SA* in reſpect of *SV*, whoſe poſition in length in reſpect of the Diſtance of the Earth's Aphelium from the Sun, is known from what has been ſaid before. But if a Comet's Trajectory be found to be Elliptical, draw (by the 3d, 6th or

7th

or 7th *Prop.* of *B.* 3. as was done in the Planet's Orbits) the right line SA in ſuch manner that theArea $VRAS$ may be to the whole Area of the Comet's Orbit, as the Interval between the given Time and the monment that the Comet is in the Perihelium, to the whole Periodical Time of the Comet, known from knowing its greater Axis; and you will have the right line SA as well in poſition as length. In either caſe, from A to the line of Nodes ☊ ☋, draw the the perpendicular AN meeting with ☋ ☊ at N; from the ſame A to the plane of the Ecliptic let fall a perpendicular meeting with it at α; join αN, αS; and the Angle $AN\alpha$ will be the Inclination of the plane of the Trajectory to the plane of the Ecliptic, and therefore known by *Prop.* 29. Becauſe the Angles VSA and VSN are kuown, the Angle NSA, which is their ſum or difference, will alſo be known: Therefore in the Triangle SNA rectangular at N, having (by the foregoing) the ſide SA expreſs'd in parts of the Mean Diſtance of the Earth from the Sun and the Angle NSA, you will have the ſides SN and NA. In theTriangle $N\alpha A$ rectangular at α, knowing the ſide NA and the angle $AN\alpha$ (which is the Inclination of the Comet's Trajectory to the Plane of the Ecliptic found by *Prop.* 29. and corrected by *Prop.* 31.) you will know the other two ſides $N\alpha$ and $A\alpha$. Again, in the Triangle $SN\alpha$ (in the Plane of the Ecliptic,) beſides the Right Angle at N, you have the ſides SN and $N\alpha$; therefore you will have the ſide $S\alpha$ and the angle $NS\alpha$; But (by *Prop.* 29.) th poſition of the right line SN is given; therefore alſo the poſition of the right line $S\alpha$ will be given; that is, the Heliocentric place of a Comet reduc'd to the Ecliptic, or the Comet's

Heliocentrical Longitude. Laſtly, in the Triangle $S\alpha A$ rectangular at α you have all the ſides, as being found before; therefore you will alſo know $AS\alpha$ the Comet's Heliocentric Latitude, if there is any reaſon of reducing the place of the Phænomenon ſeen from the Sun to the plane of the Ecliptic, rather than any other Plane.

And on the other hand this ſhews you how to find the Time that a Comet will be in a given Place of its Trajectory.

PROPOSITION XXXIV.

To find the Heliocentric Place, as to Longitude and Latitude, and the diſtance from the Earth, of a propos'd Comet for a given Time. [Fig. 37.]

The ſame things being laid down as in the foregoing *Prop.* beſides them let BT be the *Magnus Orbis*, and in it let T be the place of the Earth for the Time given, found by *Prop.* 18. *B.* 3. draw TA, $T\alpha$: and (by *Prop.* 18. *El.* 11.) let the plane of the Triangle $TA\alpha$ be perpendicular to the plane of the Ecliptic. In the Triangle $TS\alpha$ (in the plane of the Ecliptic) you have the ſides $S\alpha$, ST, the firſt of them found by the foregoing *Prop.* and the laſt from the Theory of the Earth (each expreſs'd in parts of the Mean Diſtance of the Earth from the Sun) and the Angle $TS\alpha$ comprehended by them, (for each of them is given in Poſition;) therefore you will alſo have the ſide $T\alpha$ and the Angle $ST\alpha$: But TS is given in poſition, namely, the place of the Sun for a given Time; therefore the poſition of the right line $T\alpha$ is given, that is the place of the Comet in A ſeen from the Earth, reduc'd to the Ecliptic, or the requir'd Geocentrical Longitude of the Comet. Moreover, in the Triangle $A\alpha T$

beſides the right angle $A\alpha T$ you have the two ſides $A\alpha$, $T\alpha$, this laſt lately found in the parts of the Mean Diſtance of the Earth from the Sun, and the firſt of them found by the foregoing *Prop.* in the ſame parts; therefore the angle $AT\alpha$ the requir'd Latitude of the Comet ſeen from the Earth or its Geocentrical Latitude is known; as alſo the Hypothenuſe TA the Comet's diſtance from the Earth. *Q. E. F.*

Thus may be known when the motion of the Comet will be retrograde, in reſpect of its motion before, becauſe the Earth, together with the Eye, has mov'd in the mean time, as it happens in reſpect of the Planets, when at the ſame time the Comet ſeen from the Sun appears always to go on the ſame way. And converſly that Phænomenon of the Retrogreſſion of a Comet obſerv'd from the Earth, ſhews that Comets are not very far diſtant from the Earth, but that when they are ſeen by us they come down into the Region of the Planets. Moreover, as the Eye and the Comet are carried about the Sun in different Planes, the Comet ſeen by the Eye and referr'd to the Fix'd Stars, will, from the Laws of Optics, appear not to deſcribe a great Circle among them, but to deviate differently from it; when at the ſame time, if it was ſeen from the Sun which is plac'd in the Plane of the Trajectory, it wou'd accurately deſcribe a great Circle among the Fix'd Stars. This deviation from a great Circle (which alſo agrees with the Planets, as well as the Retrogradation mention'd before) put it out of all doubt, that the Comets are in the Region of the Planets.

PROPOSITION XXXV.

TO *deſcribe Tables neceſſary for readily and exactly finding the Places of Comets.*

The Trajectory of any Comet being defin'd, if it be found to be Elliptical, (ſuch as we ſuppoſe it will be found, as being the only one of the Conic Sections which agrees with the Phænomena of Comets, if they are Bodies coeval with the World, revolving in an Orbit like the Planets) Tables, for readily finding any Comet's place for a Time given, will be of the ſame kind (and conſtructed after the ſame manner) as the Tables for the Primary Planets, and their uſe the ſame, concerning which we have ſpoken in the 39th and 40th Propoſitions of the 3d Book.

If the Orbits of all thoſe Comets, of which we have Obſervations accurate enough, are pretty nearly determin'd, their Periods are alſo known (as is ſhewn in the Scholium of *Prop.* 31.) and the Times that they will again be ſeen, and they may be corrected from Obſervations which will be made then; and at length all Comets will be reduc'd to a Rule, perhaps in a ſhorter time than has been done concerning the Planets, after their number, and their revolving in Orbits were known. I hear that it is done in 24 Comets, (whoſe Places have been obſerv'd by Aſtronomers) already by Dr. *Halley*; and I have reaſon to believe, that the number of the Comets is not vaſtly great: But it is likely that the Periods of ſome of them are ſo great, from the immenſe Interval between Saturn and the neareſt Fix'd Stars, that they are not yet deſcended into the Regions of the Planets (where only they are viſible to us) ſince Aſtronomers have begun to look upon them as celeſtial Bodies, and more accurately to determine their places; therefore the determination of the Orbits in which thoſe Comets move muſt be left to Poſterity. But ſometimes it will happen that ſome Comet will finiſh a whole Period, and yet not be obſerv'd to do ſo by Aſtronomers, upon account of its nearneſs to the Sun; which, for the ſame reaſon, does alſo ſometimes happen to *Mercury* itſelf.

Beſides, the greater **Axes** of the Comet's Orbits, upon account of the diſturbance ariſing from the attraction either of other Comets or of Jupiter and Saturn, may a little change their ſituation, and ſo the ſame Comet may be miſtaken for a different Comet: But after ſeveral Revolutions the thing will be plain enough; for ſuch a diſturbance will not produce very great effects, and tho' the Orbit of the Moon is every Day chang'd, yet it is reduc'd to an Aſtronomical Calculation, and conſider'd as if it was always the ſame.

But tho' the Orbits of the Comets are not yet determin'd, ſome Tables may be conſtructed very uſeful for the more eaſy defining of their Motions, and a more ready way of deſcribing their Trajectories: as for *Example*, a Table by which, from the given diſtance from the Sun of a Comet in its Perihelion expreſs'd in parts of the Mean Diſtance of the Earth from the Sun, may be readily defin'd the Time that a Comet, after its Perihelion, as it recedes from the Sun, will get out of our ſight. For ſuppoſing the greater Semiaxis of the Orbit of the Earth to conſiſt of 100000000 parts, an Arc of $59' \frac{1}{7}$ (which the Earth goes thro' by its mean diurnal Motion) will (by the dimenſion of a Circle) contain 1720 212 of thoſe parts, and the twenty fourth part of it, or an Arc deſcrib'd by the

Earth by its mean horary Motion will have 71675 of thoſe parts: Therefore to the different diſtances of a Comet from the Sun, when in its Perihelium, muſt reſpectively be aſcrib'd Parts, gone thro' by a Comet in its Perihelium in the ſpace of a Day or an Hour; for theſe parts will be to the former parts as the velocity of a Comet in its Perihelium to the mean velocity of the Earth, that is (by *Prop.* 17.) in a ſubduplicate Ratio of 200000000 the greater Axis of the Orbit of the Earth, to the diſtance of a Comet in its Perihelion from the Sun; or parts muſt be aſcrib'd, expreſſing the Area which a Comet by a Radius drawn to the Sun ſweeps thro' (as being made from multiplying the former in half the diſtance of the Perihelion from the Sun.) Moreover, ſince by obſervation we pretty nearly know at what diſtance from the Sun (in reſpect of the diſtance of the Earth from it) a Comet muſt be plac'd, to ceaſe to be ſeen with the naked Eye; that is, when the Ratio of that diſtance, to the diſtance of a Comet from the Sun in its Perihelion is given, you will alſo (by the inverſe of *Prop.* 33.) have the Area which a Comet with a Radius drawn to the Sun muſt deſcribe after it has paſs'd the Perihelion, that it may come to that diſtance from the Sun, that is, that it may ceaſe to be ſeen; and conſequently the Time after the Perihelium; as for *Example*, (by *Prop.* 11. *B.* 1.) for one Day as the lately found Area, to the before found Area deſcrib'd in one Day by a Comet, by a Radius drawn to the Sun. Therefore to every diſtance of the Perihelium aſcribe an agreeable Time, in which the Comet after it has paſs'd the Perihelium, as it goes from the Sun, gets out of our ſight.

Juſt in the ſame manner, and from the ſame Principles, may be calculated a Table readily ſhewing the Time that a Comet, after it has paſs'd the Perihelium, comes to a diſtance from the Sun, equal to the diſtance of the Earth from the Sun, if the Perihelium of the Comet is leſs diſtant from the Sun than the Earth itſelf. And this Table will be join'd to the firſt Table, and will, as it were, make but one Table; namely, by writing over againſt every Diſtance of the Perihelium of a Comet, expreſs'd in parts of the Semidiameter of the *Magnus Orbis* (beſides the parts gone thro' by a Comet in its Perihelium, in the ſpace of a Day, and the parts of the Area which a Radius ſweeps thro' in the ſame Time, of which we ſpoke above,) in one Column, the the Time after the Perihelium in which it comes to a diſtance from the Sun, equal to the Earth's diſtance from it; in the other, the Time that it comes to a diſtance four times or five times greater than the former, namely, that in which it ceaſeth to be ſeen by us.

Thirdly, The Table calculated by the ſecond Scholium of *Prop.* 32. will be very convenient, in which putting a round number for the diſtance of a Comet in the Perihelium from the Sun, over againſt the different bigneſs of the Area *VAS* (in the Fig. to which the ſaid Scholium refers) expreſs'd in parts of the ſquare of the right line *SV*, ſet down the quantity of the Angle *SVA* agreeable to it, and the length of the right line *SA* expreſs'd in parts of the right line *SV* alſo agreeable to it. For from that and the foregoing Table, and the known diſtance of a Comet in it Perihelium, from the Sun, you may readily know the Longitude of a Comet from the Perihelium when ſeen from the Sun, and the diſtance of a Comet from the Sun's center, agreeable to the Time before or after the Perihelium. For *SV* being given expreſs'd in parts of the mean diſtance of the Earth from the Sun (from the firſt Table) you will readily have the Area which is deſcrib'd (a Radius being drawn to the Sun) by the Comet in the ſpace of one Day or one Hour; therefore it will be given expreſs'd in parts of the ſquare of the right line *VS*, and fit for this Third Table; and therefore from the given Time before or after the Perihelium, will be given the Area *VAS* proportionable to the ſaid Time in the ſaid Parts; therefore from this Third Table you will immediately have the Angle *VSA*, which is the Heliocentrical Longitude of the Comet from the Perihelium, agreeable to the propos'd Area, and therefore proportionable to the propos'd Time, and alſo you will have in parts of *SV* the *length* of the right line *SA* agreeable to this, which by help of the Rule of Three, may be expreſs'd in parts of the Semidiameters of the *Magnus Orbis*.

But from what has been ſaid in former Propoſitions, other Tables alſo may be conſtructed, for making eaſy the calculation of the Comet's Motions, for every kind of Computation, which we muſt leave to the Induſtry of Aſtronomers.

SCHOLIUM.

By the foreſaid Method, or one like it, Dr. *Halley* has determin'd the Orbit of the Comet, which appear'd about the end of the Year 1680, and ſome of the following Months, which he aſſum'd as Parabolical, and found (as it appears from his Letters to Sir *Iſaac Newton*) that the Perihelium of its Orbit was in ♐ 27°. 22′. 30″; and therefore its Aphelium in ♊ 27°. 22′. 30″, that the principal *Latus rectum* of its Orbit was of 0, 0243 parts, of which the mean diſtance of the Earth from the Sun is 1. Beſides, he found this Orbit ſo plac'd in reſpect to the Plane of the Ecliptic, as to be inclin'd to its Plane in an Angle of 61°. 20′. 20″, and the aſcending Node in ♑ 1°. 53′, and the deſcending one in ♋ 1°. 53′, and that the Comet mov'd ſo in this Orbit as to be in its Perihelion in the Year 1680, on the 8th Day of *December* Old Stile, 0 Hour 4 Minutes after Noon. He found alſo the Orbit of the Comet of the Year 1683. figured and poſited after the following manner, *viz.* its Perihelium in ♊ 25°. 29′. 30″, which it was in on the 3d Day of *July*, Old Stile, 2 Hour. 50′; the *Latus rectum* of this Orbit of 2, 2408 parts of which the mean diſtance of the Sun from the Earth is 1, the Inclination of this Orbit to the Plane of the Ecliptic 83°. 11′, aſcending Node ♍ 23°. 23′, and deſcending Node in ♓ 23°. 23′.

THE

THE ELEMENTS OF Aſtronomy, Phyſical and Geometrical.

The SIXTH BOOK.

Of Comparative Aſtronomy.

IN the Five foregoing Books, we have explain'd Aſtronomy as it is agreeable to the Inhabitants of the Earth; that is, we have deſcrib'd the *Phænomena* of the Fix'd Stars, of the Sun, of the Planets and Comets as they are ſeen from the Earth, and we have taught how to find the reſpective Places of every one of them for a given Time, and other Things of that nature, each in its proper place: And now we ſhall in a few Words ſhew what kind of Aſtronomy the Inhabitants of the reſt of the Pla-

Plate 2. Book 5.

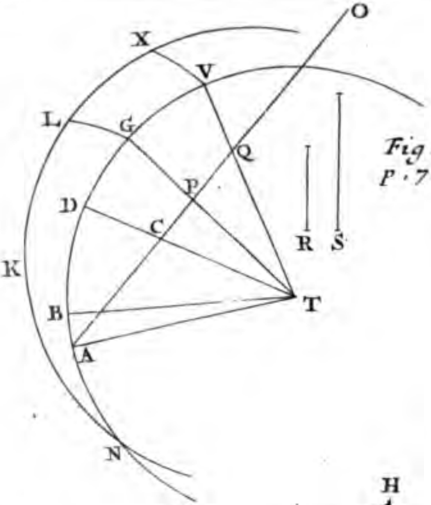


Fig. 6.
p. 726.

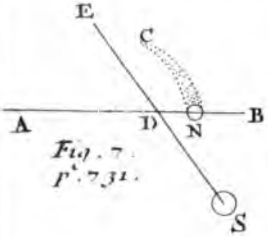


Fig. 7.
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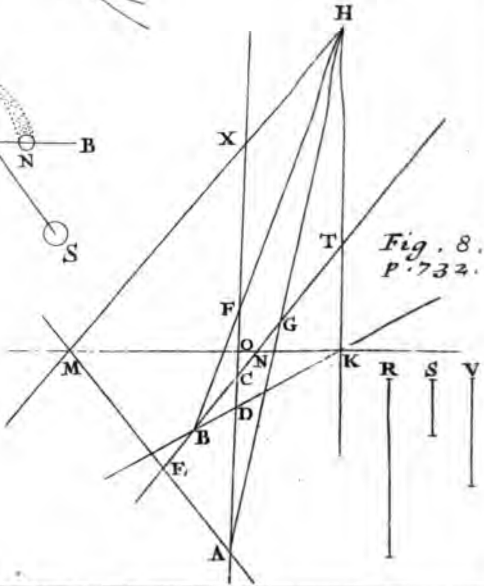

Fig. 8.
p. 732.

Plate 3. Book 5.

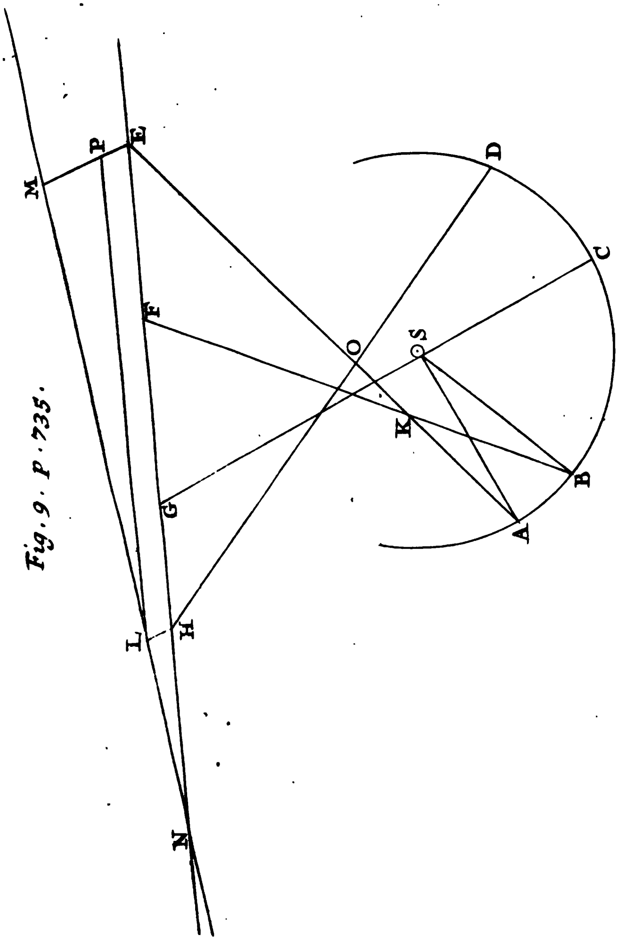

Fig. 9. P. 735.

Plate 4. Book 5.

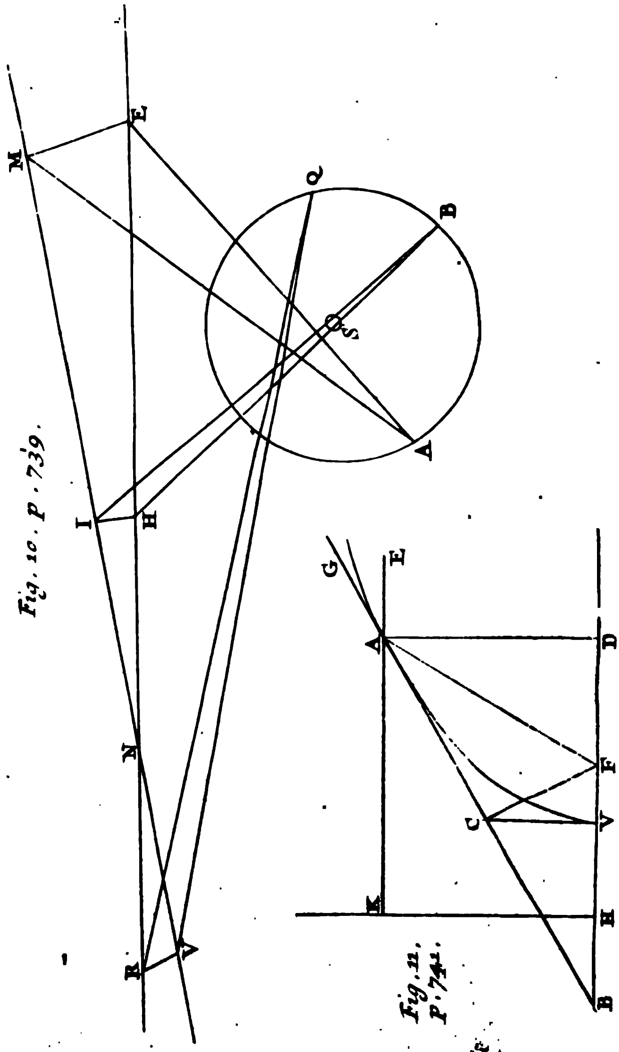

Fig. 10. p. 739.

Fig. 11. p. 741.

Plate 5. Book 5.

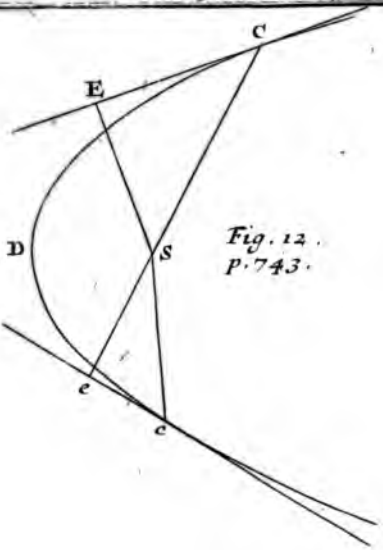

Fig. 12.
p. 743.

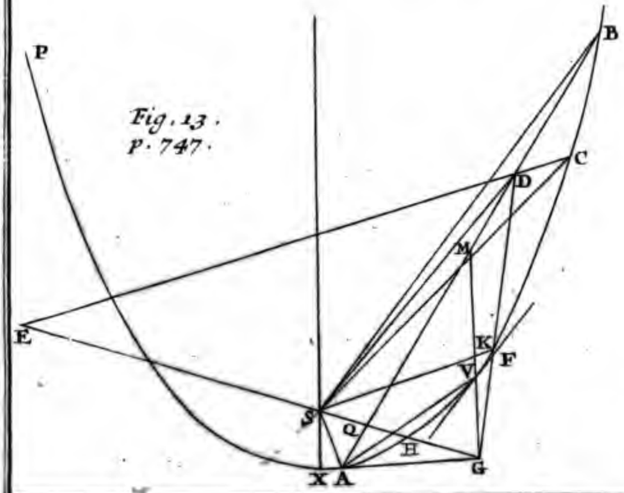

Fig. 13.
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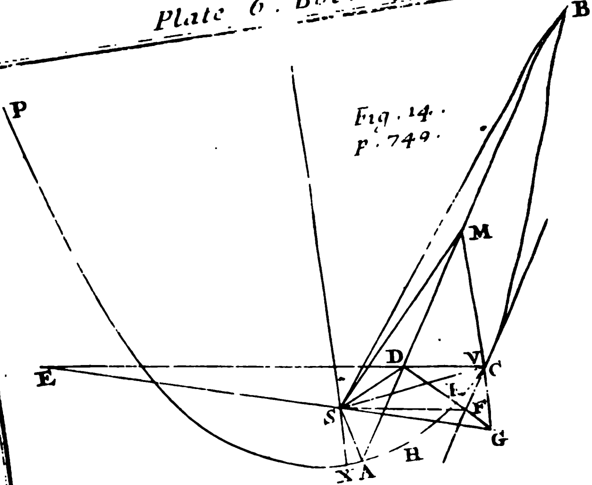

Fig. 14.
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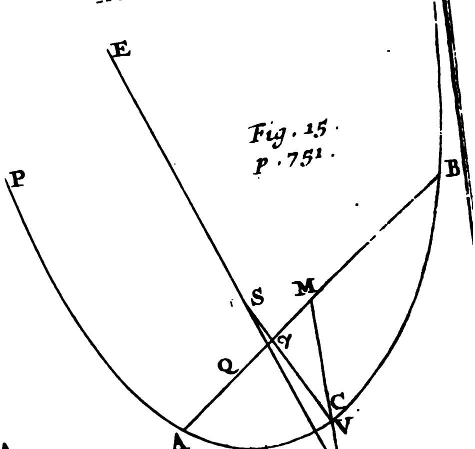

Fig. 15.
p. 751.

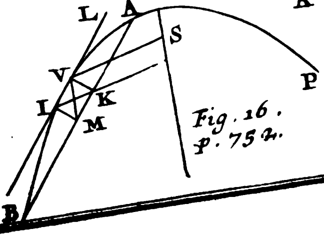

Fig. 16.
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Plate 7. Book 5.

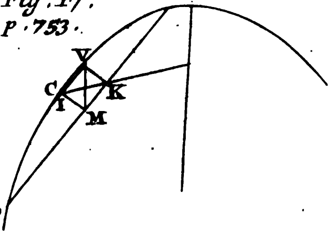

Fig. 17.
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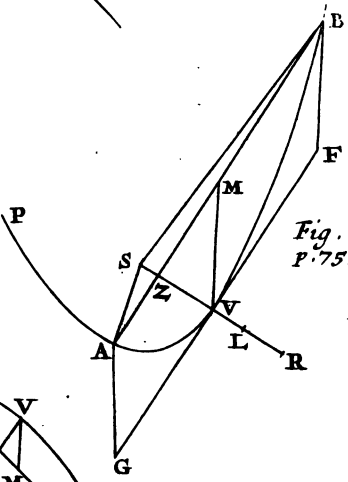

Fig. 18.
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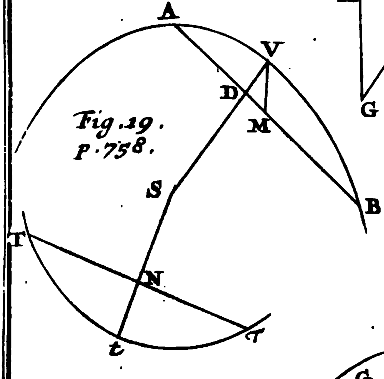

Fig. 19.
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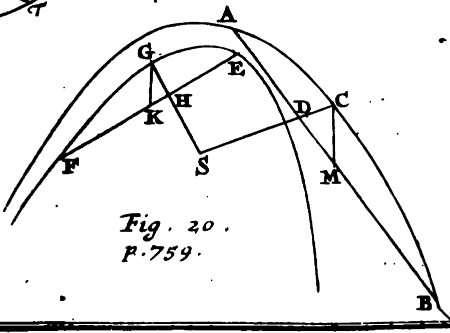

Fig. 20.
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Plate 8. Book 5.

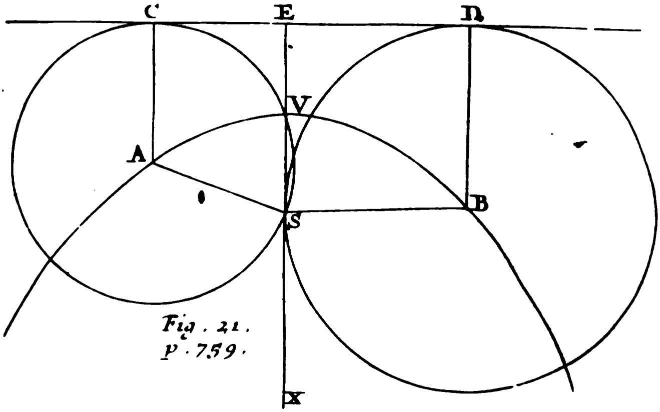


Fig. 21. p. 759.

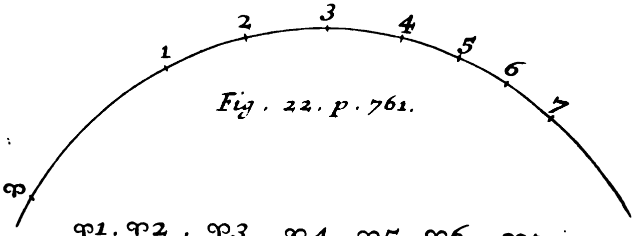


Fig. 22. p. 761.

♈1. ♈2. ♈3. ♈4. ♈5. ♈6. ♈7.

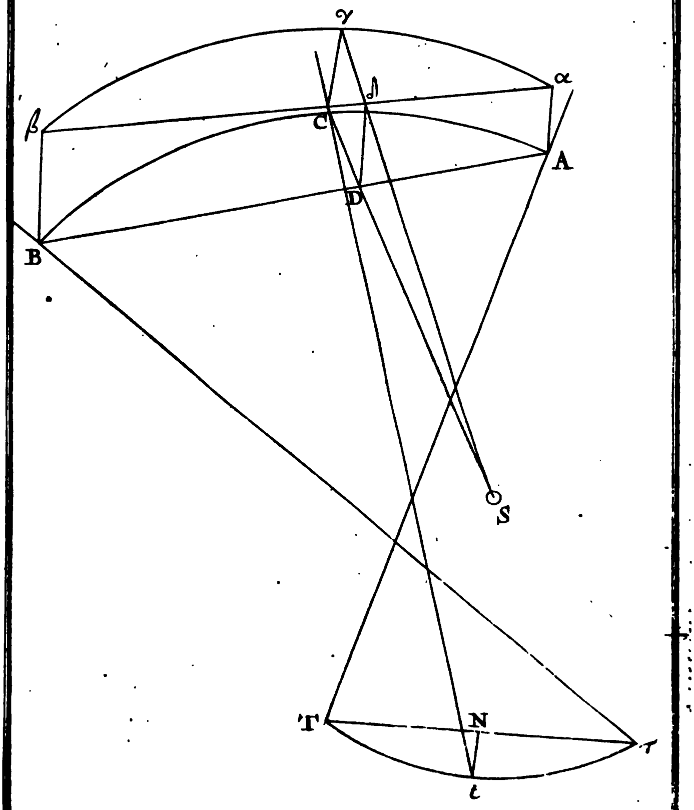
A. B. C. D. E. F.

G. H. K. L. M.

P. Q. R. S.

Plate 9. Book 5.

Fig. 23. p. 765.



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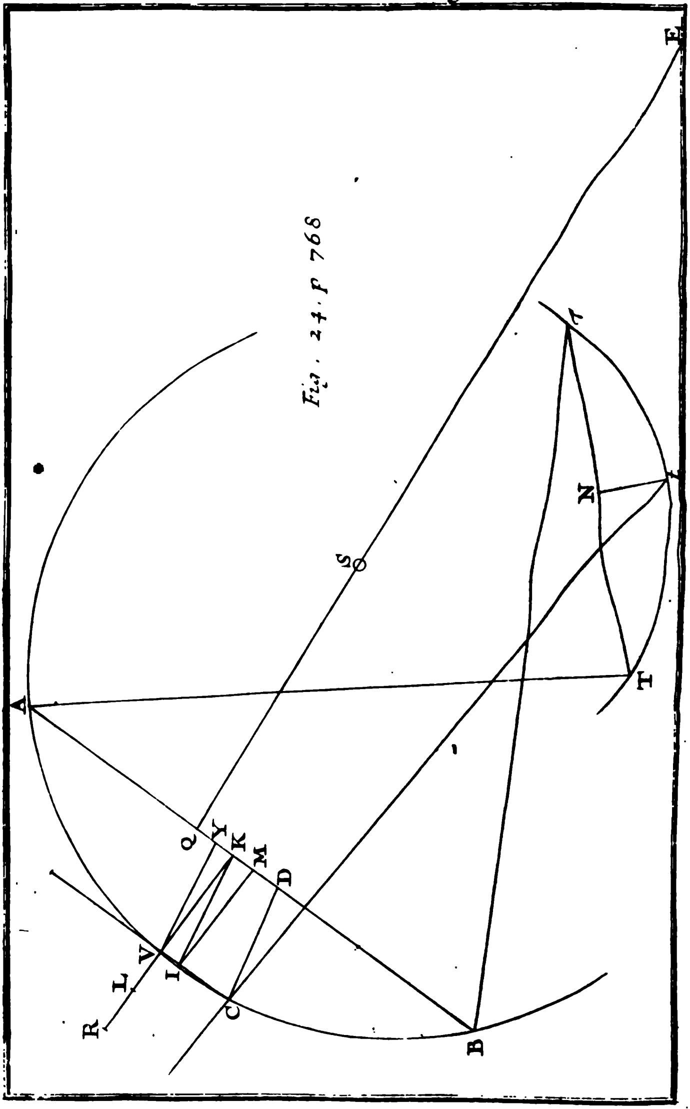


Fig. 24. p 768

Plate 11. Book 5.

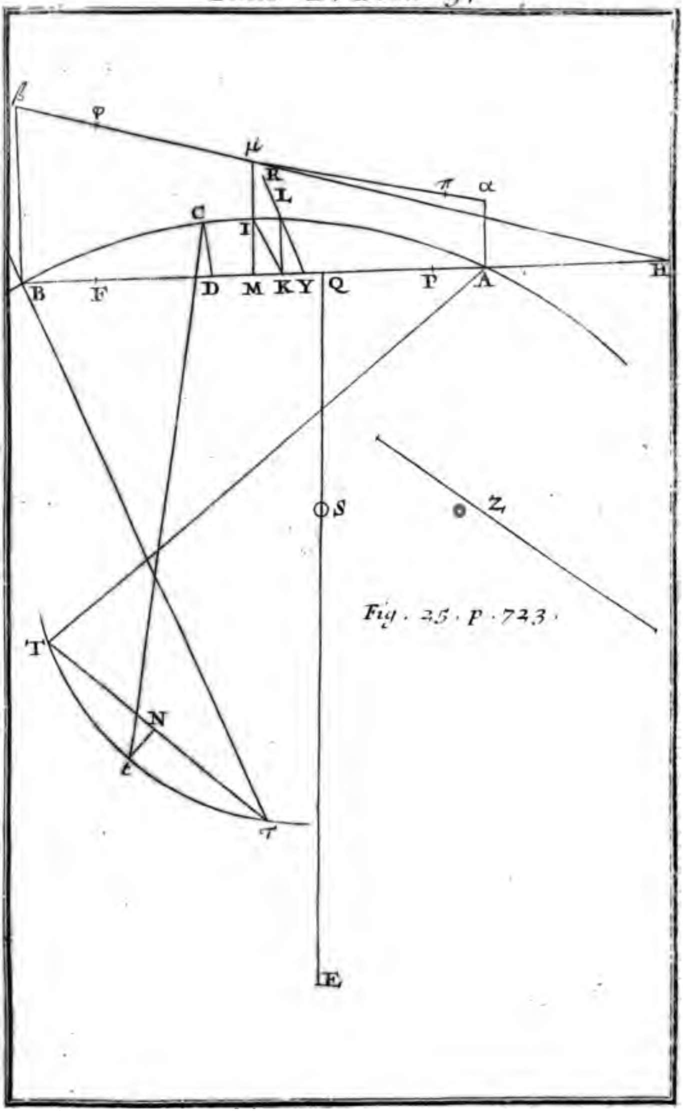


Fig. 25. P. 723.

Plate 12. Book 5.

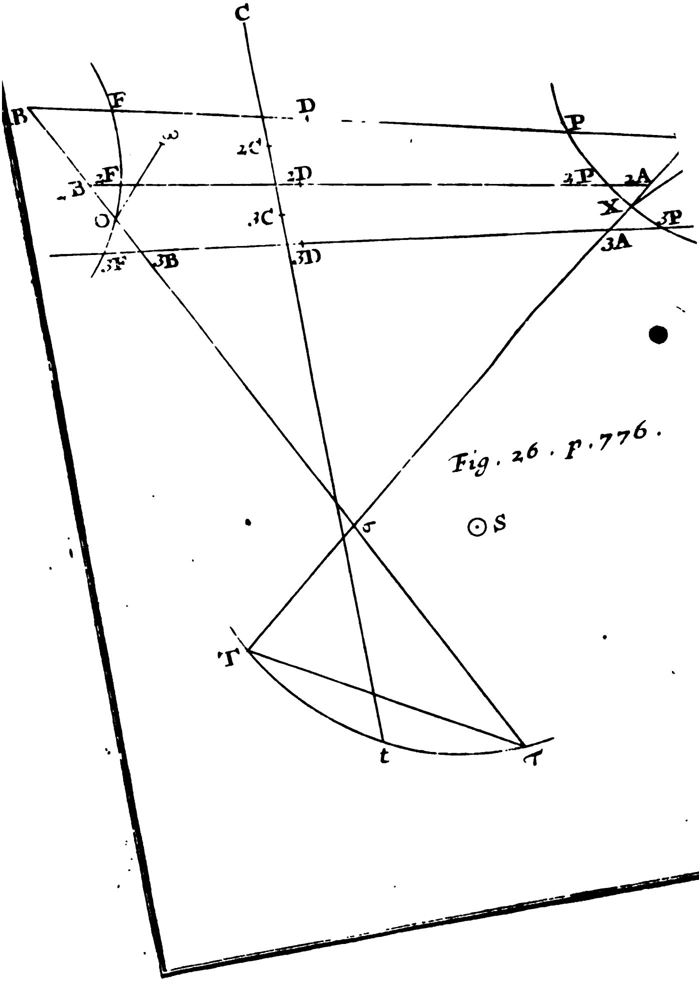


Fig. 26. p. 776.

Plate 13. Book 5.

Fig. 27. p 779.

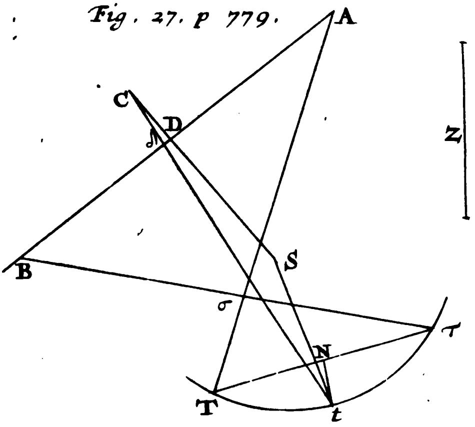


Fig. 28. p. 781.

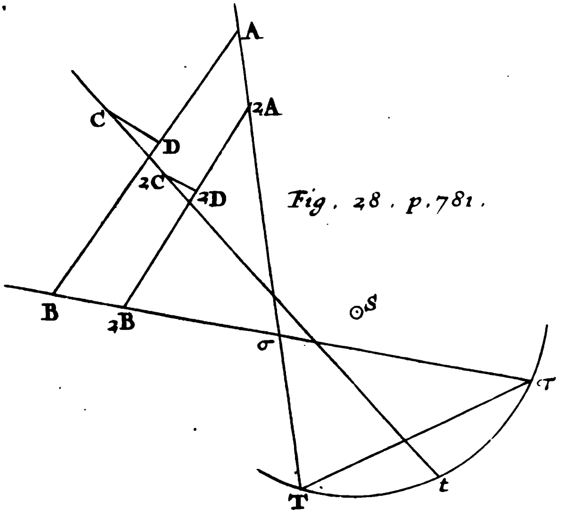


Fig. 29. p. 783.

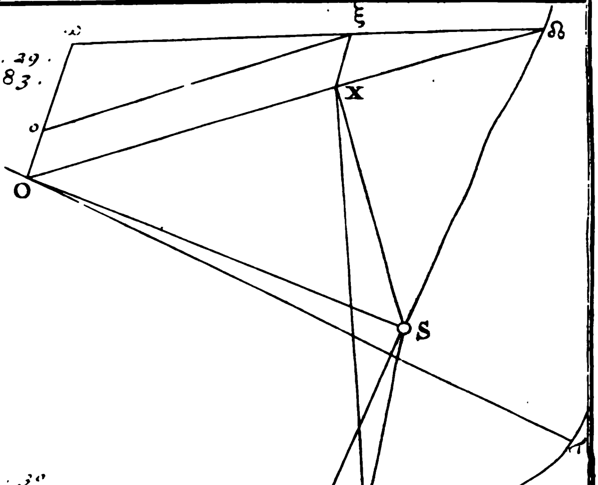
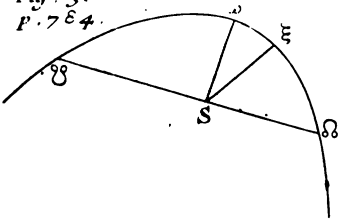


Fig. 30 p. 784.



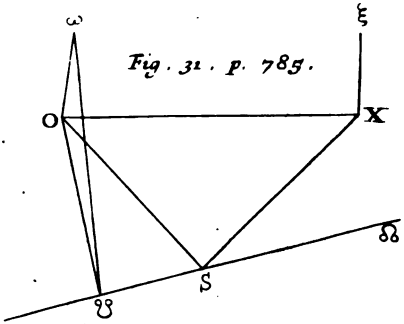
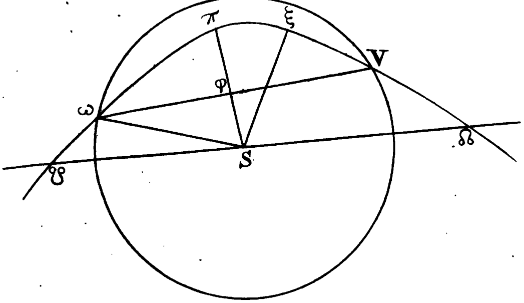


Fig. 31. p. 785.

Plate 15. Book 5.

Fig. 32. p. 785.



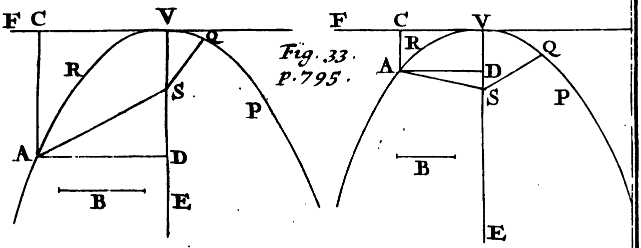


Fig. 33. p. 795.

Fig. 34. p. 797.

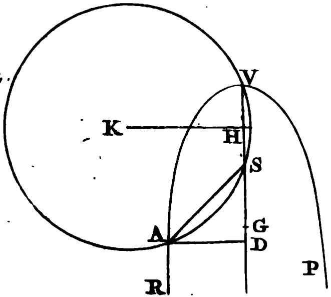


Plate 16. Book 5.

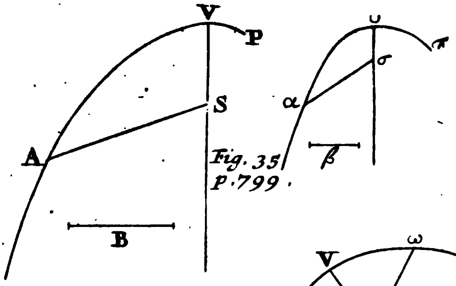


Fig. 35 p.799.

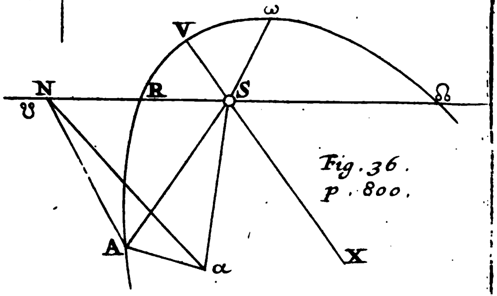


Fig. 36. p. 800.

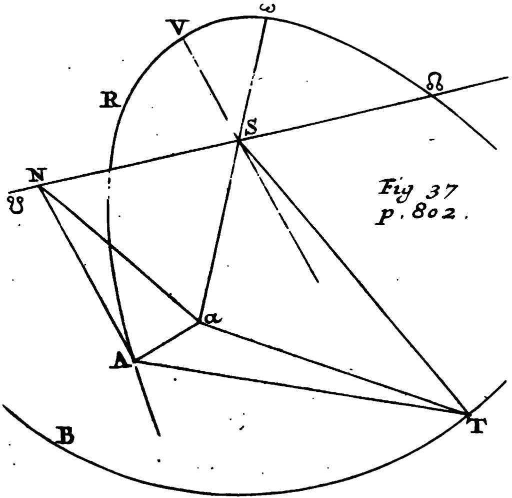


Fig 37 p. 802.

nets and other vaſt Bodies which are diſpers'd over the Univerſe, wou'd eſtabliſh. Not that we believe, and much leſs affirm, that there are any ſuch Aſtronomers any where; but by ſuppoſing the Eye to be plac'd ſometimes in one and ſometimes in another of the Celeſtial Bodies, and explaining what ſort of Aſtronomy a Spectator wou'd eſtabliſh according to his different Station, we hope to give no ſmall Light for the thorough underſtanding of thoſe Things which we have ſaid of this our Earth: For we wou'd not ſpeak of ſuch empty Appearances, and never ſeen or likely to be ſeen in any one, unleſs it was for this end. Therefore, as we go on, we ſhall not only ſuppoſe the Eye in the Planets which move about the Sun, and their Satellites, where ſeveral famous Men have formerly and do now believe that it is probable that there are Inhabitants endu'd with Senſe, but we ſhall ſuppoſe it alſo in ſome one of the Comets, in the Sun itſelf, and in the Fix'd Stars, where no body has ever ſuppos'd Inhabitants.

A great many eminent Men have taken pains to illuſtrate this. *Plutarch* ſays many things with regard to this, in his Book of the *Face in the Orb of the Moon*. *Kepler* wrote a Book of *Lunar Aſtronomy*, publiſh'd after his Death, under the Title of, *The Aſtronomical Dream*, in which he ſpeaks of ſeveral Phyſical and Aſtronomical Matters, as he always does every where elſe. But in explaining the Phænomena ſeen from the Moon, he has made ſome few Errors, becauſe he took no notice of the Libration of the Body of the Moon, or elſe he did not refer it to its proper Cauſes: Therefore I have been forc'd to alter it quite. The famous *Chriſtian Hugens* has lately publiſh'd a *Theory of the World*, full of Learned

Conjectures, in which he has ſhewn ſeveral Aſtronomical and Phyſical Conſiderations worthy of his Genius. But theſe two ingenious Men were in earneſt about the Inhabitants of the Planets; eſpecially *Hugens* who did not at all doubt but that in every one of the Planets were Inhabitants like, or at leaſt not very unlike thoſe of the Earth; and to this purpoſe it was that he wrote a Book rather of Phyſical than of Aſtronomical Conſiderations. But I, who have only undertaken to treat of Aſtronomy, have ſaid nothing concerning the Things neceſſary or convenient for the Animal or Senſitive Life, which may be found, or are wanting in particular Planets: I have only plac'd the Eye (which, with the Reader's leave, I ſhall ſometimes call a Spectator) as beholding things above and round about it.

Beginning by the Sun, where the *Aſtronomy* is moſt ſimple, I ſhall go on in order to Mercury and the other Planets: The moſt remarable Phænomena belonging to every one ſhall be treated of by themſelves; but thoſe that are common to many are referr'd to one and the ſame Propoſition. Then transferring the Eye to ſome Comet, I ſhall obſerve what would be ſeen from ſuch a Place; and, by way of Corollary, we deſcribe how our Solar Syſtem wou'd appear, if ſeen from ſome Fix'd Stars. Laſtly, ſeveral Phænomena which will appear to a Spectator in the Earth's Attendant the Moon, and beſides, I reckon up ſeveral Phænomena that wou'd appear from one of the Satellites of Jupiter and Saturn.

I mention, by the by, what will be the true and what the apparent Aſtronomy, in theſe different Stations, what Conveniencies they will have, and what Inconveniencies they will be ſubject to, who, by Obſervations made from theſe Stations, wou'd endeavour to find the true Syſtem of the Circum ſolar Planets, and meaſure their Diſtances and Magnitudes. And laſtly, I ſhew which is the moſt convenient Situation for Aſtronomy, as well among the Primary as among the Secondary Planets.

The Phænomena that are look'd upon by the Inhabitants of the Earth as principal, I alſo every where hold as principal, *viz.* the Viciſſitude and Meaſure of Day and Night; the Order of the Fix'd Stars; the apparent Magnitude of the Sun, the Quantity of its Light, and Intenſeneſs of its Heat; the Length of the Years and Months, the Summer and Winter; the Phaſes of the Moons; the Eclipſes of the Luminaries; the Number, Directions, Stations and Retrogradations of the Planets; the aptitude of Phænomena for the finding out of the true Syſtem; the exact Accuracy that is requir'd in Obſervations that are of uſe in Aſtronomy, and other things of that nature.

PROPOSITION I.

TO deſcribe what will be the chief Phænomena, when the Eye is in the Sun.

If the Eye is ſuppos'd in the Sun's Center, looking round about every way, the Fix'd Stars and all the Planets will appear to be in a Sphere concentric to the Eye, as it happens to us when we look from the Earth; and indeed the Fix'd Stars and the Milky Way will appear in the ſame Order, and diſtributed into the ſame Conſtellations as they appear to us Inhabitants of the Earth. For ſince the Order of the Fix'd Stars, and the Figures of the Conſtellations are not chang'd in reſpect to us who are carried with the Earth about the Sun, it is plain that it remains the ſame to the Eye placed any where within that Circle, and therefore to it in the Sun's Center. If the repreſentation of Things which give names to the Conſtellation be drawn forward and concave, they will become the Schemes or Conſtellations of the Ancients; but if they be drawn backwards and convex, they will become the Modern Figures of the Conſtellations, more fit for artificial Globes than for the Heaven itſelf.

It is alſo plain, that the order of the Fix'd Stars will remain the ſame if the Eye be on the ſurface of the Sun, or in Mercury, or in Venus, becauſe they are contain'd within the Orbit of the Earth. But alſo becauſe of the immenſe diſtance of the Fix'd Stars, upon account of which it happens, that tho' the place of the Obſerver is mov'd, the whole Diameter of the *Magnus Orbis*, the order of the Fix'd Stars does not appear chang'd in the leaſt, if the Eye be in Saturn, the outmoſt of the Planets of the Solar Syſtem, this Order will remain the ſame, or at leaſt not ſenſibly chang'd. For the change of Diſtance, which is inſenſible to us, if it be ten times greater (which may happen, becauſe the diſtance of Saturn from the Sun is ten times greater than that of the Earth from it) will not be very ſenſible.

Tho' the Planets ſeem plac'd among the fix'd Stars, our Obſerver will in a little time diſtinguiſh them from the Fix'd Stars, obſerve the Period of each of them, and compare them together, taking the Period of one for a meaſure, to which the Period of the reſt muſt be referr'd. And altho' we ſuppoſe the Obſerver unmov'd (becauſe he is ſuppos'd in the Sun's Center) and for that reaſon liable to no change of Situation (or any thing like it) by which he may meaſure the diſtances of the Planets, he will naturally conclude, that of two Planets compar'd together, that is the remoteſt which takes up more time to return to the ſame Fix'd Stars; and therefore that Saturn is the remoteſt, Jupiter nearer, then Mars, then the Earth below it, then Venus, and loweſt of all Mercury.

Neither is there reaſon to doubt but that the firſt Aſtronomers made uſe of that Rule for finding out the order of the Planets: (For the Opinions which *Plutarch* in his 2^{d} *Book of the Opinions of Philoſophers*, makes mention of that of *Zenocrates*, who ſuppos'd the Stars to move in one and the ſame Surface, and that of *Metrodorus* of *Chios* and of *Crates*, who believ'd the Sun to be plac'd higheſt of all, and after it the Moon, then the fix'd and wandring Stars, were either the notions of ignorant Men, or as I am more inclin'd to believe, of ſuch Men as had a mind to hide the Truth:) tho it be more difficult to apply it from the Earth than the Sun, by reaſon of the Stations and Retrogradations of the Planets ſeen from the Earth; therefore we have been oblig'd to obſerve ſeveral of their Stations and Retrogradations, to know from their mean Motions what their Periods are. But the whole Revolution of each of them was not ſo eaſily known; becauſe when ſeen from the Earth, they did not after one whole Revolution return to the ſame Fix'd Stars, becauſe the Obſerver's ſituation is not then in the ſame place. Notwithſtanding theſe Inconveniencies, they believ'd Saturn the remoteſt, becauſe the moſt ſlow, then Jupiter, then Mars, and the Moon loweſt of all. But concerning the three remaining ones, namely the Sun, Venus and Mercury, there was ſtill left room to diſpute, becauſe they found their mean Motions equal thro' the whole Zodiac. Whence ſome plac'd Venus next to Mars, Mercury next to it, and then the Sun, as *Plato* the Younger did; others put Mercury next to Mars, then Venus, and then the Sun, as *Porphyry*; *Alpetragius* makes Venus to be under Mars, then the Sun, and then Mercury; others plac'd the Sun next to Mars, then Venus, then Mercury, as *Ptolemy* himſelf, and ſo the Sun was number'd as plac'd in the middle, in the order of the Planets; (Saturn, Jupiter and Mars, being above it, and Venus, Mercury and the Moon below it;) which differ'd very little from the moſt ancient and true Syſtem, in which the Sun is in the middle of the Planets, which about that time was ſcarce out of date, at leaſt not in the Expreſſion. Others in the mean time from the equality of the Mean Motion of thoſe three Celeſtial Bodies, affirm'd, that Venus and Mercury went round the Sun, and that their Orbits were carried by the Sun as Epicycles, which Opinion was agreeable to *Cicero* and the *Ægyptian* Philoſophers; unleſs we rather think that this order of thoſe three Bodies was the remains of the true Syſtem of the World, not yet loſt among the *Ægyptian* Prieſts.

But our Solar Obſerver will have no ſuch things to hinder his Reaſonings; for to him all the Planets will be continually direct, with Periods altogether different, and will appear to return to the ſame Fix'd Stars, whence their Order will be, without heſitation ſettled from the foreſaid Principle. Which in courſe of Time will be more confirm'd, when they (being eaſily diſtinguiſh'd from one another, either by their magnitude obſerv'd before, or their different colour) an Inferior one will be eaſily ſeen to cover a Superior one.

The Spectator will alſo in a little time find, that if he compares any Planet with the Fix'd Stars it will ſometimes go ſlower and ſometimes go ſwifter, and from the ſaid Rule will conclude that it is ſometimes nearer and ſometimes further off, which will appear more plainly from the change of its apparent Diameter. But when he has obſerv'd that the Velocity is not increas'd in the ſame Ratio as its apparent Diameter, but almoſt in a duplicate Ratio, he will find that the change of Velocity is not altogether optical, but that half of it is really in the Planet, and that the other half appears ſo by reaſon of the greater nearneſs.

Tho' he eaſily has the Order of the Planets rightly eſtabliſh'd, it will not be ſo eaſy for him to compare their different diſtances from the Sun one with another: For here is wanting (what we have upon Earth) the tranſlation of the Obſerver from one place to another, or the diverſity of Stations, by which he may compare the different Diſtances from himſelf with the diſtance of the Stations.

If we remove the Eye from the Sun's Center to its Surface,then all theſe things will be plain from the Parallax of the Planets: For the horizontal Parallax of the Earth to an Eye ſo plac'd, is equal to the Sun's Semidiameter ſeen from the Earth, *viz.* 16′, and therefore ſenſible enough; and the horizontal Parallax of Saturn himſelf which is ten times farther diſtant, will exceed a Minute and a half, whilſt that of Mercury ariſes almoſt to 50′. Since then theſe Parallaxes are ſenſible enough, the diſtances of the Planets may be compar'd with the Sun's Diameter, and therefore alſo with each other. The Eye being thus plac'd, all the Fix'd Stars and Planets will appear to revolve from Eaſt to Weſt in the ſpace of twenty five of our Days. The Northern Pole of that Revolution will be in that place which We Inhabitants of the Earth refer to the tenth degree of Piſces, with a Northern Latitude of 83 or 84 Degrees, whence the arctic Polar Star will be at the ſecond flexure of the Dragon, and which is not above three Degrees from the Pole itſelf. The South Pole will be in the tenth Degree of Virgo, 83 or 84 Degrees towards the South, near to a Star of the fourth Magnitude; which according to the ingenious Dr. *Halley*, is to be ſeen in the firſt Oar of the Ship *Argo*.

And tho' the foreſaid Obſerver has no viciſſitude of Night, as being plac'd in the ſurface of a moſt lucid Body, which makes Day, yet the Fix'd Stars and Planets will make unequal Arcs above and below the Horizon, as they declin'd towards the Pole, which is rais'd above the Equator, or to that which is depreſs'd below it, juſt as it happens to us Inhabitants of the Earth.

The Planets ſeen from the Sun appear of different magnitudes. The diameter of Saturn ſubtends an Angle of $18''$, that of Jupiter of about $40''$, that of Mars of only $8''$, of Venus an Angle of $28''$, and Mercury $20''$. The famous *Hugens* lays down the Diameters of the moſt remote ones, *viz.* Jupiter and Saturn as much greater; making that of Jupiter of almoſt $54''$; and that of Saturn without his Ring, of $27''$. If we make the Earth's apparent Diameter from the Sun to be a mean between the Diameters of the neareſt, namely, the apparent Diameters of Venus and Mercury, it will be of 24″; if we make it a mean between the Diameter of the four neareſt to the Sun, (for the two ſuperior Jupiter and Saturn, are altogether different in Magnitude from the inferior Planets) it will be of about 18″ or 19″, ſuch as Mr. *Caſſini* lays it down.

Of theſe ſix Planets three have Satellites; namely, the Earth has only one, which is the Moon, which will appear to be diſtant from it, at moſt, only ten Minutes; Jupiter has four Moons, the furthermoſt of which will never be further from Jupiter than 9′; Saturn appears accompanied with five Satellites (beſides the Ring) whoſe outermoſt goes about as far from Saturn. The Ways of theſe Satellites ſeen from the Sun being ſometimes more and ſometimes leſs obliquely ſeen, will appear Ellipſes ſometimes broader and ſometimes narrower. Sometimes the Planes of thoſe Orbits produc'd go thro' the Sun, in which caſe the Ellipſes are chang'd into right lines, appearing firſt a very narrow Ellipſe, which happens twice in every revolution of that Orbit about the Sun, if ſo be that the Plane of it remains parallel to itſelf; whence the Satellite will ſometimes appear cover'd by its Primary, and ſometimes again to cover it, and at another time to move in an Ellipſis about its Diſc.

The Primary Planets and their Satellites have their Surfaces not only not ſmooth and poliſh'd, but ſo ununiform as to have ſeveral brighter and darker places, that is, which do more or leſs reflect the Rays of the Sun, by which they are enlighten'd. Theſe Spots by the Rotation of the Planets about their proper Axes, deſcribe Circles; therefore when ſeen from the Sun, their Ways on the plane of the Planetary Diſc will be ſometimes Elliptical and ſometimes ſtreight, (as the before-mentioned Satellites Ways), according as the Sun is lifted up above either ſide of the plane of the Circles, or is found in it, as it is at the time of the Æquinox of that Planet. If there are ſeries or heaps of thoſe Spots, as in Mars and Jupiter, theſe will be ſeen bent into Semi-Ellipſes (for the other half of the Ellipſis lies on the back ſide of the Planet) or extended into right lines; which alſo happens to the outward Edge of Saturn's Ring. But ſome of the Spots are ſometimes hid for ſome time, and ſometimes appear again as long; namely when a Planet approaches the Pole which is turn'd from the Sun or that which is toward it. Theſe things are very obſervable in Saturn and the Earth, but ſcarce in the others; becauſe in them the light of the Sun always reaches from Pole to Pole.

The Eye plac'd in the Sun *will ſee no* Shadow nor Eclipſe, becauſe they are always turn'd from the Sun. But ſometimes to an Eye on the ſurface of the Sun, when the Planet is in the Horizon, its Secondary will appear in the Penumbra of the Primary, or the Penumbra of the Secondary may appear caſt on the Diſc of the Primary, which will be known by the Colour, which will be ſomething duller.

Moreover, the Heliocentric places of Planets (and therefore with more difficulty their Places ſeen from a point on the Sun's ſurface) will be eaſily determin'd by what has been ſaid in the 3^{d} Book. Nay, we muſt determine every Planet's Heliocentric Place before we can determine its Geocentrical one.

Beſides, the ſix Primary Planets and the ten Satellites (for hitherto we Inhabitants of the Earth have ſeen but juſt ſo many) which to a Solar Obſerver will appear to move in a Zodiac ſcarce ſixteen Degrees broad, and not much inclin'd to the Circle of the firſt Motion, and the ſame way, and in Orbits almoſt concentric to the Sun; beſides them, I ſay, there are other Bodies of a different kind whoſe number is yet uncertain, carried in very excentric Orbits (call'd Comets) which ſometimes come near to the Sun, and ſometimes go off to immenſe Diſtances from it. Theſe Comets will appear to be carried among the Fix'd Stars not all the ſame way, but one one way and another another, (tho' every ſingle one obſerv'd by itſelf will always go the ſame Road; and for the moſt part do not go in the Zodiac or Way of the Planets, but in ways very much inclin'd to it, but always in great Circles of the Sphere.

The Comets are not only diſtinguiſh'd from the Planets by the great difference of their motion from the motion of the Planets, but by the Hairineſs, which being ſeen from the Sun, does not (as it often happens with reſpect to us) extend it ſelf like a Tail, but appears always to encompaſs the Head, as we ſee it when a Comet is in oppoſition with the Sun in reſpect to the Earth. But ſuch a Periwig is ſomething thicker towards that part among the fix'd Stars where the Comet is going, but more extended towards the contrary parts. Moreover, that Hairineſs is greater and brighter when the Comet deſcends to its Perihelium; not only upon optical accounts, becauſe it is both nearer to the Eye and the Sun which enlightens it; but chiefly becauſe the Vapour which cauſes it is thrown out in greater quantity, by reaſon of the encreaſe of the Heat which cauſes it. But the Periwig about a Comet will not even then appear to a

Solar Obſerver, to be compar'd to that which at ſuch a time wou'd appear to a Terreſtrial Obſerver.

What we have ſaid is upon ſuppoſition, that the Eye in the Sun is not hinder'd by the Atmoſphere about it from ſeeing as far and as freely as we do in the Night when the Moon does not ſhine. For if it had a ſharper or duller Sight, it might ſee other things further off or leſs enlighten'd, or not ſee the things which we have mention'd; but if it ſees them, it will ſee them in the forementioned manner.

Beſides the foreſaid Phænomena ſeen from far, it will ſee a great many changes on the ſurface of its own Habitation: For at one time it will be cover'd with great, denſe and frequent Spots or Iſlands, which in a little time will change their figures and then be altogether diſſipated and vaniſh; ſometimes for a long time there appear no ſuch Spot (at leaſt not ſo great or denſe) for a long time. But to an Eye in the place which we mention, they will not be look'd upon as Celeſtial Phænomena; therefore we ſhan't ſay much of 'em in this place.

PROPOSITION II.

TO *deſcribe what Phænomena will appear to an Eye in Mercury.*

Beſides the Phænomena of the order of the Fix'd Stars, which agree alike with an Eye plac'd any where within the Orb of Saturn, it is plain that the Eye poſited in Mercury, will ſee the Sun almoſt three times broader than we ſee it from the Earth, becauſe (by *Prop.* 40, and 42. *B.* 1.) Mercury is almoſt three times nearer to the Sun than the Earth. Whence alſo the Solar Diſc ſeen from Mercury, is ſeven times greater than the Diſc as it appears to us, and (by *Prop.* 48. *B.* 1.) Mercury has ſeven times more Light and Heat, *cæteris paribus*, than the Earth. But theſe Qualities are much more or leſs intenſe according to the different diſtance of Mercury from the Sun; for its Orbit is much more excentric than the Orbit of any other Planet.

The accelerating gravity towards the Sun is alſo ſeven times greater there than here. The denſity of Mercury, and conſequently the accelerating gravity of Bodies on its ſurface towards its Center, may be gather'd from *Prop.* 49. *B.* 3. tho' leſs accurately than that towards the Sun, becauſe it is only done analogically. But we ſhall not ſpeak of ſuch things here or in the following Pages, as not belonging to the Celeſtial Phænomena, which alone are to be conſider'd in this place.

It has not yet been found by Obſervation, whether Mercury turns upon its own Axis, and therefore it can't be certainly defin'd whether to an Eye plac'd on its ſurface, all the things without will appear to revolve with a contrary Motion; that is, whether it has the viciſſitude of Day and Night, much leſs what is the ſpace of its *Nychthemeron* or Natural Day; but one may ſafely enough conjecture that it does revolve about its Axis, ſince the other Planets do. But the Year of Mercury is ſcarce equal to a quarter of our Year, tho' it is uncertain whether it has different Seaſons, becauſe they depend upon the inclination of the Axis of its Rotation, which is unknown to the plane of the Orbit which it deſcribes about the Sun.

The Eye plac'd in Mercury looking at the Sun, will ſee the Sun's Spots (when it has any) ſometimes in a right line croſſing his Diſc from

Eaſt to Weſt, at another time their Way will appear elliptical and curve toward one ſide or t'other; as was ſhewn in the *Schol.* of *Prop.* 70. B.4. and all the variety of this Phænomenon will happen in one Year, in which the Track or Way of the Spots will appear twice rectilineal. In this caſe Mercury will ſee all the Spots on the Sun's ſurface, becauſe being in the plane of the Sun's Equator it will ſee each of its Poles in the margin or edge of the Diſc. But when the Eye is rais'd above the plane of the ſolar Equator towards either Pole; that is, when the Spots appear to deſcribe an Ellipſe, one Pole and the Spots near it are hid in the Sun's hinder Hemiſphere. But in every caſe a Spot near the Sun's Limb will appear of a Figure different from that which it had in the middle of the Diſc, as being very obliquely ſeen.

But the Way of the ſolar Spots *ſeen from* Mercury will be almoſt always ſtreight; becauſe Mercury never much declines from the Plane of the Sun's Equator,and therefore not ſenſibly from the Planes of the Circles which the Spots deſcribe by their rotation.

And as the Plane of the Orbit of the Earth is moſt of all inclin'd to the ſaid Plane of the Sun's Equator, the Way of the Spots ſeen from the Earth appears more curve than if the Sun was ſeen from any other Planet. This Curvature will be ſomething leſs ſeen from Jupiter or Mars, and yet leſs from Saturn, leſs ſtill from Venus, and leaſt of all from Mercury, as we juſt ſaid. Mercury will alſo ſee the Way as well of the Satellites as of the Spots of the other Planets, ſometimes extended in a right line, ſometimes bent in a curve, and ſome of them ſometimes hid for a great while, and ſometimes as long viſible, according as they are near to or diſtant from the hidden or ſeen Pole of the Planet.

The other five Planets are ſuperior to Mercury: therefore their Phænomena are to an Eye plac'd in Mercury, as the Phænomena of Mars, Jupiter and Saturn ſeen from the Earth, concerning which ſee the 7th, 8th and 9th Propoſitions of the firſt Book. Therefore Venus and the Earth, when they are in oppoſition with the Sun, will ſhine with a full Orb and reflecting the Sun's Rays very directly will ſhine upon Mercury all Night, and powerfully expel the Darkneſs. For ſince Venus when it is horned, and ſhews the leaſt part of its enlighten'd Hemiſphere to the Inhabitants of the Earth, is ſo bright as to caſt a Shadow, it will appear very bright to Mercury, to which it turns its whole enlighten'd Hemiſphere. The Earth alſo has a great power of reflecting the Sun's Rays, as it appears from that Light which it caſts upon that part of the Moon's Diſc which is turn'd towards it: But being as well more diſtant from Mercury, which is to be enlighten'd, as from the Sun from which it receives its Light, it will do much leſs in this caſe than Venus, unleſs it ſhou'd be more apt to reflect Light than Venus, which is not likely. The ſuperior Planets do yet enlighten Mercury with a weaker Light than they do us.

Mercury ſees no inferior Planet known to us, and therefore ſees no celeſtial Body falcated or horned; and therefore a Spectator there will want the Argument taken from ſuch Phaſes of the Planets to eſtabliſh the true Syſtem of the World: For the Phaſes of the inferior Planets have clearly ſhewn that they mov'd about the Sun; whence it was natural to think the ſame of the other Motions. But nevertheleſs, tho' we don't ſee any Planets inferior to Mercury, it does by no means follow that there are no ſuch: for Mercury himſelf is ſeldom ſeen in our oblique Sphere, and one that ſhould be much inferior to it wou'd never be ſeen upon account of its nearneſs to the Sun.

An Obſerver in Mercury, as well as we upon Earth, wants Mediums to compare the Diſtances and Diameters of the Celeſtial Bodies with the Diameter of his Habitation, or with his own Meaſures, unleſs he has very exact Inſtruments. For the Parallax of the Sun ſeen from Mercury (as far as we can gueſs from Analogy) is leſs than its Parallax when ſeen from the Earth; the apparent Diameter of Mercury ſeen from the Sun been leſs than the Diameter of the Earth ſeen from it: and Venus when in oppoſition *with* the Sun, and therefore neareſt to *Mercury*, *is* near as far from Mercury as the Sun; and therefore the Parallax of Venus (even when Mercury in its Aphelium ſees it oppoſite to the Sun, and conſequently neareſt of all to it ſelf) is not greater than the Parallax of the Sun ſeen from the Earth. And theſe are the two great Bodies of the World, near to Mercury, which have the moſt ſenſible Parallax.

But becauſe Mercury changes its place, a Spectator in Mercury, by Obſervations made from different Stations, will gather the ratio between the diſtance of Mercury from the Sun, and the diſtance of any other Planet from it, after the ſame manner as we Inhabitants of the Earth do it from the Parallax of the *Magnus Orbis.* But certainly it cannot be ſo ſafely determin'd becauſe the diſtance of Mercury from the Sun is not ſo great in reſpect of the diſtance of the other

ther Planets, as the diſtance of our Earth from the Sun. But as it is yet ſenſible enough in reſpect of them, being about the five and twentieth part of the diſtance of Saturn from the Sun; the ratio of all thoſe diſtances will conveniently enough be determin'd by Obſervations made in Mercury, if ſo be that the Obſerver knows the motion of its Habitation about the Sun. But if, truſting to his Senſes, he thinks his Mercury to be immovable, he won't be able to do even this, any more than our Aſtronomers Inhabitants of the Earth as long as they look'd upon the Earth as immovable; which is evident from the Diſtances of the Planetary Orbits aſſign'd by the Followers of *Ptolemy*, ſo vaſtly different from one another, and yet agreeing well enough with the Phænomena. For in the Syſtem whereby the Obſerver's Place is ſuppos'd unmov'd, if an Epicycle be ſuppos'd for any one of the ſuperior Planets to run thro' in one Revolution about the Sun, and ſufficient to explain its greateſt obſerv'd diſtance from the place in its excentric near the Quadrature with the Sun, the Semidiameter of the Excentric which is the Deferent of the Epicycle, may (without contradicting the Phænomena) be taken of any magnitude, ſo that it be ſuch that the Planet, when neareſt, may be further from the Earth, than what wou'd make its diſtance liable to be defin'd from the Parallax: nay, nothing hinders Mars from being plac'd above Jupiter, or Saturn below Mars, unleſs it be the Rule given in the foregoing *Prop.* and that depends upon Sagacity and not Demonſtration.

The Places of the Sun and all the Planets, ſeen from Mercury, are found the ſame way as when ſeen from the Earth, and defin'd by the help of the Tables in *B.* 3. For ſince out of them the Heliocentric Places, as well of Mercury as of any other Planet are readily defin'd, the Hermocentrical Place of this other will be defin'd by an Operation like the operation of the 40th *Prop. B.* 3. which is alſo true of any other Planet, and therefore I ſhall not repeat it in the following Propoſitions.

The Phænomena of the Comets ſeen from Mercury, or any other of the Planets, are almoſt the ſame as their Phænomena ſeen from the Earth, of which we ſpake in the foregoing Book; for they differ in nothing but their diſtance.

PROPOSITION III.

TO deſcribe the Phænomena as ſeen by an Eye in Venus.

In Venus the Sun's Diameter appears almoſt as little again as it does in Mercury, and once and a half greater than to us on Earth. Whence the Sun's Diſc ſeen from Venus is more than twice greater than from hence by us, and alſo the Light, Heat, and accelerating Gravity towards the Sun, are encreas'd in the ſame Proportion.

Venus's Year (namely, its Period about the Sun) is equal to ſeven and a half of our Months: but the Day in Venus is almoſt an Hour leſs than our Day.

Venus ſees four ſuperior Planets, whoſe Phænomena therefore are like thoſe of Mars, Jupiter and Saturn, ſeen from the Earth, or all of them ſeen from Mercury. But our Earth ſeen in oppoſition with the Sun, and for the whole Night, ſince (by *Prop.* 9. *B.* 1.) it ſhines with a full Orb, will appear very bright; and this Light will be en-

encreas'd by that of our Moon, ſhining alſo at that time with a full Orb. The Moon will appear always to accompany the Earth, as it really does, and ſometimes will be above half a degree diſtant from the Earth.

Venus has only one inferior Planet, namely, Mercury, which is never farther diſtant from the Sun than about 38 Degrees, and which will appear in the ſame manner to Venus, as Venus and Mercury do to the Inhabitants of the Earth, concerning which ſee *Book* 1. *Prop.* 4, 5, and 6.

An Obſerver in Venus will not want means to compare the diſtances of celeſtial Bodies with his own meaſures; for he will compare the Diameter of his Venus with the Meaſures which he commonly makes uſe of, as we do our Earth's Diameter with our Meaſures, by *Schol. Prop.* 17. *B.* 2. and the Parallax of the Earth in oppoſition to the Sun, is four times greater than the Parallax of the Sun ſeen from the Earth, and therefore not altogether inſenſible: Therefore he knows the ratio of the Diameter of his Venus to the Diameter of the Earth oppoſite to the Sun, and Retrograde from Venus; that is, the difference of the diſtances of Venus and the Earth from the Sun; but the Parallax of the other Planets (even when neareſt to Venus) is almoſt inſenſible. If the Obſerver in Venus knows that Venus moves about the Sun, he will from thence gather the diſtances of the Planets from the Sun, as the Inhabitants of the Earth do: And from the ratio between two Quantities, and their difference being given, the Quantities themſelves are alſo given, and conſequently thoſe others having given Ratios to one of the ſaid Quantities. But if he does not know that his Habitation moves, he will indeed (by the

Rule given in *Prop.* 1.) ſettle by conjectures the Order of the Superior Planets, but not their Diſtances, as has been ſaid in the foregoing Propoſition concerning Mercurial Aſtronomers. He will have the ſame reaſon of doubting of the order of Mercury and the Sun as the Ancients (Inhabitants of the Earth) had of doubting of the order of Mercury, Venus, and the Sun; ſome ſaying that Mercury is above the Sun in reſpect of Venus at reſt in the Center; others ſaying that it is below it; and laſtly, others ſaying that Mercury moves about the Sun as the Center of its motion. But at laſt it will be out of doubt that Mercury moves about the Sun when the Sight can be ſo far help'd as to make its Phaſes viſible. For before that, one may (without contradicting the Phænomena) err ſo much in the eſtimate of the Diſtances of the Planets, that if an Epicycle be allow'd to Mercury, whoſe Diameter ſeen from Venus is equal to two Signs and a half, and its Center be plac'd in a Line joining the Sun and Venus, or produc'd beyond the Sun, the Deferent of the Epicycle may be plac'd beyond the Sun's Orbit, or any where ſhort of it, if ſo be that it exceeds the limits of a Diurnal Parallax. Which is plain from this, that in the Syſtem of the Earth unmov'd (without contradiction to the Phænomena then known) the Deferents of the Epicycles of Mercury and Venus are plac'd far ſhort of the Sun) by almoſt all the Aſtronomers who are the followers of *Ptolemy*; tho' now there is no Body but what may of of himſelf diſcover that thoſe Planets go round the Sun.

Having once determin'd the diſtances of the Planetary Orbs from the Sun in the true Syſtem of Venus moving about the Sun, if another Aſtronomer will have Venus (his Habitation) to be in the Center of the World and unmov'd, he may ſuppoſe it two ways according to the Laws of Optics. Firſt, that Venus being in the Center, the Sun together with all the Planets, and their Orbits which they deſcribe about the Sun will move the ſame way in right lines parallel to each other; ſuch an Obſerver in Venus will have the *Tychonic* Syſtem, namely one in which the Sun is carried in an Orbit equal, ſimilar, and contrary to that which Venus does really deſcribe about the Sun, and all the Planets with their Satellites (except that they are carried by the Sun about Venus) will perform the ſame motions about the Sun which they do in the true Syſtem, the Primary Planets in Epicycles, and the Satellites in Epicyclo-Epicycles; and the Orbit of the Earth will interſect the Sun's Heaven juſt as the Orbit of Mars in the Scheme of *Tycho* on Earth. But the Orbit of Jupiter does not interſect but contain the Orbit of the Sun in ſuch a Syſtem on Mars. Secondly, An Aſtronomer may rightly eſtabliſh a Syſtem of Venus unmov'd in the Center, if after he has promoted his Venus to the central place, and at the ſame time made the Sun and all the Planets go in right lines parallel to each other as before, but not the Orbits of the Planets, which he keeps unchang'd, and the ſame which they deſcrib'd about the Sun (whilſt it remain'd in the Center) becauſe our Aſtronomer makes them move in ſuch Orbit about himſelf; and he will on his Venus have the *Ptolemaic* Syſtem, in which the Sun is carried about Venus after the manner explain'd in the *Tychonic* Syſtem, and every Planet is mov'd in an Epicycle equal, ſimilar and contrarily poſited to the true Orbit of Venus about the Sun; but the Center of the Epicycle is carried inthe ſame Orbit and after the ſame manner that the Planet it ſelf was mov'd before the Sun was thruſt out of the Central place to put Venus there, and the Epicyle of the Earth deſcends into the Heaven of the Sun after the ſame manner that the Epicyle of Mars does in ſuch a ſort of a Syſtem upon the Earth, which alſo will be true in any other Planet beſides Mars. Thus wou'd the *Ptolemaic* Syſtem be made conſiſtent with itſelf, and the motion of the Centre of the Epicycle be taken away from the Sun, and the motion of the Planet in the Epicycle be aſcrib'd to it as well in the inferior as in the ſuperior Planets; and, univerſally this Syſtem wou'd agree with all the Phænomena as well as the *True one*, or as the *Tychonic one*. Theſe two Syſtems (namely the *Ptolemaic* and the *Tychonic*) interchange the Deferents and the Epicycles: For the *Tychonic* makes the Deferent the ſame and equal for all the Planets,ſimilar, and contrarily poſited to the Orbit of the Spectator's Planet about the Sun, but with different Epicycles, ſimilar and ſimilarly poſited to the true Orbits of the reſpective Planets about the Sun; for in this Syſtem the Sun has no Epicycle, as in the True one it has no Orbit, for in ſuch a Syſtem the Epicycle, and in our true one the Orbit is only look'd upon as a point, which Figures are ſimilar and ſimilarly poſited. But the *Ptolemaic* Syſtem gives all the Planets Epicycles equal to the Orbit of the foreſaid Spectator's Planet about the Sun, ſimilar and contrarily poſited to it, but different Deferents, namely, the ſame as their reſpective Planets Orbits about the Sun in the true Syſtem. But becauſe the Syſtem aſſerted by the *Ptolemaics* is not made out of the true Syſtem,

ſtem, but built by degrees on the Phænomena without the help of the true one; therefore in it the proportions of the Deferent Orbs (as the followers of it make their Syſtem) is unknown, and only the ratio between the Deferent and Epicycle of each Planet is ſettled from its motions; whence it happens that in Saturn, Jupiter and Mars the ſemidiameter of the Deferent (from *Ptolemy*'s own Numbers) is to the Semidiameter of the Epicycle as the Diſtance of a Planet from the Sun to the Diſtance of the Earth from it. But as in the *Tychonic* Syſtem all Planets have the ſame Deferent, if from the motion of any one be found the true ratio between its Epicycle and Deferent, there will be found (*ex æquo*) the ratio between the Epicycles of them all, that is, between the Orbits of all the Planets; and nothing elſe is done by the true Syſtem. The juſt Syſtem of *Ptolemy* (that is the *Ptolemaic* Syſtem, built on the foregoing Principles) ſuppoſes the Epicycles of the inferior Planets equal to the Sun's Orbit: But as it appear'd monſtrous to make an Epicycle greater than its Deferent, therefore the *Ptolemaics* chang'd the Deferent of each of the inferior Planets into an Epicycle, but yet in ſuch manner that both (which they cou'd not hinder) the Deferent and the Epicycle, ſhou'd retain their nature; namely, that the motion of the Epicycle in the Deferent ſhou'd be join'd to the Sun, and the Planet's motion in the Epicycle ſhou'd be off from the Sun. By this change they left their genuine above-deſcrib'd Syſtem, and as it were fell into the *Tychonic*; and the *Ptolemaic* Syſtem of the inferior Planets differs from the *Tychonic* only in this, that, without knowing the true Semidiameter of the Deferent, it lays down only the true ratio between the Semidiameter of the Deferent and the Semidiameter of the Epicycle; and That according to *Ptolemy* is the ſame with the ratio of the Diſtance of the Earth from the Sun to the Diſtance of the ſaid inferior Planet in the true Syſtem. And the foreſaid two Syſtems are made by retaining the Planetary Orbit of the true Syſtem; but if you change them you may make an infinite number of Syſtems, each of which will agree with the Phænomena, nay, find the place of every Planet to be the ſame in reſpect of the Earth (that is, a right line being drawn the ſame in Magnitude and Poſition) as that in the true Syſtem; and none of theſe Syſtems wou'd be found to be falſe without the help of Natural Philoſophy.

We have more than a bare ſuſpicion to incline us to believe that Venus has a Satellite, from the Obſervations of that curious *Aſtronomer* Mr. *Caſſini*, made by chance in the two Years 1672 and 1686; in the laſt of which, on the 28th day of *Auguſt*, at a quarter after Four in the Morning, through a 34 Foot Teleſcope, he for a quarter of an Hour ſaw as it were a Satellite of Venus diſtant from Venus about $\frac{3}{5}$ of the Diameter of Venus, of the ſame Phaſis as Venus, (as it ought to be, ſince their ſituation in reſpect of the Sun enlightning, and the Earth ſeeing, was the ſame) but of no regular Figure, and of a Diameter which was about a fourth part of Venus's Diameter. That Mr. *Caſſini* did not ſee that Satellite at any other time tho' he much endeavour'd it, might be owing to this, that the ſurface of that Satellite is leſs apt to reflect the Sun's Light, as we find the ſame thing true in reſpect of the Spots of our Moon. For if our whole Moon ſhou'd as weakly reflect the Ligh as its darkeſt Spots do, it wou'd ſcarce be viſible at Venus, and appear of an irregularly terminated Figure: Yet we muſt confeſs, that ſuch a Satellite of Venus (if it has one) is of a very different nature from its Primary, which does ſo powerfully and ſtrongly reflect the Light of the Sun, that *Hugens* with a Teleſcope of 60 Foot cou'd ſee nothing like a Spot in it; which is alſo the reaſon why the poſition of the Axis of its diurnal Rotation is not defin'd. But this difference between Venus and its Satellite does perhaps ariſe from this, that it is not the ſolid Globe of Venus, but an Atmoſphere about it very denſe and full of Vapours which reflects the Sun's Light to us, ſuch an one as which is wanting to its Satellite; as it happens in our Moon, tho' the Earth is encompaſs'd with an Atmoſphere denſe enough and full of Clouds, which wou'd hinder an Obſerver in Venus from perceiving the diſtinction of Seas and Land, but he wou'd ſee its Surface almoſt ſmooth and equally enlighten'd, ſuch as Venus appears to us. If Venus has its Moon, it is almoſt in all reſpects like our Earth, from which it differs leſs in the quantity of its Day and Night, and degree of Heat, than any other Planet.

PROPOSITION IV.

TO deſcribe the Phænomena to an Eye poſited in Mars.

The Sun's Diameter appears one and a half leſs from Mars than from the Earth, and therefore it gives twice leſs Light and Heat in the Region of Mars than it does to us on Earth. But theſe things, by reaſon of the great Excentricity of the Orbit of Mars, are ſenſibly chang'd, tho' not ſo much as in Mercury.

Mars's Year is almoſt equal to two of our Years; but the natural Day but a little greater than ours: But the artificial Day, during which the Sun ſhines above the Horizon, (except the Twilight before the Sun's Riſing, and that after its Setting, according to the Height of the conſtant Atmoſphere) is almoſt always and every where equal to the Night, and therefore very little difference of the Summer and Winter is there felt; and that becauſe the Axis of Mars's diurnal Revolution is almoſt perpendicular to the Plane of its Orbit. In the mean time the Places at different diſtances from the middle Circle between the Poles have very different degrees of Heat, upon account of the different inclination of the Sun's Rays to the Horizon, as it is on the Earth at the time of the Equinox: And from hence I ſuſpect that the *Faſciæ* or Belts ariſe which run parallel in the Circle that is in the middle between the *Poles*. For ſince the ſame degree of Heat always continues in the ſame Climate, it is likely that the Spots in Mars (like the Clouds and Snow on Earth) owing their riſe to Heat and Cold, are extended according to the Climate, and make *Faſciæ* parallel to the Circles of Mars's diurnal rotation; which is alſo true of Jupiter, who like Mars has a perpetual Equinox.

The Obſerver in Mars ſees two ſuperior Planets, *viz.* Jupiter and Saturn, and two inferior Planets, namely the Earth and Venus; whoſe Phænomena are like the above-deſcribed Phænomena of the ſuperior and inferior Planets. He will never ſee Mercury, (as never being above half a Sign from the Sun) unleſs by chance like a Spot in the Sun as it goes over his Diſc; at leaſt if Mars has an Atmoſphere about it as large and as denſe as that of our Earth, as it ſeems likely by the obſervation of the Fix'd Stars which grow duller, and ſeem almoſt extinguiſhed before they appear to touch Mars's Globe. He will ſee Venus as far diſtant from the Sun as Mercury appears to us, and the Earth as far as we ſee Venus: And as the Earth ſeen from Mars, appears near and almoſt in conjunction with the Sun, he will ſee the ſame thing at the Earth as the ingenious Mr. *Caſſini* ſaw once or twice at Venus; namely the lower Planet falcated or horned, and its Companion by it of the ſame Phaſis, which at moſt will not appear above a third part of a Degree from it.

An Aſtronomer in Mars is almoſt deſtitute of means to compare the Celeſtial Bodies with his Mars, and their Diſtances with meaſures that he knows; by reaſon of the Diſtance of the neareſt Planets, *viz.* Jupiter and the Earth. For when the Earth is neareſt to it and the loweſt of all the Celeſtial Bodies, its Parallax will not be ſenſible, not amounting to above 12 Seconds, much the ſame as the Suns Parallax with us. But if he is ſenſible of the motion of his Mars about the Sun, he may compare the Diſtances of the Celeſtial Bodies one with another: If he is not ſenſible of it, he can't even know that; as has been amply ſhewn in the two foregoing Propoſitions.

PROPOSITION V.

TO deſcribe the Celeſtial Phænomena as they appear to an Eye in Jupiter.

The Diameter of the Sun ſeen from Jupiter will appear a little more than five times leſs than it does when ſeen from the Earth; and therefore its Light and Heat will be twenty-ſeven times leſs in Jupiter than with us. Jupiter's Year is almoſt equal to twelve of our Years: But on the other hand the natural Day will be leſs than ours, ſcarce ariſing to ten of our Hours; (And the motion of Jupiter about its Axis is perform'd in the leaſt Period of all the diurnal Rotations which the Aſtronomers have hitherto obſerv'd in any of the Celeſtial Bodies:) Whence in Jupiter the Year conſiſts of above ten thouſand Days. Jupiter's Day is divided into an Artificial Day and a Night almoſt equally over the whole ſurface of Jupiter; becauſe the Axis, about which Jupiter revolves, is almoſt perpendicular to the plane of the Orbit, which he deſcribes about the Sun, whence the ſame things will obtain as in Mars, concerning which we ſpoke in the foregoing Propoſition.

Tho' there are four Primary *Planets below* Jupiter, the *Eye* in Jupiter *will* ſee none of them; if we ſuppoſe it not more ſharp-ſighted than we, and believe that it has ſuch an Atmoſphere as the Earth: For Mars himſelf who goes fartheſt of all from the Sun, will not be above 18 Degrees diſtant from it; but as Mars is both ſmall, and weakly reflects the Sun's Light, it will ſcarce be ſeen at that Diſtance from the Sun. It is true indeed that the Sun's Light at Jupiter is not ſo bright as at the Earth, and therefore that the neighbouring Stars are not ſo much darken'd by it; but alſo the Light of Mars, the Earth and Venus, is diminiſh'd in the ſame ratio by reaſon of the encreas'd Diſtance of the Eye: Whence at the ſame diſtance from the Sun, when ſeen from Jupiter, they are likewiſe extinguiſh'd by the Light of the Sun. But the Planets below Jupiter may be ſeen like Spots upon the Sun's Diſc ; but that will only happen by chance : For an Aſtronomer in Jupiter being not more certain of their exiſtence than we are of the exiſtence of Planets below Mercury, will not reckon upon their motions ſo as to expect to ſee them at a due time ; an Eye therefore in Jupiter will ſee only Saturn and that above it.

But then ſuch a Spectator will ſee carried in a Circle in the middle between the Poles of its firſt Motion, his four Satellites plac'd near him and ſtrongly reflecting the Sun's Light, unleſs ſo much as they appear by the Sun's Parallax to decline towards one Pole or the other, when the Eye is plac'd towards either Pole ; ſo that the number of the Planets will appear to be Six to an Obſerver in Jupiter. For there is no doubt but that looking upon his Habitation, the Globe of Jupiter, as unmov'd (which by reaſon of the immenſe bigneſs that he eaſily finds his Globe to have above the Planets near him, he may much more reaſonably do than the Inhabitants of the Earth concerning the Earth) he will reckon his four attendants as Planets, to which number he will add the Sun and Saturn, juſt as we reckon our Moon among the other ſix : And indeed he will not err much in their Order, if he looks upon that as moſt remote, whoſe Period about Jupiter takes up the longeſt time.

Moreover, he will eaſily compare the Diſtances of the four Inferior with his own meaſures ; namely with the Diameter of Jupiter, as we do ours with the Diameter of the Earth ; and it will be eaſier for him to compare the Diſtance of the four neareſt Planets with the Diameter of Jupiter, than it is for us to compare the Diſtance of the Moon with the Diameter of the Earth : For the horizontal Parallax of the remoteſt of them ſeen from Jupiter is more than twice greater than the horizontal Parallax of the Moon ſeen from Earth, and therefore very ſenſible. And tho' the Globe of Jupiter be of an immenſe bigneſs in reſpect of our Earth, the Parallax of the Sun ſeen from Jupiter will be ſcarce ſenſible, as not ariſing to 20 Seconds; neither will the *Parallax* of Saturn (even when oppoſite to the Sun and neareſt to Jupiter) be much greater: It will therefore be difficult for the Jovial Aſtronomer to compare the Diſtances of the Sun and Saturn with his own meaſures. If he knows the motion of his Jupiter about the Sun, he will eaſily find the ratio between the Diſtances of Jupiter and Saturn from the Sun, otherwiſe not; as it has been explain'd in the 3d *Prop.*

Therefore the Jovial Obſerver relying upon his Senſes will diſtinguiſh two ſorts of Planets; namely four near him, and two very *remote, viz.* the Sun and Saturn of a ſmall, and the other four of a much greater apparent Diameter. For of the remoteſt the Sun appears about 6' broad, but Saturn even when neareſt, has ſcarce a Diameter of half a Minute. Of the others, the fourth from Jupiter will appear to be of about the ſame bigneſs as the Moon appears to the Inhabitants of the Earth, that is of a Diameter five times as big, and Diſc five and twenty times as big as the Sun to an Inhabitant of Jupiter; and if the remaining Satellites are not much leſs than the outermoſt, they will yet appear bigger, eſpecially if they are not leſs than our Earth, as that accurate Coſmographer Mr. *Hugens*, affirms that they are not. Beſides, the four neareſt and apparently greateſt, will be diſtinguiſh'd from the leſſer and moſt remote, becauſe in the neareſt Planets the Squares of the

Periodical Times are as the Cube of their Diſtances from the Center of Jupiter; but this will by no means hold if you compare any of the great ones with any of the little ones. Altho' thoſe that are neareſt appear to be biggeſt, the Sun will appear vaſtly brighter than them; nay, from their Phaſes, which depend upon their Aſpects to the Sun, they will appear as ſo many Moons. Whence the Spectator in Jupiter will have four ſorts of Months denominated from the four Moons: There will be of the leaſt Months more than 2407 in a Year; of the Months immediately greater, the number will be about as little again; of the third in order, the number will be about ſubduple in a Jovial Year, or about ſubquadruple of the leaſt; but of the greateſt Months there are about 254 in a Year. And therefore tho' the notation of the Time be ſomething more intricate on Jupiter, becauſe the Year contains a great number of Days, yet by thoſe four kinds of Months it becomes ready enough: For in the leaſt Month there will be only four Days and a quarter; but in the greateſt ſomething more than forty Days.

Beſides, theſe Moons, when they are in oppoſition with the Sun in reſpect of Jupiter, running into Jupiter's Shadow, are Eclips'd, and again, when they are in Conjunction with the Sun, as they throw their Shadows on Jupiter, they will make the appearance of an Eclipſe of the Sun to an Eye poſited on that part of Jupiter where the Shadow falls (which is a ſmall part enough of Jupiter) as our Moon does. But becauſe the Orbits of thoſe Moons about Jupiter are in a Plane (all of them indeed nearly in the ſame Plane, except that the ſecond from Jupiter does a little deflect from it) inclin'd to the Plane of [illegible]

Jupiter's Way about the Sun, the Eclipſes of theſe Moon's are central and very laſting, when the Sun is in either Node of the Moons. But when the Sun is not in it, the Eclipſes may alſo be total, tho' not central; becauſe the Shadow of Jupiter is almoſt ten times broader than any of the Satellites, and the apparent Diameter of each Moon is five times greater than the *Sun's* apparent Diameter. And this great *Inequality* of the Diameters, and the ſmall Inclination *of* the Plane of the Orbit of the Satellites to the Plane of the Orbit of Jupiter about the Sun, is the cauſe why the Eclipſes, as well of the Moons as of the Sun, happen at every Revolution, tho' the Sun is very far diſtant from the Nodes. And indeed the loweſt of them (tho' at the ſame time the Sun ſeen from Jupiter is moſt of all diſtant from the Nodes) will nevertheleſs fall into Jupiter's Shadow, and alſo, *on the other* hand, their Shadows will *fall on Jupiter*; but in that caſe the moſt remote Satellite, for the ſixth part of a Jovial Year, will miſs the Shadow when it is in Oppoſition with the Sun, and Jupiter alſo will not be any where darken'd by its Shadow when in Conjunction with the Sun: But an Eye plac'd in Jupiter will in intermediate Times and Places, ſee the Phænomena of partial Eclipſes. Moreover, an Eclipſe of the Moon is made here by a Moon, whoſe Phaſis is ſometimes quite different, nay, contrary to the before deſcrib'd Eclipſe of a Moon running into Jupiter's Shadow: For in this the Eaſtern Limb of the Moon is firſt Eclips'd, and the Weſtern one does laſt emerge out of the Shadow; but, in ſome of them, firſt the whole Weſtern Limb is Eclips'd, and the Eaſtern one does laſt of all recover his Light, in others juſt the contrary.

Tho' the Shadow of Jupiter is far extended beyond its Satellites, it does not reach ſo far as any other Planet, which alſo is true of all the other Primary Planets. For Saturn alone, even tho' it were infinite, cou'd be immers'd in it; but the Shadow of Jupiter cannot be extended ſo far as Saturn, unleſs Jupiter's Diameter ſhou'd be nearly half the Sun's Diameter; and it is ſcarce the ninth part of it.

If Jupiter's Surface has a great part cover'd with a Fluid, and we ſuppoſe that Navigation cou'd be practis'd on thoſe Seas; it wou'd be very convenient, as well by reaſon of the very ſhort Nights, (and thoſe upon account of the number of the Moons, ſcarce without a Moon, and for the moſt part enlightened by ſeveral of them, and thoſe very bright,) as becauſe by the help of the Moons the courſe may be directed. But alſo from ſo many Eclipſes the Longitude of a Place will be eaſily defin'd, and accurate Hydrographical Tables may be conſtructed; which is very convenient in ſo vaſt a Globe, whoſe Surface being an hundred times (nay, according to *Hugens*, four hundred times) bigger than the Surface of the Earth, wou'd render its meaſure very laborious. But the reciprocal Tide ariſing from the four Moons (for the Sun's Forces are of no ſenſible effect for raiſing a Tide at this vaſt diſtance) is various and uncertain, except to one who is us'd to Calculations.

PROPOSITION VI.

TO ſhew the chief Celeſtial Phænomena, which wou'd appear to an Obſerver plac'd in Saturn.

The Diameter of the Sun ſeen from Saturn is almoſt ten times leſs than the Earth, and his Diſc, Light and Heat at leaſt ninety times leſs there than here. Saturn's Year is almoſt equal to thirty of our Years: But it is uncertain what proportion a Day in Saturn bears to our Day; for the Period of Satturn's Revolution about his Axis is not yet known. *Hugens* indeed, from the Diſtance and Period of his *in*moſt Satellite, and a Compariſon of them with the Diſtance and Period of the inmoſt of the *Jo*vial Satellites, thinks it very likely that the Days are not longer there than in Jupiter. He is of that Opinion becauſe he looks upon it as an undoubted Truth, that the Rotation of the Sun and of a Primary Planet about its Axis is the cauſe of the Rotation of the Primary and Secondary Planets, (that indeed it is made towards the ſame parts is certain) or at leaſt that the Rotation of a central Body, and the Planets about it depend upon the ſame cauſe; *therefore* converſly, having obſerved the *Satellites rotated about Saturn*, he did *not at all* doubt but that Saturn revolv'd about its Axis, and nearly in the ſame time as Jupiter, becauſe the Periods of the Satellites differ but very little. But if we conſider that the cauſe of the Rotation of the Primary Planets about the Sun, or of the Satellites about a Primary is not any Vortex, or any thing like it (on which Opinion the foreſaid Argument is grounded,) but that the Planets are carried by a Motion, compounded of a projectile Motion along a right line, and of Gravity towards a central Body (about which they are moved in a free ſpace and almoſt void of Matter, and the Saturnian Satellites gravitate after the ſame manner towards Saturn, whether it revolves or is at reſt; it is plain, that from the Revolution of the Satellites one cannot rightly deduce the Rotation of Saturn about its Axis, much leſs the Period of that Rotation. It is true indeeed that it is probable that Saturn revolves about his Axis, namely, that it may ſometimes turn one and ſometimes another part of its Surface to the Sun, which is the Fountain of Light and Heat in his Syſtem: But it ſeems impoſſible to gather from hence the Period of the Rotation, theſe things being by the Allwiſe and Almighty Creator fitted to the nature of the things contained in Saturn, which is altogether unknown to us. If the Axis of Saturn be perpendicular to the Plane of Saturn's Ring and of the Orbit of the Satellites (as *Hugens* will have it) then the Situation of the Poles and of the Equator is the ſame in Saturn as it is in the Earth; therefore the ſame Polar Star will ſhine upon it, and the Conſtellations in ſimilar Latitudes riſe and fall after the ſame manner. And theſe things are thus if the Situation of the Ring be the ſame as *Hugens* has laid it down in the *Syſtem of Saturn*, which alſo we have ſuppos'd in the Fourth Book, when we ſpoke of its Phænomena: But, if it be by more accurate Obſervations, found to be different, (as he hints in his Theory of the World) any one may gather the Meaſures of the foreſaid Phænomena from what has been laid down before. The Days at Saturn are very unequal, and the differences of the Summer and Winter are very great, for they depend upon the Quantity of the Inclination of the Plane of his Equator to the Plane of the Orbit of Saturn about the Sun, which *Hugens* afterwards ſuppoſed of 31 Degrees, almoſt a third part greater than in the Earth, where thoſe Differences are very ſenſible. At Saturn in the Latitude of 59 Degrees the longeſt Day has no Night, neither has the longeſt Night any Day. And the two frigid Zones extended about the Poles (each of which is 62 Degrees broad) are ten times greater than the whole Surface of our Earth.

Of all the Primary Planets, the Eye poſited in Saturn, will be able to ſee only *Jupiter*, always accompanying the Sun, and never above 37 Degrees diſtant from it. Reckoning thoſe two amongſt his own five Companions, a number of ſeven Planets is made up to an Obſerver in Saturn, which he diſtinguiſhes into five Moons and two other Planets, as wou'd be done in Jupiter by the foregoing *Prop.*) as well for the ſame reaſons as alſo upon this account, that the five neareſt are mov'd in the ſame Plane inclin'd to the Plane of the Sun's apparent Way about Saturn in an Angle of 31 Degrees; ſo that the Zodiac to Saturn is more than the third part of Heaven in Breadth, if the Deviation of all the Planets from the Sun's Way be comprehended in it. The Phaſes of theſe Moons are ſimilar to thoſe of the Jovial Moons; except it be that in Summer and Winter, when the Sun declines very much to the Equator, each Moon oppos'd to the Sun does not ſhine with a full Orb, neither is it quite dark when in Conjunction with the Sun. For in the middle of Summer or of Winter, a Moon in Conjunction with the Sun is as far diſtant from it as our Moon is two days or more before or after the New Moon, and in oppoſition of the Sun it is as far from that didirect Oppoſition which makes a full Orb, as our Moon is two Days before or after the Full Moon: Thoſe Moons therefore will then have the ſame Phaſes as our Moon has two Days before or after the Syzygy. Whence at thoſe Seaſons of the Year of Saturn there will be no Eclipſes of the Luminaries. But at the Equinoctial Times Saturn will never be without Eclipſes, and thoſe of all the kinds mention'd in the foregoing Propoſitions.

Since the Year in Saturn is longer than that in Jupiter, there are alſo ſeveral ſorts of Months to diſtinguiſh it into parts. Of the leaſt, there are above 5700 in a Year, of thoſe immediately greater 3932, of the third Months above 2352, more than 674 of the fourth, and 135 of the greateſt.

An Inhabitant of Saturn will eaſily compare the Diſtances of his Moons with his own Meaſures; becauſe the Parallax of the fifth and laſt Moon is greater than the Parallax of our Moon. But the Sun's Parallax does not exceed 9 Seconds; therefore it is inſenſible, in compariſon with the Sun's Parallax ſeen from the Earth.

The Ring which encompaſſes Saturn, of which we ſpoke in the fourth Book, is a wonderful Phænomenon to an Eye in Saturn; for it is the only thing of that kind which is ſeen from far, and which has long ſince excited the the curioſity of all Philoſophers. This Ring gave occaſion to the learned and ingenious Dr. *Halley*, when he enquir'd into the Cauſes of the Variation of the declining magnetic Needle, to ſuppoſe the Globe of the Earth to be divided into Cruſts and concentrical Nucleus's (*See Philoſophical Tranſactions*, N°. 195.) after *Kepler*, who did the ſame in the 4th *Book, pag.* 586 of his *Epit.* to the *Copernican Aſtronomy*, when from the magnetic Direction he inveſtigated the cauſes of the Inequalities of the Planets. What if this Ring ſhou'd be the remaining part of an outward Cruſt, fallen on the Nucleus within it, and the Ruins of it ſtill to be ſeen? For if Saturn was ever of the ſame Diameter, as now his Ring is, it was of the ſame Magnitude when ſeen from the Sun, as now Jupiter's apparent Diameter ſeen from it.

Since the outward Edge of *Saturn's* Ring ſtands above Saturn, at an Height which is four times and a half the Semidiameter of *Saturn*, it will not be viſible at a diſtance from Saturn's Equator (in whoſe Plane it is) greater than 64 Degrees: Therefore a Spectator having a greater Latitude will never ſee the Ring; and there is a Zone about each Pole, about 53 Degrees broad, altogether depriv'd of the Sight of the Ring. And as the Eye goes nearer to Saturn's Pole the firſt Satellite is hid, then the ſecond, and ſo on, till the Eye at laſt being within one degree of the Pole, loſes ſight of the fifth alſo, unleſs it be by Refraction; neither will it in Winter ſee the Sun, any Moon, or any *Planet*, or any Celeſtial Body known *to the Inhabitants* of the Earth except the Fix'd Stars, and perhaps ſome Comet.

The Eye in the very Equator of Saturn, or a Zone near it, will ſee no Stars in the Equator or very near it; and therefore will never ſee any of the Moons, the Ring being in the Way: Therefore at the time of the Equinox, one that lives under the Equator will not ſee the Sun, and one that lives in another place will not ſee the Ring, becauſe in that caſe no Face of the Ring is ſhin'd upon. How broad that Zone is, is a difficult thing to determine, by Obſervations made from the Earth, becauſe the thickneſs of the Ring is ſmall; *Hugens* ſuppoſes it to be of 600 *German* Miles. It will be eaſy to determine it accurately from the Surface of Saturn, becauſe it is rais'd a little more than a Semidiameter above it. If the Eye be ſuppos'd plac'd out of the Equator, the ſpace of the Heaven which contains Stars cover'd by the Ring will be of different Magnitude and Figure, according to the different Situation of the Eye.

During half a Saturnian Year, one Face of the Ring is enlighten'd by the Sun; whence it happens that thoſe who inhabit the Hemiſphere towards which it is turn'd, that is, who have Summer, ſee that part of the Ring which is above the Horizon, ſhining with a faint Light by Day (much ſuch as we ſee the Moon ſhine with when the Sun alſo ſhines at the ſame time) if Saturn has an Atmoſphere, and that poſited towards that part where the Pole is depreſs'd; but its Light is ſtronger by Night, as alſo that of our Moon in the Sun's abſence: And from the Spots may be found whether it is at reſt or moves, and if it moves the Velocity of its Motion. After the Sun is ſet, the Eaſtern part of this lucid Arch has a Shadow (namely, the Shadow of Saturn) caſt upon it, which, as the Night comes on, riſes by degrees till in the middle of the Night it is got to its upper part or Vertex, from whence it moves by degrees towards the Weſtern part, as the Sun tends to its Riſing. This Arch will always ſhew the Meridian Line, for a Plane rais'd upon the Horizon, going thro' the Vertex of the Arch is in the Meridian. If the diſtance of the Eye from the Æquator be leſs than 52 Degrees it will ſee the Arch of the Ring as much concave as convex, like an Arch in a Building, riſing on each ſide from the Horizon; below which, namely, between the lucid Arch and the Horizon, he will ſee the Heaven and the Stars. But if the

Eye is farther diſtant than 52 Degrees, but leſs than 64, it will not at all ſee the concave part, but a lucid Body riſing as it were from the Ground, and contiguous to the Horizon, like the Morning Twilight.

During the other half of Saturn's Year, namely, when the Sun declines from the Equator towards the inviſible Pole, that is, in Winter time, the Eye will not at all ſee the Ring, becauſe it is not enlighten'd on that ſide which is towards the Eye; but the Ring will cover from the Eye part of Heaven, and the celeſtial Bodies contained in it, as has been ſaid before, and ſo by that means it will become ſenſible. But the Shadow of the Ring becomes more and more extended towards the Pole, ſo that the Eye being plac'd any where within the foreſaid ſpace, the Sun, when it has attain'd a certain Declination, will appear to be cover'd *and* eclips'd at Noon, and then *immediately* again it will become viſible. The next Day the ſame thing happens again, but the Eclipſe begins ſooner and ends later; and this Darkneſs in the middle of the Day will daily encreaſe in duration, (or perhaps come to laſt a whole Day) till the middle of Winter, from which time they will again begin to decreaſe till at laſt they are quite gone (or do not happen at all in the Day time) when the Sun returning from the Tropic is come back to the ſame degree of Declination, which it was in when the foremention'd Darkneſs began to appear. And this will be ſo if the Eye be in a greater Latitude than 25 or 26 Degrees: But if the Spectator is in a leſſer Latitude, when the Meridian Darkneſs is the moſt laſting, the Sun will ſuddenly appear juſt at Noon, and then immediately difdiſappear again, and the next Day there will again be ſuch an appearance of Light, but ſomething more laſting; and ſo on, the Meridian Light becoming daily more and more laſting till the middle of the Winter: And then again by degrees it will become of leſs and leſs continuance, till it wholly vaniſhes, when the Sun is return'd from the Winter Tropic to a declination equal to that at which the Mid-Day-Light began firſt to appear; at which time, on the ſudden the longeſt Meridian Darkneſs will again begin, and laſt a ſhorter and ſhorter time, as the Sun comes towards the Equator. And hence it follows, that there is a much greater difference between Summer and Winter on Saturn's Globe than on the Earth, as well upon account of the duration of each, and the Sun's great declination from the Equator, as upon account of the Meridian Darkneſs in Winter by reaſon of the interpoſition of the Ring which hides the Sun.

PROPOSITION VII.

TO *deſcribe the Phænomena when the Eye is ſuppos'd in one of the Comets.*

We refer the Comets to the Primary Planets, becauſe it is in the Excentricity of their Orbits that they chiefly differ from them. An Eye plac'd in a Comet will find itſelf encompaſs'd with a very great and denſe Atmoſphere, which encreaſes in its deſcent from the Aphelion, eſpecially when it is immers'd in the Region of the leſſer Planets. This Atmoſphere is ſo troubled and thick that towards its bottom it is almoſt like a Chaos. It is not known whether a Comet revolves about itſelf, but it is probable that, like all the other great Bodies of the

World, it turns all its Faces towards the Sun, that a plentiful Vapour may be rais'd from it. But whether this Rotation is about a given Axis, is difficult to determine. If the Nucleus be turn'd about, the whole Atmoſphere (eſpecially that part of it which is extended towards the Regions oppoſite to the Sun) does not revolve along with it, as the Atmoſphere of the Earth does: But that Vapour, which going out of the Comet makes the Tail, is not ſo much to be look'd upon as the Atmoſphere of the Comet join'd with it (as the denſer Atmoſphere of the Earth is join'd with it) and making part of it; as a ſeparate Body, all the parts of which perform their Motions in a free Space, like the Comet it ſelf.

As the Way of every Comet about the Sun is a very excentrical Ellipſe, it is not likely that they are made to ſerve the ſame purpoſes in the Univerſe as Planets, which are carried in Orbits nearly concentric to the Sun, and which ſeem deſign'd for the Production of Things which are almoſt always to remain in the ſame State; which Comets are by no means fit for, by reaſon of the very different Degrees of that which they ſuſtain. If therefore we may give way to conjecture, it is to be believ'd that the Comets ſerve for the renewing of the Fluid of the Sun and Planets. For ſince the Sun continually loſes a great deal of its Fluid, ſent out under the Form of Light; and the Fluid of the Planets is daily chang'd into ſolid (ſcarce becoming Fluid again) as we ſee it yearly happen in our Earth; it is plain either that the Sun's Forces will by degrees be loſt, and the Fluid of the Planets be conſum'd (which is very unlikely) or elſe that they are to be renew'd at ſtated Times. But there is no way that ſeems ſo fit for it, as to look upon Comets (which paſs thro' vaſt Spaces in the Æther) to be ſmall Bodies which are deſign'd in time to come to make Light, projected thro' thoſe Spaces, and as it were ſcatter'd and loſt in them, which likewiſe by attraction (a Quality natural and belonging to all Bodies) draw to themſelves, and as it were treaſure up the moiſt Vapours; that having rais'd 'em up in Tails they may mix them with the Atmoſpheres of Planets) which was alſo the Opinion of *Hippocrates* of *Chios*, taken notice of by *Ariſtotle*, in the *Firſt Book of Meteors*, *Chap. 6.*) and themſelves renew the Sun, the Fountain of Light, by falling into it. Hence will the Primary Planets be augmented; whence the Satellites Orbit (that is, if ſuch a Planet has a Satellite) and Period will be contracted; of which Phænomenon we ſpoke above in in its place; and other things will happen, which we mention'd when we ſpoke of Comets in the foregoing Book.

There may alſo ſometimes be another effect or uſe of a Comet. Namely, if a Comet paſſes near a Planet (its Orbit and Motion carrying it that way) it will ſo attract it that its Orbit will be chang'd (the Comet's Orbit being alſo chang'd by the mutual Action;) whence the Planet's Period will alſo be chang'd. But the Comet may alſo by its attraction ſo diſturb the Satellite as to make it leave its Primary, and itſelf become a Primary Planet about the Sun, and continually move round it. Beſides it may produce much greater Changes in the Globe of the Planet it ſelf, not only by attracting the Fluid, if it has any, but alſo by other Qualities, as if, for Example, ſo vaſt a Body goes from the Sun's Neighbourhood, and being red hot is carried near our Earth. But we have ſaid enough of this matter, eſpecially ſince it does not belong to our preſent Purpoſe.

The Years of Comets are different, according to the magnitude of the greater Axes of the Orbits which they deſcribe about the *Sun*; namely, (by the 40th and 42d Propoſitions of the firſt Book) the Year of each comet is to our Year in a ſeſquiplicate ratio of the greater Axis of the Orbit of the Comet to twice the mean Diſtance of the Sun from the Earth. The Sun's apparent Diameter, his Diſc, and alſo his Light and Heat, as well as the apparent Magnitude of the Planets, will be very much chang'd in a Comet. Beſides, the Obſerver in a Comet will find the motions of the Planets to be very different; accordingly when immers'd in their Regions, he has them ſometimes all Superior *to him*, ſometimes ſome Inferior and *ſome Superior*; or being rais'd out of *thoſe Regions* he ſees all the Planets below him; all which things may, from what has been ſaid before, be defin'd in a Comet whoſe Orbit is given. As for *Example*: If a Comet in its Aphelium aſcends to a Diſtance from the Sun ſix times greater than the Diſtance of Saturn, which will happen, if the Period of the Comet be equal to about 150 of our Years (ſuppoſing that in the Perihelium it comes extreamly near, as it happens in ſome of them) then all the Planets upon account of their nearneſs to the Sun, will then become inviſible to an Eye plac'd in a Comet; for Saturn himſelf will ſcarce be 10 Degrees diſtant from the Sun, and the Sun's Diameter will be equal to half a Minute. But to an Eye in a Comet whoſe Period is much greater, the Planets are inviſible all the while that the Comet is in the upper part of its Orbit, which is for the greateſt part of its Year; for the Velocity of the Comet towards the Aphelium is very ſmall in reſpect of that which it has when it is at, or near, the Perihelium. If the Period of the Comet be too great; that is, if the Diſtance of the Comet from the Sun in its Aphelium does often contain the diſtance of Saturn, perhaps the Order of the Fix'd Stars ſeen from ſuch a Comet will be very different in different Places of its Journey: But yet I hardly think this difference to be very ſenſible, but rather that the Orbits even of the outermoſt Comets are very ſmall when compar'd to the diſtance of the Sun from the Fix'd Stars.

COROLLARY.

Hence it appears, that the Syſtem of the Circumſolar Planets ſeen from a Fix'd Star wou'd become almoſt inviſible, and confounded with the Sun in the ſame Lucid Point, whoſe Diameter wou'd become inſenſible, if it be ſeen diſtinctly, and free from that Hairineſs or thoſe Rays which the Fix'd Stars appear to have to the naked Eye. For ſince all the Planets cannot be ſeen from a Comet in its Aphelium, by reaſon of their nearneſs to the Sun; how much more wou'd they become inviſible from ſo great a diſtance as that of the Fix'd Stars? For ſuppoſing the Parallax of the *Magnus Orbis* (or the Semidiameter of the *Magnus Orbis* ſeen from a Fix'd Star) to be equal to the Parallax of the Sun ſeen from the Earth (neither have we any Obſervations which ſhew it to be greater) the Sun's Diameter will at a Fix'd Star ſcarce ſubtend an Angle of five third Minutes, and Saturn will never be above above two firſt Minutes from the Sun. Beſides the Light which the Planets borrow by reflecting the Sun's Rays will be altogether unable to affect ſuch a Sight as ours at that immenſe diſtance; and tho' it were ſtrong enough to affect it, the Sun's Species wou'd appear to extend itſelf into a Semidiameter equal to that of the Syſtem, as *it* happens to all lucid Bodies, and eſpecially to the Fix'd Stars ſeen from the Earth; whence they *wou'd* all, as it were, fall in with the lucid Body. Therefore, on the other hand, the Planets which revolve about ſuch Suns (namely the Fix'd Stars) do altogether eſcape our Sight. There is no doubt, but that, to an Eye plac'd in any Fix'd Star, or Planet revolving about it, the Order of the other Fix'd Stars and the Figures of the Conſtellations wou'd be quite different from thoſe that we ſee; eſpecially ſince our Sun wou'd in that caſe appear as one of the Fix'd Stars.

Now this immenſe Diſtance *of the Fix'd* Stars or Suns from each other, hinders thoſe vaſt Bodies, and the Planets (very ſmall if compar'd with them) which are carried about thoſe Suns from ſenſibly exerting their attractive Forces on each other, and diſturbing each others motions. The indefinite number of thoſe Syſtems, included in no Space, is the reaſon why they don't run into one, but being ſeparated from one another, will for ever ſtand in the Univerſe as marks of the Power and Wiſdom of their Almighty Creator.

PROPOSITION VIII.

TO deſcribe the chief Phænomena of the Fix'd Stars, of the Sun, and of all the Planets, except the Earth, as they wou'd appear to an Eye in the Moon.

Since the Moon (as was ſaid in *Prop.* 56. B. 4.)

turns round itſelf from from Weſt to Eaſt, about an Axis nearly perpendicular to the Plane of the Ecliptic and always parallel to itſelf, equally and in the ſame time that the Moon revolves about the Earth, that is, in a Periodical Month; it is plain that to an Eye plac'd in its Surface, the Sun, the Fix'd Stars, and all the other things which are joyn'd to the Moon (that is all the Planets except the Earth) will ſeem nearly to revolve on the Poles of the Ecliptic from Eaſt to Weſt. And thoſe Poles are remarkable by Fix'd Stars; for the Northern one has a Star of the fourth Magnitude after the third Flexure of the Dragon, about 3 Degrees diſtant from the Pole; and the Southern Pole has near it four Stars of the Fiſh *Xiphias*, one of which is nearer the Pole than our Arctic Polar Star, but yet more remarkable by a Nebulous Star near it. And then alſo, as it happens with us, the Stars are all carried at the ſame time by the common motion, except that their Proper motions (whether true or apparent) do a little alter the caſe. And firſt, the Sun's apparent Revolution about the Moon, which is at reſt, as to ſenſe, is perform'd in a longer time, becauſe the Moon is at the ſame time together with the Earth, carried about the Sun *in conſequentia*; ſo that the Moon's natural Day is of the ſame duration as our Synodical Month, which almoſt exceeds by by two and a half of our Days, the before deſcrib'd Revolution of the ſaid Lunar Meridian in reſpect to the ſame Fix'd Star. Whence to an Obſerver in the Moon the Sun only riſes twelve times in a Year, but the Fix'd Stars thirteen times; or more accurately, in nineteen Years the Sun riſes two hundred and thirty five times but the Fix'd Stars two hundred and fifty four times: And the riſing Sun is almoſt a whole Sign forwarder than before.

This natural Day of the Inhabitants of the Moon, is equally divided into Light and Darkneſs, becauſe the Axis about which the Moon revolves is almoſt perpendicular to the plane of the Moon's Way about the Sun; and therefore there is no Twilight at the Moon before the Sun's riſing, or after its ſetting; becauſe ſuch a thing can't happen unleſs there was an Atmoſphere about the Moon, (as has been ſhewn in the 8th *Prop.* of the 2d *Book.*) Whence to the Moon's Inhabitants the Sun ſeems to move in a Circle between the Poles of the firſt Motion.

Altho' the Suns Way, ſeen from the Moon among the Fix'd Stars, be nearly the ſame as its Way ſeen from the Earth, and the Places of the Planets are nearly the ſame ſeen from the *Moon* as from the Earth; yet when *the Moon is* not in the Nodes, a nice Obſerver will ſee the Sun deviate from the Ecliptic towards the contrary ſide, and extend its Light beyond the Pole of the Moon, which is under that toward which it declines, leaving the other Pole in Darkneſs. But all theſe things are ſet right again in half a Lunar Day, when the Moon returns to its Node and the Plane of the Ecliptic, in order to go to the other ſide of the Ecliptic, and to undergo the ſame Phænomena towards the other Pole for the remaining part of the Day.

Beſides all the inequalities in the Planets Motions ſeen from the Earth, which muſt obtain in the Moon as well as the Earth, becauſe it is carried along with it; there are others peculiar to the Moon, the chief of which are theſe. When it is day to the Inhabitants of the middle of that Hemiſphere of the Moon which is turn'd from the Earth, the Sun appears to go ſlowly under the Fix'd Stars; and very ſlowly at the point of Noon; but the next day it will be found to have gone faſter under the Fix'd Stars in the Night time. And on the contrary to an Obſerver in the middle of that Hemiſphere of the Moon which is toward the Earth, the Sun will appear by Day to move ſwiftly under the Fix'd Stars towards the Eaſt, and very ſwift at Noon: But at Night the Sun goes very ſlowly under the Fix'd Stars, and ſloweſt of all at Midnight. The ſame things are alſo true of Venus and Mercury, becauſe they are always near the Sun; and alſo of Mars when it is in conjunction with the Sun, tho' thoſe things are leſs remarkable then, upon account of its great Diſtance from the Moon at that time; but they are inſenſible in Jupiter and Saturn when their poſition is toward the Sun. If any one of the ſuperior Planets ſeen from the Moon be in oppoſition to the Sun, in which caſe it is retrograde, it will appear to an Inhabitant in the middle of the Hemiſphere turn'd from the Earth, to be more retrograde than ordinary by Night, but leſs by Day; and leaſt of all, or moſt of all juſt at Noon or juſt at Midnight. So likewiſe an Obſerver living in that part of the Moon which the Earth ſeems to hang over, will ſee a Planet in oppoſition to the Sun to be the leaſt retrograde of all at Midnight, and moſt of all at Noon. And theſe things in Mars are twice more ſenſible than in the Sun: But in Jupiter and Saturn theſe Differences almoſt vaniſh. Likewiſe, if any Planet which from the Earth appears ſtationary, appears to be in oppoſition to the Earth in reſpect to the Moon, it will be retrotrograde; but direct when in conjunction with the Earth, which will be very ſenſible in Venus.

An Aſtronomer plac'd in the Moon, and ignorant of the motion of its Habitation about the Earth, or the motion of the Earth about the Sun, for making good the Theory of every Planet, will to all the Epicycles invented by a Terreſtrial Aſtronomer ſuperadd a new Epicycle, or Epicyclo-Epicycle, to explain the abovemention'd inequalities. As for *Example*: To the Excentric of the Sun deſcrib'd about the Earth made uſe of by an Inhabitant of the Earth, (for an Inhabitant of the Earth that admits an Excentric, does not want an Epicycle for explaining the Sun's motion) will add over and above an Epicycle, and will ſuppoſe that the Sun is carried round in it in the ſpace of one of the Lunar Days, whilſt at the ſame time the center of the Epicycle is carried in the Excentric by the Annual Motion: Thus will the Suns diurnal Inequality obſerv'd by a Lunar Aſtronomer be explain'd, if whilſt it is Night with the Lunar Inhabitant which is directly under the Earth, he ſuppoſes the Sun to be in the lower prrt of the Epicycle, and in the upper part of it whilſt it is Day with him. So alſo to the excentric of Mars about the Earth and its Epicycle contriv'd by a Terreſtrial Aſtronomer, by which the motion of Mars ſeen from the Earth (ſuppos'd at reſt) is explain'd, he will ſuperadd another Epicycle carried in the circumference of the firſt, which Mars will be imagin'd to go thro' in the ſpace of a *Nychthemeron* or Natural Day. And this is the only way that our Lunar Aſtronomer can ſettle his Aſtronomy, if he be ignorant of the motion of his Habitation, and aſſerts it is at reſt: Namely by transferring the Circle which is really deſcrib'd by himſelf and his Habitation in the Heavens, and ſuppoſing the Motion to be made in it in the ſame time and in an inverted Order; and retaining its true Magnitude, if the true Diſtance is known; but if it is not (and there are no means to find it out in that ſuppoſition of the Moon at reſt) at leaſt the Epicycle is ſuppos'd of ſuch a Magnitude, that it may from the Obſerver's Place appear of the ſame Magnitude as the true Orbit of the ſaid Place wou'd appear at the ſame diſtance. If the Inhabitant of the Moon's Hemiſphere which the Earth hangs over, be ſuppos'd to know the motion of his Habitation about the Earth, and of the Earth about the Sun, and capable of making Obſervations as accurately as we make them on Earth; he will be able exactly enough to compare the Magnitudes and Diſtances of the Sun and the Planets of the Solar Syſtem with ſuch Magnitudes and Diſtances as are familiar to him. For he may meaſure the Diameter of his Habitation after the ſame manner as we do that of our Earth. Moreover, he will eaſily compare the Diſtance of the Earth with the Moon's Diameter; for the Moon's Diameter ſubtends an Angle ſenſible enough at the Earth, namely an Angle of half a Degree: And he will make uſe of the Diameter of his Orbit as a Scale to find the Diſtances of the other Bodies of the World. For that ſubtends ſan Angle at the Sun of the third part of a Degree, ſuppoſing that the Sun's Parallax ſeen from the Earth is only 10″; but ariſes to almoſt half a Degree at Mars when it is in oppoſition with the Sun, and at Venus in conjunction with the Sun, and retrograde exceeds a fifth and a half of a Degree. Moreover, an Inhabitant of the Moon will after the ſame manner find out the Ratios between the Diſtances of the Primary Planets from the Sun, as an Inhabitant of the Earth does; but knowing the Diſtances, he will eaſily find their Magnitudes from their apparent Diameters.

PROPOSITION IX.

To deſcribe the chief Phænomena in the Earth ſeen by an eye in the Moon.

There are in the Moon two Hemiſpheres altogether different, nay, even contrary, in reſpect of the Phænomena, of which we are to ſpeak in this Propoſition. For they conſtantly ſee the Earth in one Hemiſphere of the Moon, and never in the other; unleſs that near the Limits of them there is a Tract to whoſe *Inhabitants* the Earth ſometimes riſes a *little above* the Horizon, and ſometimes going back as it were ſubſides again under it. In the Hemiſphere from which the Earth is ſeen, it appears as it were nail'd up,in the ſame point of the Heavens (except that it appears to move from thence a little towards one ſide or the t'other, and then to return by a libratory Motion) whilſt,in the mean time, in the ſpace of a Natural Day, the Sun and the other Celeſtial Bodies move towards it from the Eaſt, to go afterwards from it towards the Weſt. And a Star coming to the Earth hanging in the Æther, is not extinguiſh'd ſuddenly, but by little and little is firſt cover'd with a Miſt and at laſt getting behind the Earth, ceaſes to be ſeen by a Lunar Obſerver; which is occaſion'd by the Atmoſphere about the Earth, by which the Rays going from a Star to the Eye are almoſt choak'd up. Moreover, to ſome of the Inhabitants of the ſaid Lunar Hemiſphere, the Earth appears continually to be fix'd directly over their Heads; and that is to ſuch of them as live in the very middle of the ſaid Hemiſphere: But to ſome of them it appears to decline towards the North, to others towards the South, to ſome towards the Eaſt, and to others towards the Weſt, more or leſs, according as their Habitation is nearer to or farther from the ſaid middle or ſome ſide or other of it; till at laſt, to the Inhabitants of the foreſaid Limits which divide the Moon's two Hemiſpheres, the Earth will appear to be in the Horizon, one half of it ſtanding up like a Flaming Mountain: But they that dwell in a Circle of the Moon, which goes thro' the middle of the ſaid Hemiſphere and the Poles, have always the Earth in the Meridian.

They that obſerve the Earth ſee its Orb fifteen times greater than we ſee the Moon, at reſt in its place, but all the while turning about its Axis, and appearing to carry its Spots (which they will in a rude manner diſtribute into ſome Figures, after the manner that the common People ſuppoſe a thing like a Man's Face in the Orb of the Moon) from Eaſt to Weſt, and then return to its firſt Situation, in about the twenty-ninth part of a lunar Natural Day. *Kepler* from that Rotation of it ſuppoſes, in his *Lunar Aſtronomy*, that the Lunar Aſtronomers call the Earth *Volva*, (Whirler) and the Moon ſuppos'd fix'd *Veſta*, as never to be mov'd from its Place, and that they look upon our Globe, which they ſee hanging in the Æther, as a Celeſtial Body, which therefore they muſt not call by the Name of *Earth*: Therefore he calls the Inhabitants of that Hemiſphere under this *Volva*, *Subvolvæ* and *Subvolvani*,

vani, and the others which are depriv'd of the ſight of it, *Privolvæ*; which Names I ſhall alſo make uſe of hereafter. Beſides the ſaid Revolution of the *Volva*, there is alſo as it were another nodding Motion of it, by which ſometimes it ſhews the *Lunarians* its Northern, and ſometimes its Southern Pole, and that in the ſpace of one Day. For when by the Firſt Motion, or Motion of the *Primum Mobile*, *Sagittarius*'s Bow comes towards the *Volva*, the Lunarians (eſpecially thoſe which are at the North) will ſee the Northern Pole of the *Volva*, and the Spots and brighter parts about it, which for ſome Revolutions of the *Volva* will not retire behind it, when at the ſame time thoſe that are about the Southern Pole do not offer themſelves to ſight: After half a *Nychthemeron* is over, and that *Caſtor* is come to the *Volva*, the Northern Spots of the *Volva* will be hid, and the *Southern* ones will come in view.

The Lunarians ſee this *Volva* encreaſe or decreaſe in the ſame manner, with the ſame Phaſes, and for the ſame reaſons as our Moon; but the ſame Period of Phaſes which we call a Month, they call a Natural Day. They that ſee the *Volva* in their Meridian have the *New-Volva* or New-Earth at Noon, the Firſt Quarter at Night, the Full-*Volva* at Midnight, and the laſt Quarter in the Morning. They that dwell under any other Meridian at a diſtance from the foreſaid Meridian (which is the principal one, and, upon account of its Situation, may according to *Kepler*, be ſuppos'd to be call'd, *Circulus Medivolvanus*, by the Lunarians) towards the Eaſt or Weſt, have the Moment of each Phaſis at a different Hour according to the diſtance of their Meridian from the Medivolvan

Circle, and the number of Hours into which they divide their Day; therefore the Phaſes of the *Volva* denote the Hours, which alſo is done by the diſtance of the Sun from the *Volva* obſerv'd by Day. Whence the *Volva* ſerves for a Dial to *all* the *Subvolvani* : For by the diverſity of its *Phaſes*, by the approach of the Sun or Fix'*d* Stars towards it, by the name of the Pole that it ſhews, and laſtly, by the ſwift Motion of the Spots; (for the remarkable parts of the *Volva*, our Mountains, Lakes, Iſlands, Seas, *&c.* even tho' they are not immediately ſhin'd upon by the Sun, become viſible, eſpecially before and after the New-*Volva*, as the Moon's parts to us before and after the New-Moon, tho', *cæteris paribus*, not ſo diſtinctly, becauſe the Moon being a leſs Body, muſt reflect leſs Light to the Earth, than the Earth, which is a greater Body, does to the Moon;) it divides the Day and Night (which are indeed very long) into a ſufficient number of parts. This *Volva* alſo does ſhew the parts of the Year diſtinctly enough in the following manner: Since in Summer-time to the Inhabitants of *Europe* the Arctic Pole points towards the Sun, and the whole Arctic frozen Zone is enlighten'd by the Sun for ſeveral Days, and that our Earth is the *Volva* of the Lunarians, whoſe Arctic Pole, and a great Region about it, is enlighten'd during ſome part of every *Nychthemeron*; it is plain that when a great Region of the *Volva* lying about its Arctic Pole appears enlighten'd to a Lunarian, the Sun will be ſeen from the Moon at the Conſtellation of *Gemini*, and that will be the time of the Year which an Inhabitant of the Northern Hemiſphere of the Earth calls Summer, and in that caſe the Lunarians will clearly ſee the dry Land and Seas about the North Pole, which are yet undiſcover'd by us. After a quarter of a Year our Lunar Obſerver will ſee the Light terminated at each Pole of the *Volva*; a quarter of a Year after that, he will ſee the Regions about the South Pole of the Earth, which are altogether unknown to us, enlighten'd by the Sun; and after another Quarter the Light will again reach each Pole. He will alſo diſtinguiſh the Seaſons of the Year according to the different brightneſs of *Scandinavia* and *Scythia*, for example, as being greater, *cæteris paribus*, in Winter, when it is cover'd with Snow, than in Summer: And this difference of Brightneſs will be more viſible in that Sea which is near the Pole of the *Volva*; becauſe in our Summer it looks like a Spot, and in Winter it is as bright as the moſt ſhining parts.

The Lunarians have Eclipſes of the *Sun* and of the *Volva*, as the *Inhabitants of the* Earth ſee Eclipſes of the Sun and Moon, and at the ſame time, but with ſome difference. For as to us an Eclipſe of the Moon happens at the time of the Full-Moon, ſo to them an Eclipſe of the *Volva* happens at the time of the Full-*Volva*; and as the Sun appears to us to be in an Eclipſe at the time of the New-Moon, it appears ſo to them at the time of the New-*Volva*: But tho' to the Inhabitants of the Earth the whole Moon appears Eclips'd, the Lunarians never have a total Eclipſe of the *Volva*; for they ſee a certain reddiſh Spot paſs from Eaſt to Weſt over the Diſc of the *Volva*, making more haſte than its native Spots, and ſometimes going over the Center of the Diſc, when it is four Hours in going over; oftner beſide it, when the appearance is leſs laſting.

Their ſolar Eclipſe is caus'd by the *Volva*, juſt as our ſolar Eclipſe is by the Moon; and when we have an Eclipſe of the Moon, the Sun is totally Eclips'd to the Lunarians that inhabit the darken'd part of the Moon: But it often happens that an Inhabitant of the Moon (eſpecially one that dwells near the Limits of the *Subvolvæ* and *Privolvæ*) ſees a partial Eclipſe of the Sun, when the Moon appears to us with a full Orb, the Earth's *Penumbra* not ſenſibly extinguiſhing the Moon's Light. And it likewiſe happens, that to a certain Tract of the Earth the Sun appears to ſuffer a partial Eclipſe, when at the ſame time a Lunarian does not perceive the *Volva* to be Eclips'd at all: But when we have a total Eclipſe of the Moon, the Sun is wholly Eclips'd to all the Lunarians. Since the Diameter of the *Volva* is four times greater than the Sun's Diameter, the Sun will be fully hidden by the *Volva*, and that for a long time, which cannot happen to us by the interpoſition of the Moon; nay, the Sun's Eclipſe may be total and laſting when it is not Central. The Inhabitant of the Lunar Hemiſphere, over which the *Volva* hangs, ſees frequenter Eclipſes of the Luminaries than the Inhabitants of a given Hemiſphere of our Earth; for the *Subvolvans* alone ſee all the Eclipſes, the *Privolvans* none, ſince a great part of the Eclipſes ſeen by the Inhabitants of the Earth, belong to the oppoſite Hemiſphere. The before-deſcrib'd total Eclipſe of the Sun, tho' it happens often to the Lunarians, cannot but be very remarkable; for that is the only time that the *Subvolvans* are altogether depriv'd of Light: For it has been ſhewn before, that the *Medivolvans*, or thoſe who are in that part of the Moon which is directly under the Earth, have the firſt Quarter of the *Volva* at Sun-ſet, and the laſt Quarter at Sun-riſe; whence the *Volva* ſhining all Night is to them fifteen times greater than our Moon, and powerfully diſpels the Darkneſs. But in the total Eclipſe of the Sun, both the Sun and the *Volva* are extinguiſh'd at the ſame time, and all is dark. But this Inconveniency has its Remedies; for the Sun's *R*ays, being refracted by the Atmoſphere which is about the *Volva*, make the Body of the Sun appear enlarg'd, and as it were of a greater Diameter, but at the edges, (which only are ſeen) of an Iron-colour, to thoſe Inhabitants of the Moon which receive thoſe refracted Rays; for which reaſon there is hardly a total Darkneſs, except when the Sun's Eclipſe is nearly central, and that only for a little time about the middle of the Eclipſe. But if it happens that a very high Mountain ſtanding almoſt out of the Atmoſphere of the *Volva*, is in the edge of it; he on whom the Shadow of that Mountain falls, will wholly loſe the Sun's Light, and not enjoy the benefit of the refraction of the ſolar Rays made at the Atmoſphere of the *Volva*: And he that from a great diſtance ſees the Shadow of the *Volva* receiv'd on the Moon's Diſc, will perceive a very black Spot in that place. On the other hand, there is here no diminution of the Darkneſs in a ſolar Eclipſe upon account of the illumination of the Spectator's Atmoſphere, becauſe he has none.

Moreover, at the time of the Sun's Eclipſe, the Light that ends in a point extended along the Ecliptic from each ſide of the Sun (of which we ſpoke in the *Schol.* of *Prop.* 8. *B.* 2.) will be clearly ſeen by a Lunarian, if there be at that time a heap of ſmall Bodies in the ſhape of a Lens to produce it; nay, that light may alſo be ſeen when there is no Eclipſe, if the Sun be cover'd from the Eye by the interpoſition of ſome opake Body; for the Eye in the Moon is not affected by the Rays which are reflected from the ambient enlighten'd Atmoſphere, becauſe the Moon has no Atmoſphere about it; which is likely from obſerving that it is ſo evenly circumſcrib'd,and not vaniſhing at the Edges as if it had Down upon them; and becauſe the Stars near its Limb are not at all darkned, but vivid and ſhining till they are wholly hidden. By reaſon of that want of an Atmoſphere, the Lunarians will have no Twilight before the riſe of the Sun(except the ſhining of the Solar Atmoſphere) or after its ſetting; becauſe the cauſe which occaſions it upon Earth ceaſes here: Moreover, the Celeſtial Bodies will have no refractions, and therefore no change of Place upon that account.

We have hitherto ſuppos'd that to the Lunarians the *Volva* is unmov'd in the Heaven, becauſe for the moſt part it it is really ſo: But it has a certain libratory motion, by which it appears to any Lunarian ſometimes to move a little towards one ſide, then to return to the ſame Place, and afterwards to move to the other ſide. But this libratory motion of the *Volva* is eaſily divided into two: For either the *Volva* appears to librate towards the South and North upon the ſame Circle of Latitude; or towards the Eaſt and Weſt upon the Ecliptic. Let us ſuppoſe the Sun and the *Volva* to be in a central Conjunction when the Sun is ſeen in the Ecliptic it ſelf, or that a right line joining the Centers of the Sun and *Volva* goes thro' the Moon; that is, that to the Inhabitants of the Earth the Moon is in oppoſition to the Sun when in one nineteen Years before the *Volva* be found again in the Ecliptic, in Conjunction with the ſame Fix'd Stars, juſt as it is about to decline Northward. From the different magnitude of the Period of the libration of the *Volva* in Latitude and of the *Nychthemeron*, it happens, that when half a *Nychthemeron* is elaps'd after the Sun's central Eclipſe, the Luminaries will have no Eclipſe of the *Volva*. For the Sun being come to the Oppoſition with the *Volva* (that is in the *Plenivolvium* or Full-*Volva*, in which alone ſuch an Eclipſe may happen) the *Volva* having juſt paſs'd the Ecliptic (for it was come to it a little before the Sun was in Oppoſition with the *Volva*) will be got ſo far Southward by its Libration as to eſcape the Moon's ſhadow. An Inhabitant of the *Volva* will indeed ſee the Sun's Eclipſe at that time if he lives towards the North Pole of it, but not total; in which caſe alone the *Volva* appears to a Lunarian to have an Eclipſe. Likewiſe in the following *Novivolvium* or New-*Volva*, tho' centrally ecliptical, the Moon will eſcape the ſhadow of the *Volva*.

Thoſe Lunarians which are nice Obſervers, will ſometimes ſee the *Volva* bigger and ſometimes leſſer, and that whole variety is gone thro' nearly in the ſpace of a Lunar natural Day; but more accurately in the ſpace that the Fix'd Stars take up to return to the ſame Meridian. Let us ſuppoſe that to a Lunarian the *Volva* appears to have the leaſt Diameter when it is ſeen in the ſaid Circle of Latitude on which it has its North and South Libration. The Diameter of the *Volva* being by degrees encreas'd, the *Volva* itſelf will deviate from the ſaid Circle towards the Weſt during almoſt a quarter of the natural Day, and return to it the next quar-

ter,

ter, and then the Diameter of the Volva will appear the greateſt; and then from that Place it will again deviate towards the Eaſt, and its Diameter will grow leſs, and it will perform one entire Libration by returning to the Ecliptic (at which time its Diameter will again be leaſt of all) in one Revolution of the Fix'd Stars. This Deviation, when greateſt, will be of about 5 Degrees; and juſt ſo much will the Circle which divides the *Subvolvans* from the *Privolvans* be carried ſometimes in the Weſt leaving the Eaſt, ſometimes in the Eaſt leaving the Weſt, as by the former Libration it was carried to the North or to the South. And the Volva, after having perform'd one whole Libration in Longitude of this kind, returning to its firſt Circle of Latitude, namely the Mean between between the Extremes of its Libration, (during which time from appearing *leaſt*, it gradually encreaſes till it *appears biggeſt*, and returns again to its firſt appearance) will appear in Conjunction with more Eaſterly Fix'd Stars than thoſe with which it was in Conjunction before when it was in the ſame Circle of Latitude: And again after the next Libration, to Stars ſtill more Eaſterly; and ſo on, till after almoſt nine Years the *Volva* appears both leaſt and in conjunction with the ſame Fix'd Stars, and in its proper Circle of Latitude, ready to go from it by librating towards the Weſt. But neither of theſe Librations is found to be always equal to itſelf, but the Libration in Latitude is the greateſt on thoſe Days which are near to the Eclipſe of either of the Luminaries; and leaſt on thoſe Days which are the moſt remote from the Eclipſes: and the Libration in Longitude is greateſt of all when the *Volva* in the

Sun's Syzygy appears greateſt or leaſt; but leaſt of all when the *Volva* in the Sun's Quadrature appears greateſt or leaſt.

PROPOSITION X.

To determine the principal Phænomena that wou'd appear to an Eye plac'd in ſome one of the Satellites *of* Jupiter *or* Satun.

The Phænomena of the Sun and of the other Primary Planets, except that to which the Satellite belongs, when ſeen from a Satellite of Jupiter or Saturn, will be like or ſimilar to their Phænomena ſeen from our Moon, if thoſe Satellites are like it turn'd about their own Axes; namely each of them upon an Axis almoſt perpendicular to the Plane of the Orbit of his Primary about the Sun, and in a Period equal to the Period of the ſaid Satellites about its Primary, as it is very likely. In the mean time we muſt obſerve, that tho' from Jupiter we can ſee no Primary Planet beſides Saturn, and from Saturn none but Jupiter; yet from ſome of the Satellites of Jupiter or Saturn they may all be ſeen. For we ſaid that Mars and the Planet below Mars cou'd not be ſeen from Jupiter for this reaſon, *viz.* Becauſe we ſuppos'd Jupiter to have an Atmoſphere, which being enlighten'd by the Sun, hinder'd ſuch Planets as were nearer to the Sun than 20 Degrees from being ſeen. But if we ſuppoſe the Satellites of thoſe Primary Planets to be like the Satellite of the Earth, they will alſo be without an Atmoſphere as our Moon is: Therefore an Eye in one of 'em will ſee the Planets when they are near the Sun, unleſs the Sun's ſhining Atmoſphere hinders it; but even Mercury ſeen from Saturn will ſometimes appear 3 Degrees diſtant from the Sun.

And as the Sun's apparent Diameter from Saturn is about $90''$, it is plain that Mercury goes farther from the Sun than the limits of the Atmoſphere; and then that it may be ſeen by an Eye that is not hinder'd by an encompaſſing Atmoſphere; that is, by an Eye plac'd in a Satellite of Saturn, to which Mercury may yet (tho' but a little) appear farther diſtant from the Sun than to an Eye in Saturn itſelf. Moreover the Phænomena of Jupiter ſeen from Jupiter's Satellite, or of Saturn ſeen from a Satellite of Saturn, will be the ſame as the Phænomena of our Earth ſeen from the Moon, and differ in nothing but the meaſure of their Digreſſions and Librations: As for *Example*, To the *Subvolvans* of any Satellite of Jupiter the *Volva* does not appear to librate from the Ecliptic towards the North or South above 3 Degrees; tho' the Libration in Latitude of the *Volva* ſeen from one of Saturn's Satellites is of more than 30 Degrees. But in the latter as well as in the former Satellites, there is no Libration in Longitude, or if there is any it is not ſenſible, becauſe the Orbits of the Satellites of Jupiter and Saturn are nearly circular.

The Phænomena which appear to an Obſerver in any of the ſaid Satellites, very different from the Phænomena which appear in our Moon, are ſuch as appear from their Companions. For if the *Volva* has any Satellite nearer to itſelf than that in which the Eye is, that Satellite will never appear in oppoſition to the *Volva*, but always near it, librated like a Pendulum, ſometimes towards the Weſt and ſometimes towards the Eaſt of the *Volva*, deſcribing the Arc of a great Circle, which the *Volva* divides into two equal parts; and if there are more ſuch Satellites, one will run out into a leſſer Arc on each ſide, and the other into a greater; and theſe Satellites will never Set to the middle *Subvolvans*, and never riſe to the middle *Privolvans*. But thoſe which have the *Volva* a little below the Horizon to the Eaſt or Weſt, will ſee theſe Satellites Riſe in the Eaſt or Weſt, and go towards the Meridian, then go back again and ſet in the ſame place. If the *Volva* has a Satellite more remote from it than that in which the Eye is, it will appear to the *Privolvans* once in their natural Day, to the middle *Privolvans* and middle *Subvolvans* it riſes and ſets very ſlowly, and when it comes to the Meridian, it appears to the *Subvolvans* to move faſter towards the Weſt, and to the *Privolvans* to move ſlower towards it; from which it is eaſy to gueſs what appearance the Inhabitants of the Satellites between both will have. In the mean time any one of thoſe Companions or Satellites of the *Volva*, make the Sun to be Eclips'd, and itſelf is alſo Eclips'd, either when it runs into the ſhadow of the *Volva*, or into the ſhadow of the Satellite where the Obſerver is, or of any other of the Satellites. But any one may eaſily find the the various Symptoms and Times of theſe Eclipſes, and ſeveral other things relating to this matter.

COROLLARY.

From the ten foregoing Propoſitions it follows that Aſtronomy is the moſt ſimple of all to an Eye plac'd in the Sun. For if the Eye be plac'd in its Center, the true Syſtem does not differ from the apparent; if it be rais'd up to its Surface, the Obſerver may alſo compare the Diſtances of the ſix Primary Planets from his Habitation with his own Meaſures, and

But the Syſtem being diſcover'd, the Magnitudes and Diſtances of the Planets will be very hard to be found out by the Inhabitants of any of the Primary Planets, and will not exactly be compar'd with their common Meaſures. Jupiter ſeems the fitteſt for that Work, then Venus, then Saturn, next Mercury, and Mars is the unfitteſt of any: What place we muſt give to the Earth in this caſe is uncertain, becauſe the Angle at the Sun, which the Earth's Diameter ſubtends, has been hitherto found rather by Analogy than Obſervation. Perhaps the Magnitude of the ſolar Syſtem will in time be firſt diſcover'd by Obſervations of Comets: For if, by any of the Methods given in the Second or Third Book, you can find the diſtance of a Comet paſſing near the Earth, whoſe Orbit is given in poſition and Magnitude (from the Fifth Book,) you will find the Magnitude of the Orbit of any Planet, or Comet, whoſe Period is known.

The Aſtronomy is more intricate and difficult to an Obſerver in one of the Secondary Planets. For in order to eſtabliſh it, ſo as it may agree with the Phænomena, a *Primum Mobile* muſt be introduc'd to carry all Celeſtial Bodies from Eaſt to Weſt in the Space of a natural Day: But that will not be enough; for the *Volva* is not carried by that Firſt Motion altho' it be a Celeſtial Body; and in one of Jupiter's or Saturn's Satellites, ſome of the Celeſtial Bodies (inferior to the *Volva*) are carried round by the *Primum Mobile*, and others not. Moreover, for demonſtrating of the Second Motions there muſt be Deferents, Epicycles, and Epicyclo-Epicycles, and the Aſtronomer will often be obliged to change them all; whence his Syſtem will knows that this Habitation moves about the *Volva*, and that the *Volva* revolves about the Sun, he is ſituated in the beſt place of any for improving Aſtronomy: For beſides an accurate Geography very neceſſary to Aſtronomy, there being no Atmoſphere, no Miſts or Clouds do ever hinder Obſervations, and he may obſerve the Celeſtial Bodies by Day as well as by Night, and all the Planets, even thoſe that are near the Sun; therefore, if there are any Planets below Mercury, which we can't ſee, he will ſee them, and encreaſe Aſtronomy ſo much the more. Beſides, he will obſerve the Celeſtial Bodies without Refraction, (becauſe it is occaſion'd by an ambient Atmoſphere) and this alone does ſo much diſturb the Obſervations of the Inhabitants of the Earth, that the taking off ſmall Angles is a matter almoſt deſpair'd of. Moreover, the *Volva* rotated in its place ſerves as a Clock for all the *Subvolvans* to look at, and by help of it the Moments of Obſervations are eaſily known. The Earth's Companion the Moon is the moſt convenient of all the Satellites for this uſe, for its apparent Aſtronomy (ſuch as the firſt beginnings of Aſtronomy muſt be every where, unleſs the true Syſtem be divinely reveal'd) will be leaſt compounded or perplex'd, becauſe the Earth has no other Companion; whence an Obſerver there will ſooner come to the knowledge of the true Syſtem. And tho' the Diameter of the Orbit of the outmoſt Satellite of Jupiter or Saturn ſubtends an Angle at the Sun almoſt equal to that which the Diameter of the Moon's Orbit ſubtends, and ſo the diſtance of the Sun from Jupiter or Saturn may, by Obſervations made in their outermoſt Satellites, be almoſt as exactly defin'd, as its diſtance from the Earth, by Obſervations made in the Moon; yet ſince the diſtance of the Sun is known from knowing the diſtance of any Primary Planet, an Inhabitant of one of Jupiter's Satellites will, in order to this Enquiry, chooſe the next Primary to it ſe*lf*, namely, Mars, when it is in conjunction with the Sun and retrograde; as we do Venus, when *it* is ſimilarly poſited: But the Angle which the Diameter of the Moon's Orbit ſubtends at Venus, when it is near the Earth, is almoſt three times greater than the Angle which the Diameter of the Orbit of the outermoſt of Jupiter's Satellites ſubtends at Mars, when it is neareſt to Jupiter.

A SY-

A SYNOPSIS OF THE Aſtronomy of COMETS.

By EDMUND HALLEY L.L.D. *Savilian* Profeſſor of Geometry at *Oxford*.

THE ancient Egyptians and Caldeans (if we may credit *Diodorus Siculus*) by a long courſe of obſervations, were ſaid to be able to predict the Apparitions of Comets. But ſince they are alſo ſaid, by the help of the ſame Arts, to have prognoſticated Earthquakes and Tempeſts, 'tis paſt all doubt, that their knowledge in theſe matters, was the reſult rather of meer Aſtrological Calculation, than of any Aſtronomical Theories of the Celeſtial motions. And the Greeks, who were the conquerors of both thoſe People, ſcarce found any other ſort of Learning amongſt them, than this. So that 'tis to the Greeks themſelves as the Inventors (and eſpecially the great *Hipparchus*) that we owe the Aſtronomy we have, and which is now improv'd to ſuch a heigth. But yet, amongſt the Greeks, the opinion of *Ariſtotle* (who wou'd have Comets to be nothing elſe, but Sublunary Vapours, or Airy Meteors) prevail'd ſo far, that this moſt difficult part of the Aſtronomical Science lay altogether neglected; for no Body thought it worth while to take notice of, or write about, the wandring uncertain Motions of what they eſteemed Vapours floating in the Æther; whence it came to paſs, that nothing certain, concerning the Motion of Comets, can be found tranſmitted from them to us.

But *Seneca* the Philoſopher, having conſider'd the Phænomena of two remarkable Comets of his time, made no ſcruple to place them amongſt the Celeſtial Bodies; believing them to be Stars of equal duration with the World, tho' he owns their Motions to be govern'd by Laws not as then known or found out. And at laſt (which was no untrue or vain Prediction) he foretells, that there ſhould be Ages ſometime hereafter, to whom Time and Diligence ſhou'd unfold all theſe Myſteries, and who ſhou'd wonder how 'twas poſſible the Ancients cou'd be ignorant of them, after ſome lucky Interpreter of Nature had ſhewn, *in what parts of the Heavens the Comets wander'd, what ſort of Beings, and how great they were.* Yet almoſt all the Aſtronomers differ'd from this opinion of *Seneca*; neither did *Seneca* himſelf think fit to ſet down thoſe Phænomena of the Motion, by which he was enabled to maintain his opinion; nor the Times of thoſe Appearances, which might be of uſe to poſterity, in order to the determining theſe things. And indeed, upon the turning over very many Hiſtories of Comets, I find nothing at all that can be of ſervice in this affair, before *A. D.* 1337. at which time *Nicephorus Gregoras*, a Conſtantinopolitan Hiſtorian and Aſtronomer, did pretty accurately deſcribe the Path of a Comet amongſt the Fix'd Stars, but was too laxe as to the account of the Time; ſo that this moſt doubtful and uncertain Comet, only deſerves to be inſerted in our Catalogue, for the ſake of its appearing near four-hundred Years ago.

The next of our Comets was in the Year 1472, which being the ſwifteſt of all, and neareſt to the Earth, was obſerv'd by *Regiomontanus*. This Comet (ſo fearful upon the account both of the magnitude of its Body, and the Tail) mov'd forty degrees of a great circle in the Heavens, in the ſpace of one Day; and was the firſt, of which any proper obſervations are come down to us. But all thoſe that conſider'd Comets, until the time of *Tycho Brahe* (that great Reſtorer of Aſtronomy) believ'd them to be below the Moon, and ſo took but little notice of them, reckoning them no other than Vapours.

But in the Year 1577, (*Tycho* ſeriouſly purſuing the ſtudy of the Stars, and having gotten large Inſtruments for the performing Celeſtial menſurations, with far greater care and certainty than the Ancients cou'd ever hope for) there appear'd a very remarkable Comet; to the obſervation of which *Tycho* vigorouſly applied himſelf; and found by many juſt and faithful trials, that it had no Diurnal Parallax that was perceptible: And conſequently was not only no Aerial vapour, but alſo much higher than the Moon; nay, might be plac'd amongſt the Orbs of the *Planets*, for any thing that appear'd to the contrary; the cavilling oppoſition made by ſome of the School-men in the mean time, being to no purpoſe.

Tycho was ſucceeded by the moſt Sagacious *Kepler*. He having the advantage of *Tycho's* labours and obſervations, found out the true Phyſical Syſtem of the World, and vaſtly improv'd the Science of Aſtronomy.

For he demonſtrated that all the Planets perform their revolutions in *Elliptic Orbits, whoſe Plains paſs thro' the Center of the Sun*, obſerving this Law, that *the Area's of the Elliptic Sectors, taken at the Center of the Sun (which he proved to be in the common Focus of theſe Ellipſes) are always proportional to the Times in which the correſpondent Elliptical Arcs are deſcrib'd.* He diſcover'd alſo that *the Diſtances of the Planets from the Sun are in the Seſquialtera ratio of the Periodical Times*, or (which is all one) that *the Cubes of the Diſtances are as the Squares of the Times.* This great Aſtronomer had the opportunity of obſerving two Comets, one of which was a very remarkable one. And from the obſervations of theſe (which afforded ſufficient indications of an Annual Parallax) he concluded, That *the Comets mov'd freely thro' the Planetary Orbs, with a Motion not much different from a Rectilinear one; but of what kind he cou'd not preciſely determine.* Next, *Hevelius* (a noble emulator of *Tycho Brahe*) following in *Kepler's* ſteps, embraced the ſame Hypotheſis of the Rectilinear Motion of the Comets, himſelf accurately obſerving many of them. Yet he complain'd that his calculations did not perfectly agree to what he obſerved in the Heavens: And was aware, that *the Path of a Comet*

was

as bent into a Curve Line concave towards the Sun.
t length, came that prodigious Comet of the
ear 1680: which deſcending (as it were from
infinite Diſtance) perpendicularly towards
e Sun, aroſe from him again with as great a
elocity.

This Comet, (which was ſeen for four months
ntinually) by the very remarkable and pecu-
r Curvity of its Orb (above all others) gave
e fitteſt occaſion for inveſtigating the Theory
its Motion. And the Royal Obſervatories
Paris and *Greenwich* having been for ſome time
unded, and committed to the care of moſt
cellent Aſtronomers, the apparent Motion of
is Comet was moſt accurately (perhaps as far
humane skill cou'd go) obſerv'd by Mrs.
ſſini and *Flamſteed.*

Not long after, that great Geometrician the
uſtrious *Newton,* writing his *Mathematical*
inciples of Natural Philoſophy, demonſtrated not
ly that what *Kepler* had found, did neceſſarily
tain in the Planetary Syſtem; but alſo, that
the Phænomena of Comets wou'd naturally
llow from the ſame Principles; which he a-
ndantly illuſtrated by the example of the a-
reſaid Comet of the Year 1680. ſhewing at
e ſame time, a method of delineating the
rbits of Comets Geometrically; therein ſol-
ng (not without meriting the higheſt admi-
tion of all Men) a Problem, whoſe intricacy
nder'd it ſcarce acceſſible to any but himſelf.
his Comet he prov'd to move round the Sun
a Parabolical Orb, and to deſcribe Area's
aken at the Center of the Sun) proportional
the Times.

Wherefore (following the ſteps of ſo Grea a Man) I have attempted to bring the ſame Method to Arithmetical Calculation; and tha with all the ſucceſs I cou'd wiſh. For, havin collected all the obſervations of Comets I cou'd I have fram'd this following Table, the reſult o a prodigious deal of Calculation: which, tho' but ſmall in bulk, will be no unacceptable preſent to Aſtronomers. For theſe Numbers ar capable of repreſenting all that has been ye obſerv'd about the Motion of Comets, by the help only of the annext General Table; in the making of which I ſpar'd no labour, that it migh come forth perfect, as a Thing conſecrated t Poſterity, and to laſt as long as Aſtronomy i ſelf.

The Aſtronomical Elements of the Motions in a Parabolic Orb of all the Comets that have been hitherto duely obſerv'd.

Com. An.	Nodus Aſcend. gr. ′ ″	Inclin. Orbitæ. gr. ′ ″	Perihelion. gr. ′ ″	Diſtan. Periheli. à Sole.	Log. Diſt. Periheliæ à Sole.	Temp. equat. Perihelii. d. h. ′	Perihelion à Nodo. gr. ′ ″	
1337	♊ 24.21. 0	32.11. 0	♉ 7.59. 0	40666	9.609236	*June* 2. 6.25	46.22. 0	Retrog.
1472	♑ 11.46.20	5.20. 0	♉ 5.33.30	54273	9.734584	*Feb.* 28.22.23	123.47.10	Retrog.
1531	♉ 19.25. 0	17.56. 0	♒ 1.39. 0	56700	9.753583	*Aug.* 24.21.18½	107.46. 0	Retrog.
1532	♊ 20.27. 0	32.36. 0	♋ 21. 7. 0	50910	9.706803	*Oct.* 19.22.12	30.40. 0	Direct
1556	♍ 25.42. 0	32. 6.30	♑ 8.50. 0	66390	9.666424	*Apr.* 21.20. 3	103. 8. 0	Direct
1577	♈ 25.52. 0	74.32.45	♌ 9.22. 0	18342	9.263447	*Oct.* 26.18.45	103.30. 0	Retrog.
1580	♈ 18.57.20	64.40. 0	♋ 19. 5.50	59628	9.775450	*Nov.* 28.15.00	90. 8.30	Direct
1585	♉ 7.42.30	6. 4. 0	♈ 8.51. 0	109358	0.038850	*Sept.* 27.19.20	28.51.30	Direct
1590	♍ 15.30.40	29.40.40	♏ 6.54.30	57661	9.760882	*Jan.* 29. 3.45	51.23.50	Retrog.
1596	♒ 12.12.30	55.12. 0	♏ 18.16. 0	51293	9.710058	*Juli* 31.19.55	83.56.30	Retrog.
1607	♉ 20.21. 0	17. 2. 0	♒ 2.16. 0	58680	9.768490	*Oct.* 16. 3.50	108.05. 0	Retrog.
1618	♊ 16. 1. 0	37.34. 0	♈ 2.14. 0	37975	9.579498	*Oct.* 29.12.23	73.47. 0	Direct
1652	♊ 28.10. 0	79.28. 0	♈ 28.18.40	84750	9.928140	*Nov.* 2.15.40	59.51.20	Direct
1661	♊ 22.30.30	32.35.50	♋ 25.58.40	44851	9.651772	*Jan.* 16.23.41	33.28.10	Direct
1664	♊ 21.14. 0	21.18.30	♌ 10.41.25	102575½	0.011044	*Nov.* 24.11.52	49.27.25	Retrog.
1665	♏ 18.02. 0	76.05. 0	♊ 11.54.30	10649	9.027309	*Apr.* 14. 5.15½	156. 7.30	Retrog
1672	♑ 27.30.30	83.22.10	♉ 16.59.30	69739	9.843476	*Feb.* 20. 8.37	109,29. 0	Direct
1677	♏ 26.49.10	79.03.15	♌ 17.37. 5	28059	9.448072	*Apr.* 26.00.37½	99.12. 5	Retrog.
1680	♑ 2. 2. 0	60.56. 0	♐ 22.39.30	00612½	7.787106	*Dec.* 8.00. 6	9.22.30	Direct
1682	♉ 21.16.30	17.56. 0	♒ 2.52.45	58328	7.765877	*Sept.* 4.07.39	108.23.45	Retrog.
1683	♍ 23.23. 0	83.11. 0	♊ 25.29.30	56020	9.748343	*Juli* 3. 2.50	87.53.30	Retrog.
1684	♐ 28.15. 0	55.48.40	♏ 28.52. 0	96015	9.982339	*Mai* 29.10.16	29.23.00	Direct
1686	♓ 20.34.40	31.21.40	♊ 17.00.30	32500	9.511883	*Sept.* 6.14.33.	86.25.50	Direct
1698	♐ 27.44.15	11.46. 0	♑ 0.51.15	69129	0.829660	*Oct.* 8.16.57	3. 7. 0	Retrog.

This Table needs little explication, ſince 'tis plain enough rom the Titles, what the Numbers mean. Only it may be bſerv'd, that the Perihelium Diſtances, are eſtimated in uch Parts, as the Middle Diſtance of the Earth from the un, contains 100000.

A General Table for Calculating the Motions of Comets in a Parabolical Orbit.

Med. mot.	Angul. à perihelio.	Logar. pro diſt. à Sole.	Med. mot.	Ang. à perihelio.	Logar. pro diſt. à Sole.
°	gr. ′ ″		°	gr. ′ ″	
1	1.31.40	0.000077	41	53.29.44	0.098300
2	3. 3.15	0.000309	42	54.27.32	0.102019
3	4.34.43	0.000694	43	55.24.21	0.105752
4	6. 6. 0	0.001231	44	56.20.12	0.109490
5	7.37. 1	0.001921	45	57.15. 6	0.113240
6	9. 7.43	0.002759	46	58. 9. 3	0.116995
7	10.38. 2	0.003745	47	59. 2. 4	0.120756
8	12. 7.54	0.004876	48	59.54.11	0.124518
9	13.37.17	0.006151	49	60.45.25	0.128278
10	15. 6. 7	0.007564	50	61.35.45	0.132035
11	16.34.20	0.009115	51	62.25.14	0.135792
12	18. 1.54	0.010798	52	63.13.52	0.139544
13	19.28.47	0.012609	53	64. 1.40	0.143291
14	20.54.54	0.014550	54	64.48.38	0.147029
15	22.20.14	0.016607	55	65.34.50	0.150762
16	23.44.44	0.018783	56	66.20.13	0.154482
17	25. 8.22	0.021072	57	67.04.50	0.158192
18	26.31. 8	0.023470	58	67.48.42	0.161890
19	27.52.55	0.025969	59	68.31.50	0.165578
20	29.13 47	0.028570	60	69.14.16	0.169254
21	30.33.40	0.031263	61	69.55.58	0.172914
22	31.52.32	0.034045	62	70.36.56	0.176557
23	33.10.23	0.036916	63	71.17.16	0.180188
24	34.27.12	0.039864	64	71.56.56	0.183803
25	35.42.59	0.042892	65	72.35.57	0.187404
26	36.57.41	0.045989	66	73.14.15	0.190978
27	38.11.20	0.049154	67	73.51.59	0.194540
28	39.23.54	0.052382	68	74.29. 6	0.198085
29	40.35.23	0.055668	69	75.05.38	0.201614
30	41.45.47	0.059009	70	75.41.35	0.205122
31	42.55.06	0.062400	71	76.16.56	0.200612
32	44. 3.20	0.065838	72	76.51.43	0.212080
33	45.10.29	0.069319	73	77.25.57	0.215529
34	46 15.35	0.072839	74	77.59.41	0.218963
35	47.21.36	0.076396	75	78.32.54	0.222378
36	48.25.33	0.079984	76	79. 5.35	0.225769
37	49.28.27	0.083600	77	79.37.45	0.229142
38	50.30.19	0.087244	78	80. 9.23	0.232488
39	51.31. 8	0.090910	79	80.40.34	0.235809
40	52.30.56	0.094596	80	81.11.16	0.239127

The General Table continued.

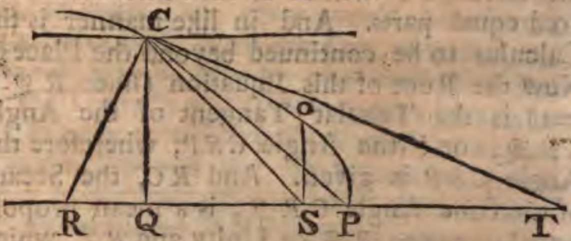
Med. mot.	*Angul. à perihelio.*	*Logar. pro diſt. à Sole.*	*Med. mot.*	*Ang. à perihelio.*	*Logar. pro. diſt. à Sole.*
o	gr. ′ ″		o	gr. ′ ″	
81	81.41.31	0.242416	142	102.32.41	0.407380
82	82.11.19	0.245684	144	103.00.31	0.411784
83	82.40.40	0.248933	146	103.27.47	0.416132
84	83. 9.34	0.252159	148	103.54.31	0.420430
85	82.38. 4	0.255366	150	104.20.43	0.424676
86	84. 6. 8	0.258552	152	104.46.22	0.428866
87	84.33.49	0.261720	154	105.11.33	0.433012
88	85. 1. 5	0.264865	156	105.36.16	0.437110
89	85.27.58	0.267989	158	106.00.32	0.441164
90	85.54.27	0.271092	160	106.24.23	0.445178
91	86.20.34	0.274176	162	106.47.47	0.449144
92	86.46.20	0.277239	164	107.10.44	0.453060
93	87.11.43	0.280284	166	107.33.17	0.456936
94	87.36.45	0.283306	168	107.55.27	0.460772
95	88.01.27	0.286308	170	108.17.14	0.464208
96	88.25.49	0.289293	172	108.38.37	0.468318
97	88.48.40	0.292252	174	108.59.39	0.472030
98	89.13.32	0.295201	176	109.20.20	0.475705
99	89.36.54	0.298122	178	109.40.40	0.479340
100	90.00.00	0.301030	180	110.00.40	0.482937
102	90.45.14	0.306782	182	110.20.20	0.486498
104	91.29.18	0.312469	184	110.39.41	0.490022
106	92.12.14	0.318060	186	110.58.44	0.493512
108	92.54. 4	0.323587	188	111.17.28	0.496965
110	93.34.52	0.329042	190	111.35.55	0.500384
112	94.14.40	0.334424	192	111.54.05	0.503769
114	94.53.30	0.339736	194	112.11.58	0.507121
116	95.31.22	0.344979	196	112.29.34	0.510441
118	96. 8.22	0.350153	198	112.46.55	0.513729
120.	96.44 30	0.355262	200	113. 4.00	0.516984
122	97.19.48	0.360306	204	113.37.25	0.523406
124	97.54.19	0.365284	208	114. 9.52	0.529705
126	98.28.00	0.370200	212	114.41.23	0.535886
128	99.00.57	0.375052	216	115.12.02	0.541958
130	99.33.11	0.379842	220	115.41.51	0.547922
132	100. 4.43	0.384576	224	116.10.52	0.553782
134	100.35.45	0.389252	228	116.39. 7	0.559538
136	101. 5.48	0.393868	232	117.16.38	0.565[illegible]99
138	101.35.22	0.398428	236	117.33.27	0.570762
140	102. 4.19	0.402930	240	117.59.35	0.576233

The General Table continued.

Med. mot.	Ang. à perihelio.	Logar. pro dist. à Sole.	Med. mot.	Ang. à perihelio.	Logar. pro dist. à Sole:
°	gr. ′ ″		°	gr. ′ ″	
244	118.25. 5	0.581616	620	137.33.13	0.882575
248	118.49.57	0.586912	640	138. 3.58	0.892649
252	119.14.14	0.592122	660	138.33.21	0.902401
256	119.37.56	0.597252	680	139. 1.29	0.911866
260	120. 1. 6	0.602301	700	139.28.25	0.921012
264	120.23.44	0.607274	720	139.54.16	0.929907
268	120.45.52	0.612174	740	140.19. 5	0.938549
272	121. 7.30	0.616998	760	140.42.56	0.946951
276	121.28.39	0.621750	780	141. 5.55	0.955124
280	121.49.22	0.626438	800	141.28. 3	0.963082
284	122. 9.38	0.631056	820	141.49.24	0.970836
288	122.29.28	0.635608	840	142.10.00	0.978397
292	122.48.54	0.640098	860	142.29.56	0.985771
296	123. 7.57	0.644525	880	142.49.10	0.992970
300	123.26.36	0.648893	900	143. 7.48	0.100000
310	124 11.40	0.659559	920	143.25.51	1.006871
320	124.54.36	0.669880	940	143.43.21	1.013586
330	125.35.34	0.679876	960	144.00.18	1,020155
340	126.14.44	0.689568	980	144.16.46	1.026583
350	126.52.12	0.698970	1000	144.32.46	1.032876
360	127.28. 6	0.708104	1500	149.26. 8	1.158188
370	128. 2.33	0.716976	2000	152.26.15	1,246058
380	128.35.38	0.725606	2500	154.32.20	1.313703
390	129. 7.27	0.734006	3000	156. 7.27	1.368678
400	129.38. 4	0.742186	3500	157.22.49	1.414974
410	130. 7.34	0.750160	4000	158.24.36	1.454950
420	130.36. 2	0.757930	4500	159.16.36	1.490125
430	131. 3.30	0.765516	5000	160. 1.12	1.521521
440	131.30. 2	0.772918	5500	160.40. 5	1.549874
450	131.55.41	0.780148	6000	161.14.24	1.575718
460	132.20.30	0.787216	6500	161.45.00	1.599460
470	132.44.32	0.794122	7000	162.12.34	1.621417
480	133. 7.50	0.800882	7500	162.37.34	1.641838
490	133.30.25	0.807494	8000	163.00.23	1.660922
500	133.52.20	0.813969	8500	163.21.20	1.678834
520	134.34.18	0.826522	9000	163.40.42	1.695708
540	135.14. 0	0.838600	9500	163.58.38	1.711662
560	135.51.28	0.850187	10000	164.15.20	1.726784
580	136.27. 6	0.861369	50000	170.52. 0	2.197960
600	137.00.57	0.872155	100000	172.45.44	2.399655

The Conſtruction and Uſe of the general Table.

AS the Planets move in Elliptic Orbs, ſo do the Comets in Parabolic ones, having the Sun in their common Focus, and deſcribe equal Areas in equal Times. Now ſince all Parabola's are ſimilar to one another, therefore if any determinate part of the Area of a given Parabola, be divided into any number of parts at liberty, there will be a like diviſion made in all Parabola's under the ſame angles, and the diſtances will be proportional: Conſequently this one Table of ours will ſerve for all Comets. Now the manner of the calculation of this Table is thus: In the *Fig.*



Let *S* be the Sun, *POC* the orbit of a Comet, *P* the Perihelion, *O* the Place where the Comet is 90 gr. diſtant from the Perihelion, *C* any other Place. Draw the Right lines *CP*, *CS*, and make *ST*, *SR*, equal to *CS*; and having drawn the Right lines *CR*, *CT* (whereof the one is a Tangent, and the other a perpendicular to the Curve) let fall *CQ* Perpendicular to the Axis *PSR*.

Now, any Area, as *COPS*, being given, 'tis requir'd to find the Angle *CSP*, and the Diſtance *CS*. From the nature of the Parabola

RQ is ever $= \frac{1}{2}$ the Parameter or *Latus rectum* of the Axis, and conſequently if the Parameter be put $=2$, then $RQ = 1$. Let $CQ = z$; then PQ ſhall $= \frac{1}{2}zz$, and the Parabolic ſegment $COP = \frac{1}{12}zzz$, But the Triangle CSP will $= \frac{1}{4}z$, and ſo the Mixtilineal Area $COPS = \frac{1}{12}z^3 + \frac{1}{4}z = a$, whence $z^3 + 3z = 12a$. Wherefore reſolving this Cubical Equation, z or the Ordinate CQ will be known. Now, let the Area OPS be propos'd to be divided into one hundred parts; this Area is $\frac{1}{12}$ of the Square of the Parameter, and conſequently $12a$ is $=$ that Square $= 4$. If therefore the Roots of theſe Equations $z^3 + 3z = 0{,}04 : 0{,}08 : 0{,}12 : 0{,}16$, *&c.* be ſucceſſively extracted, there will be obtain'd ſo many z or Ordinates CQ reſpectively, and the Area SOP will be divided into one hundred equal parts. And in like manner is the Calculus to be continued beyond the Place *O*. Now the Root of this Equation (ſince RQ is $=1$) is the Tabular Tangent of the Angle CRQ, or $\frac{1}{2}$ the Angle CSP, wherefore the Angle CSP is given. And RC, the Secant of the ſame Angle CRQ, is a mean proportional between RQ or Unity and RT, which is the double of SC, as is plain from the Conics. But if SP be put $=1$, and ſo the *Latus rectum* $=4$ (as in our Table) then RT will be the Diſtance ſought, *viz.* the double of SC in the former Parabola. After this manner therefore, I compos'd the foregoing Table, which ſerves to repreſent the motions of all our Comets; of which hitherto there has been none obſerved, but come within the Laws of the Parabola.

It remains now, to give the Rules for the calculation, and to ſhew the way of determining the Place of a Comet ſeen, by theſe Numbers.

bers. *The Velocity of a Comet moving in a* Parabola, *is every where to the Velocity of a Planet deſcribing a Circle about the Sun, at the ſame Diſtance from the Sun, as* $\surd 2$ to 1. as appears from *Cor.* 7. *Prop.* 16. *Lib.* 1. of the *Princip. Phil. Nat. Math.* If therefore a Comet in its Perihelium were ſuppos'd *to* be as far diſtant from the Sun as the Earth *is*, then the Diurnal Area which the Comet wou'd deſcribe, wou'd be to the Diurnal Area of *the* Earth, as $\surd 2$ to 1. And conſequently, the Time of the Annual revolution, is to the Time in which ſuch a Comet wou'd deſcribe the Quadrant of its Orbit from the Perihelium, as 3.14159, *&c.* (that is the Area of the Circle) to $\surd \frac{8}{9}$. Therefore the Comet wou'd deſcribe that Quadrant in 109 Days, 14 Hours, 46 Minutes; and ſo the Parabolic Area (analogous to the Area *POS*) being divided into 100 parts, to each Day there wou'd be allotted 0.912280 of thoſe parts, the Log. of which, *viz.* 9.960128, is to be kept for continual uſe. But then the Times in which Comets, at a greater or leſs Diſtance, wou'd deſcribe ſimilar Quadrants, are as the Times of the revolutions in Circles, that is, in the Seſquiplicate ratio of the Diſtances: Whence the Diurnal Areas eſtimated in Centeſimal parts of the Quadrant (which parts we put for meaſures of the mean Motion, like degrees) are in each, in the Subſeſquialtera proportion of the Diſtance from the Sun in the Perihelion.

Theſe things being neceſſarily premis'd, let it be propos'd to compute the apparent Place of any one of the forementioned Comets for any given Time. Therefore,

1. *Let the Sun's Place be had, and the Log. of its Diſtance from the Earth,*

2. *Let the difference between the Time of the Perihelion and the Time given be gotten, in Days and Decimal parts of Days. To the Log. of this Number, let there be added the conſtant Log.* 9.960128, *and the Complement Arithmetical of three halves of the Log. of the Perihelion Diſtance of the Comet from the Sun: The Sum will be the Log. of the mean Motion, to be ſought in the firſt Column of the general Table.*

3. *With the Mean Motion let there be taken the correſpondent Angle from the Perihelion in the Table, and the Log. for the Diſtance from the Sun: Then in Comets that are Direct, add, and in Retrograde ones ſubtract; if the Time be after the Perihelion, the Angle thus found, to or from the Perihelion: or in Direct Comets, ſubſtract; and in Retrograde ones add; if the Time be before the Perihelion, the foreſaid Angle to or from the Place of the Perihelion; and ſo we ſhall have the Place of the Comet in its Orbit. And to the Log. for the Diſtance found, let there be added the Log. of the Diſtance in the Perihelion, and the ſum will be the Log. of the true Diſtance of the Comet from the Sun.*

4. *The Place of the Node, together with the Place of the Comet in its Orbit, being given, let the Diſtance of the Comet from the Node be found; then the Inclination of the plane being given, there will be given alſo (from the common Rules of Trigonometry) the Comet's Place reduced to the Ecliptic, the Inclination or Heliocentric Latitude, and the Log. of the Curtate Diſtance.*

5. *From theſe things given (by the very ſame Rules that we find the Planet's Places, from the Sun's Place and Diſtance given) we may obtain the Apparent or Geocentric Place of the Comet, together with the Apparent Latitude.* And this it may be worth while to illuſtrate by an Example or two.

EXAMPLE I.

EXAMPLE I.

Let it be requir'd to find the Place of the Comet of the Year 166$\frac{4}{5}$, March 1^{d}, 7^{h}, 00$'$, *P. M.* London. *That is,* 96^{d}, 19^{h}, 8$'$, *after the Perihelion, which happen'd* Novemb. 24°, 11^{h}, 52$'$.

Log. Diſt. Perihel.		0. 011044
Log. Seſquialt.		0. 016566
Comp. Arith.		9. 983434
		9. 960128
Log. Temp.		1. 985862
Log. Med. Mot.		1. 929424
Medius Motus		85.001
Perihel.	♌	10. 41. 25
Ang. Correſp.		83. 38. 05
Comet in Orb.	♉	17. 3. 20
Aſcend. Nod.	♊	21. 14. 00
Com. à Nodo		34. 10. 40
Red. ad Eclip.		32. 19. 05
Com. Helioc.	♉	18. 54. 55
Incl. Bor.		11. 46. 50
Log. pro diſt.		0. 255369
Log. Perihel.		0. 011044
Co-ſin. Incl.		9. 990754
Log. diſt. Curt.		0. 257167
Log. diſt. ☉		9. 997918
☉	♓	21. 44. 45
Com. Viſus	♈	29. 18. 30
Lat. Viſa.		8. 36. 15

EXAM.

EXAMPLE II.

Let it be requir'd to find the Place of the Comet of the Year 1683, July 23°, 13^{h}, 35′, *P. M.* London: Or, 13^{h}, 40′ *Equat. Time. That is*, 21^{d}, 10^{h}, 50′ *after the Perihelion.*

Log. Diſt. Perihel.	9. 748343
Log. Seſquialt.	9. 622514
Comp. Arith.	0. 377486
	9. 960128
Log. Temp.	1. 310723
Log. Med. Mot.	1. 648337
Medius Motus	44. 498
Perihel. ♊	25. 29. 30
Ang. Correſp.	56. 47. 20—
Comet. in Orb. ♈	28. 42. 10
Nod. Deſcend. ♓	23. 23. 00
Com. à Nodo	35. 19. 10
Red. ad Eclip.	4. 48. 30
Com. Helioc. ♓	28. 11. 30
Incl. Bor.	35. 2. 00
Log. pro diſt.	0. 111336
Log. Perihel.	9. 748343
Co-ſin. Incl.	9. 913187
Log. diſt. Curt.	9. 772866
Log. diſt. ☉.	0. 006104
☉ Locus ♌	10. 41. 25
Com. Viſus ♋	5. 11. 50
Lat. Bor.	28. 52. 00

At the inſtant of Time ſpecified in the firſt Example, 'twas obſerv'd (at *London*) that the Comet applied to the ſecond Star of *Aries*; ſo that it was found to be 9′ more Northerly, and 3′ to the Eaſt, according to Dr. *Hook*'s obſervation. But at that of the ſecond Example, I my ſelf (near *London*, with the ſame Inſtruments whereby I formerly obſerv'd the Southern Conſtellations) found the Place of the Comet to be ♋, 5°, 11′½, and 28°, 52′ North Latitude, which agreed exactly with the obſervation made at *Greenwich*, almoſt at the very ſame Moment.

As for the Comet of the Year 1680, which came almoſt to the very Sun itſelf (being in its Perihelion, not above one third of the Semidiameter of the Sun diſtant from the ſurface of it,) ſince the *Latus Rectum* of its Orb is ſo very ſmall, it could hardly be contained within the limits of the general Table, becauſe of the exceſſive Velocity of the Mean Motion. Wherefore in this Comet, the beſt way will be (after the Mean Motion is found) to get from thence (by the help of the foregoing Equation $z^3 + 3z = \frac{4}{100}$ of the Mean Motion) the Tangent of half the angle from the Perihelion, together with the Log. for the Diſtance from the Sun. Which being found, we are to proceed by the ſame Rules, as in the reſt.

After this manner therefore, the Aſtronomical reader may examine theſe Numbers, which I have calculated with all imaginable care, from the obſervations I could meet with. And I have not thought fit to make them publick before they have been by my ſelf duly examin'd, and made as accurate as 'twas poſſible, not without the labour of many Years. I have publiſh'd this Specimen of Cometical Aſtronomy, as a *Prodromus* of a future Work I have in deſign, leſt, happening to be prevented by Death, By comparing together the Elements of the Motions of theſe Comets, 'tis apparent, their Orbits are diſpos'd in no manner of order; nor can they, as the Planets are, be comprehended within a Zodiac, moving indifferently every way, as well retrograde as direct; from whence it is clear, they are not carry'd about or mov'd in a Vortical Syſtem. Moreover, the Diſtances in their Perihelia are ſometimes greater, ſometimes leſs; which makes me ſuſpect, there may be a far greater number of them, which may move in Regions more remote from the Sun, and being therefore very obſcure; and wanting Tails, may paſs by us unſeen.

Hitherto I have conſider'd the Orbits of Comets as exactly Parabolic; upon which ſuppoſition it wou'd follow, that Comets being impell'd towards the Sun by a Centripetal Force, would deſcend as from ſpaces infinitely diſtant, and by their ſo falling acquire ſuch a Velocity, as that they may again fly off into the remoteſt parts of the Univerſe, moving upwards with a perpetual tendency, ſo as never to return again to the Sun. But ſince they appear frequently enough, and ſince none of them can be found to move with an Hyperbolic Motion, or a Motion ſwifter than what a Comet might acquire by its Gravity to the Sun, 'tis highly probable they rather move in very Excentric Elliptic Orbits, and make their returns after long periods of Time: For ſo their number will be determinate, and, perhaps, not ſo very great. Beſides, the ſpace between the Sun and the Fix'd Stars is ſo immenſe, that there is room enough for a Comet to revolve, tho' the Period of its revolution be vaſtly long. Now, the *Latus Rectum* of an Ellipſis, is to the *Latus Rectum* of a Parabola, tical to a Parabolical one. And I am the more confirmed in my opinion of its being the ſame; for that in the Year 1456, in the Summer time, a Comet was ſeen paſſing Retrograde between the Earth and the Sun, much after the ſame manner: Which tho' nobody made obſervations upon it, yet from its Period and the manner of its Tranſit, I cannot think different from thoſe I have juſt now mention'd. And ſince looking over the Hiſtories of Comets I find, at an equal interval of Time, a Comet to have been ſeen about Eaſter in the Year 1305, which is another double Period of 151 Years before the former. Hence I think I may venture to foretel, that it will return again in the Year 1758. And, if it ſhould then ſo return, we ſhall have no reaſon to doubt but the reſt may return alſo: Therefore Aſtronomers have a large field wherein to exerciſe themſelves for many ages, before they will be able to know the number of theſe many and great Bodies revolving about the common Center of the Sun, and to reduce their Motions to certain Rules. I thought indeed that the Comet which appear'd in the Year 1532, might be the ſame with that obſerv'd by *Hevelius* in the Year 1661. But *Apian*'s obſervations, which are the only ones we have concerning the firſt of theſe Comets, are too rude and inaccurate for any thing of certainty to be drawn from them, in ſo nice a matter. But as far as probability from the equality of Periods, and ſimilar appearance of Comets, may be urged as an argument, the late wondrous Comet of $168\frac{0}{1}$. ſeems to have been the ſame, which was ſeen in the time of our King *Henry* I. *anno* 1106, which began to appear in the Weſt about the middle of February, and continued for many days after, with ſuch a Tail as was *ſub Septentrionibus*, as if the Comet had appear'd in the North, but that it was ſeen under the *Septem triones* or brighter Stars of *Urſa major*. And as to its riſing *Hora undecima diei*, it can no ways be underſtood, unleſs the word *diei* be left out, as it is by *Suetonius*; for it muſt have been very far from the Sun, either to riſe at five in the Afternoon, or at eleven at Night; in which caſes it muſt have appeared for a long time, and its Tail have been ſo little remarkable, that it could by no means be call'd, *clarum & omnibus Terris conſpicuum Sydus.* But ſuppoſing this Comet to have traced the ſame Path with that of the Year 1680, the aſcending part of the Orb will exactly repreſent, all that *Auguſtus* hath ſaid concerning it; and is yet an additional argument to that drawn from the equality of the Period. Thus 'tis not improbable but this Comet may have four times viſited us at intervals of about 575 years: Whence the Tranſverſe diameter of its Elliptic Orb will be found $\sqrt[3]{575\times575}$ times greater than the annual Orb; or 138 times greater than the mean diſtance of the Sun; which diſtance tho' immenſely great, bears no proportion to that of the Fix'd Stars.

I have lately found out a ready method to compute the motion of Comets in theſe Elliptic Orbs, of which perhaps ſhortly we may exhibit a Specimen, giving this Comet for an Example. In the mean time, thoſe that deſire to know how to conſtruct Geometrically the Orb of a Comet, by three accurate obſervations given, may find it at the end of the third Book of Sir *Iſaac Newton*'s Principles of Natural Philoſophy, entituled *De Syſtemate Mundi*, in the words of its renowned Inventor. Which have ſince been

Way of the Earth. At which time, had the Earth been there, the Comet would have had a Parallax equal to that of the Moon, as I take it. This is ſpoken to Aſtronomers: But what might be the Conſequences of ſo near an appulſe; or of a contact, or, laſtly, of a ſhock of the Celeſtial Bodies, (which is by no means impoſſible to come to paſs,) I leave to be diſcuſs'd by the Studious of Phyſical matters.

FINIS.

ERRATA.

PAge 34. line 14. read, *be no bigger.* p. 36. l. 4. dele *no.* p. 37. l. 26. r. *Phænomena.* p. 48. l. 9. r. *CD is to EF.* p. 61. l. 26. r. *from the.* p. 70. l. 10. r. *which is leaſt.* l. 11. r. *and greateſt.* p. 480. l. 28. inſert, *Fig.* 8. p. 496. l. antepenult. r. *at the Equator.* p. 504. l. penult. r. *Fig.* 13. p. 773. l. 14. inſert, *Fig.* 25. p. 768. l. antepenult. r. *Fig.* 24.

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